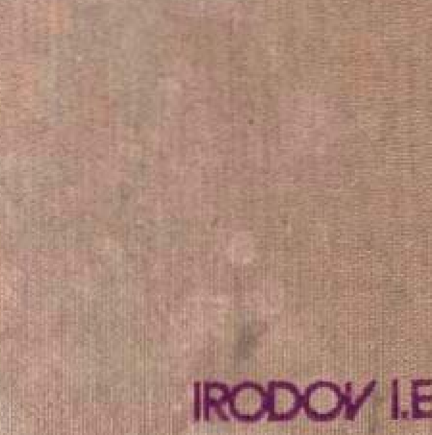
IRODOV I.E.



PROBLEMS IN ATOMIC AND NUCLEAR PHYSICS

MENDELEEV'S PERIODIC TABLE OF THE ELEMENTS

Periods	Series	Groups of Elements I	II	III	IV	V	VI	VII	VIII	VIII	VIII	VIII
1	1							1 H Hydrogen 1.00797				2 He Helium 4.00260
2	2	Li 3 Lithium 6.94	Be 4 Beryllium 9.01218	5 B Boron 10.81	6 C Carbon 12.01	7 N Nitrogen 14.0067	8 O Oxygen 15.9994	9 F Fluorine 18.9984				10 Ne Neon 20.179
3	3	Na 11 Sodium 22.98977	Mg 12 Magnesium 24.305	13 Al Aluminium 26.9815	14 Si Silicon 28.086	15 P Phosphorus 30.97376	16 S Sulphur 32.06	17 Cl Chlorine 35.453				18 Ar Argon 39.948
4	4	K 19 Potassium 39.098	Ca 20 Calcium 40.08	Sc 21 Scandium 44.9559	Ti 22 Titanium 47.90	V 23 Vanadium 50.9414	Cr 24 Chromium 51.996	Mn 25 Manganese 54.9380	Fe 26 Iron 55.847	Co 27 Cobalt 58.9332	Ni 28 Nickel 58.70	
	5	29 Cu Copper 63.546	30 Zn Zinc 65.38	31 Ga Gallium 69.72	32 Ge Germanium 72.59	33 As Arsenic 74.9216	34 Se Selenium 78.96	35 Br Bromine 79.904				36 Kr Krypton 83.80
5	6	Rb 37 Rubidium 85.4673	Sr 38 Strontium 87.62	Y 39 Yttrium 88.9059	Zr 40 Zirconium 91.22	Nb 41 Niobium 92.9064	Mo 42 Molybdenum 95.94	Tc 43 Technetium 98.9062	Ru 44 Ruthenium 101.07	Rh 45 Rhodium 102.9055	Pd 46 Palladium 106.4	
	7	47 Ag Silver 107.868	48 Cd Cadmium 112.40	49 In Indium 114.82	50 Sn Tin 118.69	51 Sb Antimony 121.75	52 Te Tellurium 127.66	53 I Iodine 126.9045				54 Xe Xenon 131.30
6	8	Cs 55 Cesium 132.9054	Ba 56 Barium 137.34	La* 57 Lanthanum 138.9055 (* 58–71)	Hf 72 Hafnium 178.49	Ta 73 Tantalum 180.9479	W 74 Tungsten 183.85	Re 75 Rhenium 186.207	Os 76 Osmium 190.2	Ir 77 Iridium 192.22	Pt 78 Platinum 195.09	
	9	79 Au Gold 196.9665	80 Hg Mercury 200.59	81 Tl Thallium 204.37	82 Pb Lead 207.2	83 Bi Bismuth 208.9804	84 Po Polonium [209]	85 At Astatine [210]				86 Rn Radon [222]
7	10	Fr 87 Francium [223]	Ra 88 Radium 226.0254	Ac 89 Actinium [227] (** 90–103)	Ku 104 Kurchatovium [261]	105						

* LANTHANIDES

Ce 58	Pr 59	Nd 60	Pm 61	Sm 62	Eu 63	Gd 64	Tb 65	Dy 66	Ho 67	Er 68	Tm 69	Yb 70	Lu 71
Cerium	Praseodymium	Neodymium	Promethium	Samarium	Europium	Gadolinium	Terbium	Dysprosium	Holmium	Erbium	Thulium	Ytterbium	Lutetium
140.12	140.9077	144.24	[145]	150.4	151.96	157.25	158.9254	162.50	164.9304	167.26	168.9342	173.04	174.97

** ACTINIDES

Th 90	Pa 91	U 92	Np 93	Pu 94	Am 95	Cm 96	Bk 97	Cf 98	Es 99	Fm 100	Md 101	(No) 102	Lr 103
Thorium	Protactinium	Uranium	Neptunium	Plutonium	Americium	Curium	Berkelium	Californium	Einsteinium	Fermium	Mendelevium	(Nobelium)	Lawrencium
232.0381	231.0359	238.02	237.0482	[244]	[243]	[247]	[247]	[251]	[254]	[257]	[258]	[255]	[256]

ИЗДАТЕЛЬСТВО МИР

СБОРНИК ЗАДАЧ ПО АТОМНОЙ И ЯДЕРНОЙ ФИЗИКЕ

Атомиздат Москва

IRODOV I.E.

PROBLEMS IN ATOMIC AND NUCLEAR PHYSICS

Translated from the Russian
by Yuri Atanov

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На английском языке

PREFACE TO THE ENGLISH EDITION

This book is intended primarily for students taking general courses in atomic and nuclear physics. It contains, however, a sufficiently large number of problems lying beyond the general course to make it also useful in the study of some special courses.

The volume contains over 1000 problems. The solutions of the most complicated of them are provided with detailed explanation. A brief summary of the basic terms and definitions at the beginning of each chapter also makes the solving easier. The fundamental physical constants and reference tables and graphs are given in the Appendix. Both the Periodic Table of the elements and the table of elementary particles are also provided.

The Gaussian system of units is employed throughout the book. All initial data and numerical answers are given with due regard for the accuracy of appropriate values and the rules of operation with approximate numbers.

In conclusion, the author takes pleasure in expressing his deep appreciation to his colleagues from the Moscow Engineering Physics Institute and all those who submitted their comments on certain problems and thereby contributed to the improvement of the book.

I. Irodov

NOTATION

Vectors are designated in boldfaced Roman type, e.g., **v**, **H**; the same letters printed in lightfaced Italic type (v, H) denote the modulus of a vector.

Mean quantities are denoted by French quotes $\langle\ \rangle$, e.g., $\langle\lambda\rangle$, $\langle x\rangle$.

The terms L and C frame designate the laboratory frame of reference and the frame of the centre of inertia respectively. All quantities in the C frame are marked with the ~ (tilde) sign over a letter, e.g., $\tilde{\mathbf{p}}$, $\tilde{E}$.

Energy: T kinetic, U potential, and E total.

$B\rho$ is the product of the magnetic field and the radius of curvature of a particle's trajectory.

Wave numbers: spectroscopic $\bar{\nu} = 1/\lambda$,
adopted in theory $k = 2\pi/\lambda$,
where λ is the wavelength.

All operators (with the exception of coordinates and functions of coordinates only) are marked with the sign "^" over a letter, e.g., $\hat{A}$, $\hat{p}_x$.

The designations of antihyperons indicate the sign of the electric charge of antihyperons themselves, not of the corresponding hyperons.

CONTENTS

1

QUANTUM NATURE OF ELECTROMAGNETIC RADIATION

● The radiant exitance M is related to the volume density u of thermal radiation as

$$M = \frac{c}{4} u. \tag{1.1}$$

● Wien's formula and Wien's displacement law

$$u_\omega = \omega^3 f(\omega/T); \qquad \lambda_{pr} T = b, \tag{1.2}$$

where ω is the radiation frequency, s^{-1}; T is the absolute temperature; λ_{pr} is the most probable wavelength in the radiation spectrum; b is a constant.

● Stefan-Boltzmann law (for blackbody radiation):

$$M = \sigma T^4. \tag{1.3}$$

● Planck's formula for the spectral concentration of radiant exitance:

$$u_\omega = \frac{\hbar\omega^3}{\pi^2 c^3} \cdot \frac{1}{e^{\hbar\omega/kT} - 1}. \tag{1.4}$$

● Relation between the total energy E and the momentum p of a relativistic particle with rest mass m:

$$E^2 = p^2c^2 + m^2c^4. \tag{1.5}$$

● The Compton change in wavelength of a scattered photon:

$$\Delta\lambda = 4\pi\Lambda \sin^2(\vartheta/2); \qquad \Lambda = \hbar/mc, \tag{1.6}$$

where Λ is the Compton wavelength of a particle with rest mass m.

THERMAL RADIATION

1.1. Demonstrate that Wien's formula permits one to calculate the energy distribution curve $u_1(\omega)$ for thermal radiation at the temperature T_1, if the distribution $u_2(\omega)$ at the temperature T_2 is known.

1.2. Using Wien's formula demonstrate that: **(a)** the most probable radiation frequency $\omega_{pr} \propto T$; **(b)** the radiant exitance $M \propto T^4$ (the Stefan-Boltzmann law).

1.3. Using Wien's formula demonstrate that in the thermal radiation energy distribution over wavelengths: **(a)** the most probable wavelength $\lambda_{pr} \propto 1/T$ (Wien's displacement law); **(b)** the maximum spectral density of radiation energy $(u_\lambda)_{max} \propto T^5$.

1.4. The initial temperature of thermal radiation is equal to 2000 K. By how many degrees does this temperature change when the most probable wavelength of its spectrum increases by 0.25 μm?

1.5. Find the most probable wavelength in the spectrum of thermal radiation with the radiant exitance 5.7 W/cm^2.

1.6. The solar radiation spectrum resembles that of a black body with $\lambda_{pr} = 0.48$ μm. Find the thermal radiation power of the Sun. Evaluate the time interval during which the mass of the Sun diminishes by 1% (due to thermal radiation). The mass of the Sun is equal to $2.0 \cdot 10^{30}$ kg and its radius is $7.0 \cdot 10^8$ m.

1.7. There are two cavities *1* and *2* with small holes of equal radii $r = 0.50$ cm and perfectly reflecting outer surfaces. The cavities are oriented so that the holes face each other and the distance between them is $R = 10.0$ cm. A constant temperature $T_1 = 1700$ K is maintained in cavity *1*. Calculate the steady-state temperature inside cavity *2*.

1.8. Considering that the thermal radiation pressure $p = u/3$, where u is the volume density of radiation energy, find: **(a)** the pressure of thermal radiation from the Sun's interior, where the temperature is about $1.6 \cdot 10^6$ K; **(b)** the temperature of totally ionized hydrogen plasma of density 0.10 g/cm^3, at which the thermal radiation pressure is equal to the gas pressure of plasma's particles. (At high temperatures substances obey the equation of state for the ideal gas.)

1.9. A copper sphere of radius $r = 1.00$ cm with perfectly black surface is placed in an evacuated vessel whose walls are cooled down to the temperature close to 0 K. The initial temperature of the sphere is $T_0 = 300$ K. How soon will its temperature decrease $n = 1.50$ times? The heat capacity of copper is $c = 0.38$ J/g·K.

1.10. Wien proposed the following formula to describe the energy distribution in the thermal radiation spectrum: $u_\omega = A\omega^3 e^{-a\omega/T}$, where $a = 7.64 \cdot 10^{-12}$ s·K/rad. Using this formula, find for $T = 2000$ K: **(a)** the most probable radiation frequency; **(b)** the mean radiation frequency.

1.11. Using the formula of the foregoing problem, find in the thermal radiation energy distribution over wavelengths at the temperature $T = 2000$ K: **(a)** the most probable wavelength; **(b)** the mean wavelength.

1.12. A piece of copper located in a closed cavity is in equilibrium with its radiation. The system's temperature is $T = 300$ K. Resorting to Dulong and Petit's law, find the ratio of the volume density of vibration energy of copper to that of radiation energy.

1.13. The thermal radiation filling up a certain cavity can be treated as a set of oscillators, that is, natural oscillations with different frequencies. Find the mean energy of the oscillator with frequency ω and the volume density of energy within the interval $(\omega, \omega + d\omega)$, assuming the energy ε of each oscillator to take on: **(a)** any value (continuous spectrum); **(b)** only discrete values $n\hbar\omega$, where n is an integer.

The energies of the oscillators are supposed to be distributed according to Boltzmann's formula $N(\varepsilon) \propto e^{-\varepsilon/kT}$.

1.14. Derive the approximate expressions of Planck's formula for the extreme cases $\hbar\omega \ll kT$ and $\hbar\omega \gg kT$.

1.15. Transform Planck's formula to obtain the distribution over: **(a)** linear frequencies; **(b)** wavelengths.

1.16. In what wavelength interval does Wien's formula taken in the form $u_\omega = (\hbar\omega^2/\pi^2c^3)\, e^{-\hbar\omega/kT}$ describe the energy distribution to an accuracy better than 1.0% at the temperature $T = 2000$ K?

1.17. Using Planck's formula, calculate: **(a)** by what factor the spectral density of radiation with wavelength $\lambda = 0.60$ μm increases when the temperature T grows from 2000 to 2300 K; **(b)** the radiation power emitted from a unit area of the black body surface in the interval of wavelengths whose values differ less than 0.50% from the most probable value at $T = 2000$ K.

1.18. Using Planck's formula, find the numerical values of: **(a)** the Stefan-Boltzmann constant; **(b)** the constant b in Wien's displacement law.

1.19. From Planck's formula determine: **(a)** the mean frequency value ω in the thermal radiation spectrum at $T = 2000$ K; **(b)** the temperature of thermal radiation whose mean wavelength is equal to 2.67 μm.

CORPUSCULAR THEORY

1.20. Making use of Planck's formula, obtain: **(a)** the expression giving the number of photons per 1 cm^3 within spectral intervals $(\omega, \omega + d\omega)$ and $(\lambda, \lambda + d\lambda)$; **(b)** the total number of photons per 1 cm^3 at the temperature $T = 300$ K.

1.21. Using Planck's formula, calculate: **(a)** the most probable energy of photons; **(b)** the mean energy of photons at $T = 1000$ K.

1.22. Demonstrate that the number of thermal radiation photons falling on a unit area of cavity's wall per unit time is equal to $nc/4$, where c is the velocity of light and n is the number of photons in a unit volume. See that the product of this value and the mean energy of the photon is equal to the radiant exitance.

1.23. Find the photon flux density at the distance 1.0 m from a point light source 1.0 W in power, if light: **(a)** is monochromatic with a wavelength of 0.50 μm; **(b)** contains two spectral lines with wavelengths of 0.70 and 0.40 μm whose intensities relate as 1:2.

1.24. The wavelengths of photons are equal to 0.50 μm, 2.5×10^{-8} cm, and 0.020 Å. Calculate their momenta in units of eV/c, where c is the velocity of light.

1.25. On the basis of the corpuscular theory demonstrate that the momentum transferred by the plane luminous flux Φ is independent of its spectral composition.

1.26. A laser emits a light pulse of duration $\tau = 0.13$ ms and energy $E = 10$ J in the shape of a narrow beam. Find the pressure, averaged over the pulse duration, that such a beam would develop when it is focused into a stop of diameter $d = 10$ μm on a surface with reflectance $\rho = 0.50$. The beam falls at right angles to the surface.

1.27. A short light pulse with energy $E = 7.5$ J falls in the form of a narrow beam on a mirror plate whose reflectance is $\rho = 0.60$. The angle of incidence is $\theta = 30°$. Find the momentum transferred to the plate.

1.28. From the concepts of the corpuscular theory find the force of light pressure that a plane luminous flux of intensity J W/cm² exerts, when it illuminates: **(a)** a flat mirror surface at the incidence angle θ; **(b)** a mirror hemisphere; **(c)** a flat perfectly matted surface at right angles.

In all cases the area of illuminated surface is equal to S and reflectance to unity.

1.29. A point light source of power $N = 60$ W is located above the centre of a round perfectly mirror plate whose radius is $r = 10$ cm. The distance between the plate and the source is equal to $l = 10$ cm. Employing the concepts of the corpuscular theory, find the force that light exerts on the plate. Consider the cases $r \ll l$ and $r \gg l$.

1.30. On the basis of the corpuscular theory demonstrate that the thermal radiation pressure $p = u/3$, where u is the volume density of radiation energy.

1.31. An atom moving with velocity v ($v \ll c$) emits a photon at the angle ϑ to its motion direction. Using the conservation laws, find the relative magnitude of the Doppler shift in the frequency of the photon.

1.32. A photon is emitted from the surface of a star whose mass is M and radius R. Assuming the photon to possess a mass with its intrinsic gravitational properties, find the relative decrease in the photon's energy at a great distance from the star. Calculate the gravitational wavelength shift ($\Delta\lambda/\lambda$) of the radiation emitted from the surface of: **(a)** the Sun ($M = 2.0 \cdot 10^{30}$ kg, $R = 7.0 \cdot 10^8$ m); **(b)** a neutron star whose mass equals that of the Sun and whose mean density is $1.0 \cdot 10^{14}$ times that of the Sun.

1.33. Explain the existence of the short-wave limit in the X-ray continuous spectrum. Calculate the magnitude of the constant C in the relation $\lambda_{min} = C/V$, if λ is expressed in Å and V in kV.

1.34. Find the wavelength of the short-wave limit of the X-ray continuous spectrum, if it is known that it shifts by 0.50 Å when the voltage applied to an X-ray tube increases 2.0 times.

1.35. A narrow X-ray beam falls on a NaCl single crystal. The least grazing angle at which the mirror reflection from the natural face of the crystal is still observed is equal to 4.1°. The corresponding interplanar distance is 2.81 Å. How high is the voltage applied to the X-ray tube?

1.36. Calculate the velocity of electrons flying up to the target cathode of an X-ray tube, if the wavelength of the short-wave limit of the X-ray continuous spectrum is $\lambda_{min} = 0.157$ Å.

1.37. With a thin metal foil used as a target cathode of an X-ray tube the spectral distribution of bremsstrahlung has the form $J_\lambda = 10^{-5}PZ/\lambda^2$, W/Å, where P is the tube current power, W; Z is the atomic number of the element used as the target; λ is the radiation wavelength, Å.

(a) Draw the approximate graphs of the functions $J_\lambda(\lambda)$ and $J_\omega(\omega)$.

(b) Calculate the tube efficiency, if the applied voltage is $V = 80$ kV and the target cathode is made of golden foil.

1.38. Find the most probable wavelength of bremsstrahlung with spectral distribution of the form $J_\omega \propto (\omega_{max} - \omega)$, where ω_{max} is the limit frequency of the spectrum. The voltage applied to the tube is equal to 31 kV.

1.39. Using the tables of the Appendix, calculate: **(a)** the photoelectric threshold wavelengths for Cs and Pt; **(b)** the highest velocities of electrons liberated from the surface of zinc, silver, and nickel by light of the wavelength 0.270 μm.

1.40. Up to what maximum potential will a copper ball, remote from all other bodies, be charged when illuminated by light of the wavelength 0.20 μm?

1.41. At a certain maximum value of retarding potential difference the photoelectric current from lithium surface illuminated by light of wavelength λ_0 cuts off. The increase in the wavelength of light by a factor of $n = 1.5$ results in the increase of the cut-off potential difference by a factor of $\eta = 2.0$. Calculate λ_0.

1.42. Find the maximum kinetic energy of photoelectrons liberated from the surface of lithium by electromagnetic radiation whose electric component varies with time as $E = a(1 + \cos\omega t)\cos\omega_0 t$, where a is a constant, $\omega = 6.0 \cdot 10^{14}$ s⁻¹, $\omega_0 = 3.60 \cdot 10^{15}$ s⁻¹.

1.43. There is a vacuum photocell one of whose electrodes is made of cesium and the other of copper. The electrodes are shorted outside the cell. The cesium electrode is illuminated by monochromatic light. Find: **(a)** the wavelength of light at which the current appears in the cell's circuit; **(b)** the highest velocity of photoelectrons approaching the copper electrode, if the wavelength of light is equal to 0.220 μm.

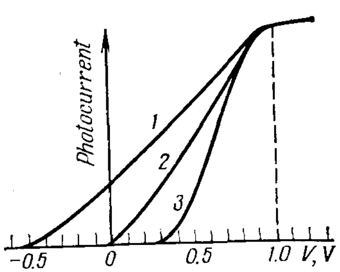

Fig. 1

1.44. A photoelectric current emerging in the circuit of a photocell when its zinc electrode is illuminated by light of wavelength 2620 Å is cancelled, if the external retarding potential difference 1.5 V is applied. Find the magnitude and polarity of the contact potential difference of the given photocell.

1.45. A nickel sphere serving as an inner electrode of a spherical vacuum photocell is illuminated by monochromatic light of various wavelengths. Figure 1 illustrates how the photoelectric current depends on the applied voltage V. Using these graphs, find the corresponding wavelengths.

1.46. A photon with $\lambda = 0.170$ Å knocks out of a stationary atom an electron whose binding energy is $E = 69.3$ keV. Find the momentum transferred to the atom in this process, if the electron is ejected at right angles to the direction of the incident photon.

1.47. Making use of the conservation laws, demonstrate that a free electron cannot absorb a photon.

1.48. A photon of energy $\hbar\omega$ is scattered at the angle ϑ by a stationary free electron. Find: **(a)** the increment of wavelength of the scattered photon; **(b)** the angle φ at which the recoil electron moves.

1.49. A photon with energy 0.46 MeV is scattered at the angle $\vartheta = 120°$ by a stationary free electron. Find: **(a)** the energy of the scattered photon; **(b)** the energy transferred to the electron.

1.50. A photon with momentum 60 keV/c, having experienced the Compton scattering at the angle 120° by a stationary free electron, knocks out of a Mo atom an electron whose binding energy is equal to 20.0 keV. Find the kinetic energy of the photoelectron.

1.51. On irradiation of a substance by hard monochromatic X-rays the highest kinetic energy of Compton electrons was found to be $T_{max} = 0.44$ MeV. Determine the wavelength of the incident radiation.

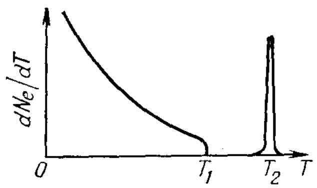

Fig. 2

1.52. Figure 2 shows the energy spectrum of electrons ejected from a sample made of a light element, when it is exposed to hard monochromatic X-ray radiation (T is the kinetic energy of the electrons). Explain the character of the spectrum. Find the wavelength of the incident radiation and T_1 and T_2, if $T_2 - T_1 = 180$ keV.

1.53. A photon with energy 374 keV is scattered by a stationary free electron. Find the angle between the directions in which the recoil electron and scattered photon move. The Compton shift in wavelength of the scattered photon is equal to 0.0120 Å.

1.54. A photon is scattered by a stationary free electron. Find the momentum of the incident photon if the energy of the scattered photon is equal to the kinetic energy of the recoil electron with the divergence angle of 90°.

1.55. At what angle is a gamma quantum with energy 0.80 MeV scattered after collision with a stationary free electron, if the velocity of the recoil electron is equal to $0.60c$?

1.56. A photon with momentum 1.02 MeV/c is scattered by a stationary free electron. Calculate the Compton shift in wavelength of the scattered photon, if the recoil electron: **(a)** moves at the angle 30° to the direction of the incident photon; **(b)** obtains the momentum 0.51 MeV/c.

1.57. Find the energy of an incident photon that is scattered at the angle $\vartheta = 60°$ by a stationary free electron and transfers to the latter a kinetic energy $T = 0.450$ MeV.

1.58. A photon with energy $\hbar\omega = 1.00$ MeV is scattered by a stationary free electron. Find the kinetic energy of the recoil electron, if in the process of scattering the photon's wavelength changes by $\eta = 25\%$.

1.59. After collision with a relativistic electron a photon was scattered at an angle of 60° while the electron stopped. Find: **(a)** the Compton shift in wavelength of the scattered photon; **(b)** the kinetic energy that the electron possessed prior to collision, if the energy of the striking photon is equal to the rest energy of an electron.

1.60. Explain the following features of the Compton effect emerging on irradiation of a substance by monochromatic X-rays: **(a)** the Compton shift equation can be verified when sufficiently hard radiation is used; **(b)** the magnitude of the shift is independent of the nature of the substance; **(c)** the presence of the non-shifted component in the scattered radiation; **(d)** the increase in intensity of the shifted component of the scattered light as the atomic number of the substance decreases and the scattering angle increases; **(e)** the broadening of both components of the scattered light.

2

RUTHERFORD-BOHR ATOM

● The angle ϑ at which a charged particle is scattered by the Coulomb field of a stationary atomic nucleus is defined by the formula

$$\tan\frac{\vartheta}{2}=\frac{q_1q_2}{2bT}, \qquad (2.1)$$

where q_1 and q_2 are the charges of the interacting particles, T is the kinetic energy of the incoming particle, b is the aiming parameter.

In the general case this expression is valid in the C frame as well, provided that the substitution $\vartheta \to \tilde{\vartheta}$ and $T \to \tilde{T}$ is made, where $\tilde{\vartheta}$ and $\tilde{T}$ are the scattering angle and the total kinetic energy of interacting particles in the C frame:

$$\tilde{T}=\frac{\mu v_{rel}^2}{2}=\frac{\tilde{p}^2}{2\mu}; \qquad \mu=\frac{m_1m_2}{m_1+m_2}. \qquad (2.2)$$

Here μ is the reduced mass, v_{rel} is the relative velocity of the particles, and $\tilde{p}$ is their momentum in the C frame.

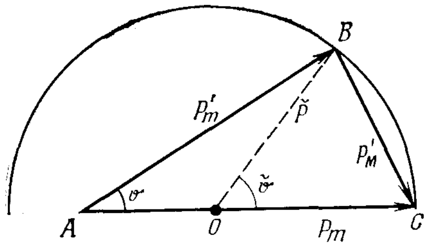

Fig. 3

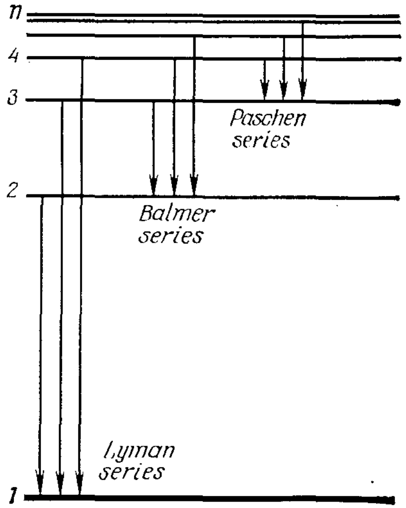

Fig. 4

● Vector diagram of momenta for elastic scattering on non-relativistic particle of mass m by an initially stationary particle of mass M is shown in Fig. 3. In this figure p_m and p'_m denote the momenta of the incoming particle before and after scattering, p'_M is the momentum of the recoil particle, O is the centre of a circle whose radius equals the momentum $\tilde{p}$ of particles in the C frame; the point O divides the line segment AC into two parts in the ratio $AO:OC = m:M$, and $\tilde{\vartheta}$ is the scattering angle of the incoming particle in the C frame.

● Rutherford formula. The relative number of particles scattered into an elementary solid angle $d\Omega$ at an angle ϑ to their initial propagation direction equals

$$\frac{dN}{N}=n\left(\frac{q_1q_2}{4T}\right)^2\frac{d\Omega}{\sin^4(\vartheta/2)}, \qquad (2.3)$$

where n is the number of nuclei per unit area of the foil surface, T is the kinetic energy of the incoming particles, $d\Omega = \sin\vartheta\, d\vartheta\, d\varphi$.

● Generalized Balmer formula (Fig. 4)

$$\omega=R^*Z^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right); \qquad R^*=\frac{\mu e^4}{2\hbar^3}, \qquad (2.4)$$

where ω is the transition frequency (in s^{-1}) between energy levels with quantum numbers n_1 and n_2, Z is the atomic number of atom (ion), R^* is the Rydberg constant, and μ is the reduced mass.

● Resonance line is a line resulting from the transition of atoms from the first excited state to the ground one.

SCATTERING OF PARTICLES. RUTHERFORD FORMULA

2.1. Employing the Thomson model, calculate: **(a)** the radius of a hydrogen atom whose ionization energy is equal to 13.6 eV; **(b)** the oscillation frequency of the electron if the radius of a hydrogen atom is r. At what value of r is the wavelength of emitted light equal to 0.6 μm?

2.2. To what minimum distance will an alpha-particle with kinetic energy $T = 40$ keV approach, in the case of the head-on collision: **(a)** a stationary Pb nucleus; **(b)** a stationary Li^7 nucleus?

2.3. Using the laws of conservation of energy and angular momentum, derive formula (2.1).

2.4. An alpha-particle with momentum 53 MeV/c (c is the velocity of light) is scattered at the angle 60° by the Coulomb field of a stationary uranium nucleus. Find the aiming parameter.

2.5. An alpha-particle with kinetic energy T strikes a stationary Pb nucleus with the aiming parameter $0.90\cdot10^{-11}$ cm. Find: **(a)** the modulus of the momentum vector increment of the scattered alpha-particle if $T = 2.3$ MeV; **(b)** at what value of T the modulus of the momentum vector increment of the scattered alpha-particle will be the greatest for the given aiming parameter. What is the magnitude of the scattering angle in this case?

2.6. To what minimum distance will a proton with kinetic energy $T = 0.87$ MeV approach a stationary Hg nucleus, if the scattering angle is equal to $\vartheta = \pi/2$? Compare this distance with the corresponding value of aiming parameter.

2.7. A non-relativistic particle of mass m and kinetic energy T is elastically scattered by initially stationary nucleus of mass M. Find the momentum of each particle and their combined kinetic energy in the C frame.

2.8. Substantiate the construction of the vector diagram of momenta shown in Fig. 3. Draw the similar diagrams for the cases $m = M$ and $m > M$.

2.9. A non-relativistic particle of mass m_1 and kinetic energy T undergoes a head-on collision with initially stationary particle of mass m_2. Find the kinetic energy of the incoming particle after the collision.

2.10. Find the maximum value of the angle at which an alpha-particle is scattered by an initially stationary deuteron.

2.11. A non-relativistic deuteron is elastically scattered by an initially stationary H^4 nucleus through the angle ϑ. Find this angle if in the C frame the corresponding angle is equal to $\tilde{\vartheta} = 45°$.

2.12. A deuteron with kinetic energy 15.0 keV and aiming parameter $0.60 \cdot 10^{-10}$ cm is scattered by the Coulomb field of a stationary He^4 nucleus. Find the deuteron's scattering angle in the L frame.

2.13. A proton with the aiming parameter $b = 2.5 \cdot 10^{-11}$ cm is elastically scattered at right angles by the Coulomb field of a stationary deuteron. Find the relative velocity of the particles after scattering.

2.14. As a result of elastic scattering of a proton with kinetic energy $T = 13.0$ keV by the Coulomb field of a stationary He^4 nucleus the latter recoils at an angle $\vartheta' = 60°$ to the motion direction of the incoming proton. Calculate the aiming parameter.

2.15. An alpha-particle with kinetic energy $T = 5.0$ keV is elastically scattered by the Coulomb field of a stationary deuteron. Find the aiming parameter corresponding to the greatest possible scattering angle of the alpha-particle in the L frame.

2.16. After scattering of an alpha-particle with kinetic energy $T = 29$ keV by the Coulomb field of a stationary Li^6 nucleus the latter recoils at an angle $\vartheta = 45°$ to the motion direction of the incoming particle. To what minimum distance do both particles approach in the process of interaction?

2.17. A stationary sphere of radius R is irradiated with parallel flux of particles of radius r. Assuming the collision of a particle with the sphere to be perfectly elastic, find:

(a) the deflection angle ϑ of a particle as a function of its aiming parameter b;

(b) the fraction of particles which after collision with the sphere are scattered in the angular interval from ϑ to $\vartheta + d\vartheta$, and also the probability of a particle being scattered into the front hemisphere ($\vartheta < \pi/2$).

2.18. Using formula (2.1) derive the expressions for the relative number of alpha-particles scattered in the angular interval $(\vartheta, \vartheta + d\vartheta)$ and for the corresponding cross-section of a nucleus.

2.19. A narrow beam of protons with kinetic energy 100 keV falls normally on a golden foil of thickness 1.0 mg/cm^2. The protons scattered through the angle 60° are registered by a counter with round inlet of the area 1.0 cm^2 located at the distance 10 cm from the scattering section of the foil and oriented normally to the motion direction of incident protons. What fraction of the scattered protons reaches the counter inlet?

2.20. Calculate the cross-section of Au nucleus causing protons with kinetic energy $T = 1.20$ MeV to scatter through the angular interval from $\vartheta = \pi/3$ to π.

2.21. Alpha-particles with kinetic energy $T = 1.70$ MeV are scattered by the Coulomb field of Pb nuclei. Calculate the differential cross-sections of these nuclei, $d\sigma/d\vartheta$ and $d\sigma/d\Omega$, corresponding to scattering through an angle $\vartheta = \pi/2$.

2.22. The differential cross-section of scattering of alpha-particles by the Coulomb field of a stationary nucleus is equal to $d\sigma/d\Omega = 7.0 \cdot 10^{-22}$ cm^2/sr for an angle $\vartheta_0 = 30°$. Calculate the cross-section of scattering of alpha-particles for the angles $\vartheta > \vartheta_0$.

2.23. Find the probability for an alpha-particle with energy $T = 3.0$ MeV to be scattered after passing through a lead foil 1.5 μm in thickness into the angular interval **(a)** 59-61°; **(b)** 60-90°.

2.24. A narrow beam of alpha-particles with kinetic energy 1.00 MeV and intensity $3.6 \cdot 10^4$ particles per second falls normally on a golden foil of thickness 1.0 μm. Find the number of alpha-particles scattered by the foil during 10 min into the angular interval **(a)** 59-61°; **(b)** $\vartheta > \vartheta_0 = 60°$; **(c)** $\vartheta < \vartheta_0 = 10°$.

2.25. A narrow beam of protons with kinetic energy $T = 1.0$ MeV falls normally on a brass foil whose mass thickness is $\rho \cdot d = 1.5$ mg/cm^2. Find the fraction of the protons scattered through the angles exceeding $\vartheta_0 = 30°$ if the weight ratio of copper to zinc in the foil is 7:3.

2.26. A narrow beam of alpha-particles of equal energy falls normally on a lead foil with mass thickness 2.2 mg/cm^2. The fraction of the original flux scattered through angles exceeding $\vartheta = 20°$ is equal to $\eta = 1.6 \cdot 10^{-3}$. Find the differential cross-section $d\sigma/d\Omega$ of a Pb nucleus corresponding to a scattering angle $\vartheta_0 = 60°$.

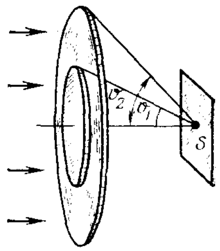

Fig. 5

2.27. A plane flux of alpha-particles with kinetic energy T falls normally on a thin gold-en foil cut in the shape of a flat ring (Fig. 5). The flux density of alpha-particles is equal to N_0 particles per cm^2 per second. The foil contains n nuclei per 1 cm^2 area. Find N', the number of alpha-particles reaching the screen near the point S per 1 second per 1 cm^2 area. The angles ϑ_1 and ϑ_2 are known, and scattering through these angles obeys the Rutherford formula.

HYDROGEN-LIKE ATOMS

2.28. Estimate the time interval during which an electron moving in a hydrogen atom along an orbit of radius $0.5 \cdot 10^{-8}$ cm would have fallen to the nucleus, if it had been losing energy through radiation in accordance with classical theory: $dE/dt = -(2e^2/3c^3)\,\mathbf{w}^2$, where $\mathbf{w}$ is the acceleration of the electron. The vector $\mathbf{w}$ is assumed to be permanently directed toward the centre of the atom.

2.29. A particle of mass m moves along a circular orbit in the central-symmetry potential field $U = \varkappa r^2/2$. Using the Bohr quantization condition, find the permissible orbital radii and energy levels of the particle.

2.30. Calculate the ratio of the Coulomb and gravitational forces acting between an electron and a nucleus in a hydrogen atom.

2.31. Assuming the nucleus to be stationary, calculate for a hydrogen atom and He^+ and Li^{++} ions: **(a)** the radii of the first and second Bohr orbits and the velocities of an electron moving along them; **(b)** the kinetic energy and binding energy of the electron in the ground state; **(c)** the first excitation potential and wavelength of resonance line.

2.32. Demonstrate that the photon frequency ω corresponding to the electronic transition between neighbouring orbits of hydrogen-like ions satisfies the inequality $\omega_n > \omega > \omega_{n+1}$, where ω_n and ω_{n+1} are the circular frequencies of the electron moving along these orbits. Make sure that $\omega \to \omega_n$, if $n \to \infty$.

2.33. In the spectrum of some hydrogen-like ions the three lines are known, which belong to the same series and have the wavelengths 992, 1085, and 1215 Å. What other spectral lines can be predicted?

2.34. Calculate and draw on the wavelength scale the spectral intervals in which the Lyman, Balmer, and Paschen series for atomic hydrogen are confined. Indicate the visible portion of the spectrum.

2.35. **(a)** Calculate the wavelengths of the first three spectral lines of the Balmer series for atomic hydrogen. **(b)** What is the minimum number of slits needed for a diffraction grating to resolve the first 30 lines of the Balmer series of atomic hydrogen in the second order?

2.36. The emission spectrum of atomic hydrogen has two lines of the Balmer series with wavelengths 4102 and 4861 Å. To what series does a spectral line belong, if its wave number is equal to the difference of the wave numbers of the two lines? What is the magnitude of its wavelength?

2.37. Atomic hydrogen is excited to the nth energy level. Determine: **(a)** the wavelengths of emission lines if $n = 4$; to what series do these lines belong? **(b)** how many lines does hydrogen emit when $n = 10$?

2.38. What lines of the atomic hydrogen absorption spectrum fall within the wavelength range from 945 to 1300 Å?

2.39. Find the quantum number n corresponding to the excited state of a hydrogen atom, if on transition to the ground state the atom emits: **(a)** a photon with $\lambda = 972.5$ Å; **(b)** two photons with $\lambda_1 = 6563$ Å and $\lambda_2 = 1216$ Å.

2.40. What hydrogen-like ion has the difference of wavelengths of the main lines of Balmer and Lyman series equal to 593 Å?

2.41. Find the binding energy of an electron in the ground state of hydrogen-like ions in whose spectrum the third line of the Balmer series is equal to 1085 Å.

2.42. The binding energy of an electron in a He atom is equal to $E_0 = 24.6$ eV. Find the energy required to remove both electrons from the atom.

2.43. Find the velocity of electrons liberated by light with wavelength $\lambda = 180$ Å from He^+ ions in the ground state.

2.44. A photon emitted by He^+ ion passing from the first excited state down to the ground one ionizes a hydrogen atom in the ground state. Find the velocity of the photoelectron.

2.45. At what minimum kinetic energy of a moving hydrogen atom will its inelastic head-on collision with another, stationary, hydrogen atom produce a photon emitted by one of the atoms? Both atoms are supposed to be in the ground state prior to the collision.

2.46. Determine the velocity which a stationary hydrogen atom obtains due to photon emission resulting from transition of the atom from the first excited state down to the ground one. How much (in per cent) does the energy of the emitted photon differ from the transition energy?

2.47. When observed at the angle 45° to the motion direction, a beam of excited hydrogen atoms seems to radiate the resonance line whose wavelength is shifted by 2.0 Å. Find the velocity of the hydrogen atoms.

2.48. A He^+ ion approaching a hydrogen atom emits a photon corresponding to the main line of the Balmer series. What must be the minimum approach velocity to enable the photon to excite the hydrogen atom from the ground state? *Instruction*: make use of the precise formula for the Doppler effect.

2.49. Taking into account the motion of the nucleus in a hydrogen atom, find the expressions for the electron's binding energy in the ground state and for the Rydberg constant as a function of nuclear mass. How much (in per cent) do the binding energy and Rydberg constant, obtained when neglecting the motion of the nucleus, differ from the more accurate corresponding values of these quantities?

2.50. Calculate the proton to electron mass ratio if the ratio of Rydberg constants for heavy and light hydrogen is equal to $\eta = 1.000272$ and the ratio of their nuclear masses is $n = 2.00$.

2.51. For atoms of light and heavy hydrogen find the difference: **(a)** of the binding energies of their electrons in the ground state; **(b)** of the first excitation potentials; **(c)** of the wavelengths of the resonance lines.

2.52. For a mesonic hydrogen atom (in which an electron is replaced by a meson whose charge is the same and mass is 207 that of electron) calculate: **(a)** the distance between a meson and a nucleus in the ground state; **(b)** the wavelength of the resonance line; **(c)** the ground state binding energies of the mesonic hydrogen atoms whose nuclei are a proton and a deuteron.

2.53. For a positronium consisting of an electron and a positron revolving around their common centre of masses find: **(a)** the distance between the particles in the ground state; **(b)** the ionization potential and first excitation potential; **(c)** the Rydberg constant and wavelength of the resonance line.

2.54. According to the Bohr-Sommerfeld postulate the following quantization rule has to be satisfied in the case of a particle moving in a potential field:

$$\oint p_q \, dq = 2\pi\hbar \cdot n,$$

where q and p_q are the generalized coordinate and projection of generalized momentum (x, p_x and φ, L_z), n is an integer. Using this rule, find the allowed energy values E for a particle of mass m moving:

(a) in a unidimensional rectangular potential well of width l with infinitely high walls;

(b) in a unidimensional potential field $U = \varkappa x^2/2$, where $\varkappa$ is a positive constant;

(c) along the circle of permanent radius r;

(d) along a round orbit in a central field, where the potential energy of the particle is equal to $U = -\alpha/r$ (α is a positive constant).

3

WAVE PROPERTIES OF PARTICLES

● de Broglie relations for energy and momentum of a particle:

$$E = \hbar\omega; \quad p = \hbar k, \tag{3.1}$$

where ω is the frequency of the de Broglie wave, and $k = 2\pi/\lambda$.

● Uncertainty principle

$$\Delta x \cdot \Delta p_x \geqslant \hbar. \tag{3.2}$$

● Schrödinger equation in the time-dependent and time-independent form

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + U\Psi;$$
$$\nabla^2 \psi + \frac{2m}{\hbar^2}(E - U)\psi = 0, \tag{3.3}$$

where Ψ is the total wave function, ψ is its coordinate part, ∇^2 is the Laplace operator, E and U are the total and potential energies of the particle.

● Energy eigenvalues and eigenfunctions of a particle of mass m in the unidimensional potential field $U(x) = \varkappa x^2/2$ (a harmonic oscillator with frequency $\omega = \sqrt{\varkappa/m}$):

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right); \qquad \psi_n(\xi) = a_n (-1)^n e^{\xi^2/2} \frac{d^n}{d\xi^n}(e^{-\xi^2}), \tag{3.4}$$

where $n = 0, 1, 2, \ldots$; $\xi = \alpha x$; $\alpha = \sqrt{m\omega/\hbar}$; a_n is the normalizing factor.

● Coefficient of transparency D of the potential barrier $U(x)$:

$$D \approx \exp\left[-\frac{2}{\hbar}\int_{x_1}^{x_2} \sqrt{2m(U - E)}\, dx\right], \tag{3.5}$$

where x_1 and x_2 are the coordinates of the points between which $U > E$.

DE BROGLIE WAVES. UNCERTAINTY PRINCIPLE

3.1. Calculate the de Broglie wavelengths of an electron and proton moving with kinetic energy 1.00 keV. At what values of kinetic energy will their wavelengths be equal to 1.00 Å?

3.2. The 200 eV increase in electron's energy changes its de Broglie wavelength by a factor of two. Find the initial wavelength of the electron.

3.3. Calculate the wavelength of hydrogen molecules moving with the most probable velocity in gas at the temperature 0°C.

3.4. Determine the kinetic energy of a proton whose wavelength is the same as that of an alpha-particle with $B\rho = 25$ kG·cm.

3.5. What amount of energy should be added to an electron with momentum 15.0 keV/c to make its wavelength equal to 0.50 Å?

3.6. A proton with wavelength $\lambda = 0.017$ Å is elastically scattered through the angle 90° by an initially stationary particle whose mass is $n = 4.0$ times larger than the proton's. Find the wavelength of the scattered proton.

3.7. A neutron with kinetic energy $T = 0.25$ eV collides elastically with a stationary He^4 nucleus. Find the wavelengths of both particles in the C frame before and after the collision.

3.8. Two atoms, H^1 and He^4, move in the same direction, with the same de Broglie wavelength $\lambda = 0.60$ Å. Find the wavelengths of both atoms in the C frame.

3.9. A relativistic particle with rest mass m possesses the kinetic energy T. Find: **(a)** the de Broglie wavelength of the particle; **(b)** the values of T at which the error in wavelength obtained from the non-relativistic formula does not exceed 1.0% for an electron and a proton.

3.10. At what value of kinetic energy is the de Broglie wavelength of an electron equal to its Compton wavelength?

3.11. Find the wavelength of relativistic electrons reaching the anticathode of an X-ray tube, if the short wavelength limit of the continuous X-ray spectrum is equal to 0.100 Å.

3.12. Using Maxwell's distribution of velocities find the distribution of molecules of gas over de Broglie wavelengths and the most probable wavelength of hydrogen molecules at $T = 300$ K.

3.13. The velocity distribution function of atoms in a beam has the form $f(u) \sim u^3 e^{-u^2}$, where u is the ratio of the atom's velocity in the beam to the most probable velocity v_{pr} in the source ($v_{pr} = \sqrt{2kT/m}$). Find the distribution function in terms of de Broglie wavelengths. Calculate the most probable wavelength in the beam of He atoms provided the source temperature is 300 K.

3.14. Determine the kinetic energy of electrons falling on a diaphragm with two narrow slits, if on a screen located at a distance $l = 75$ cm from the diaphragm the separations between neighbouring maxima Δx and between the slits d are equal to 7.5 and 25 μm respectively.

3.15. A narrow stream of monochromatic electrons falls at a grazing angle $\vartheta = 30°$ on the natural facet of an aluminium single crystal. The distance between neighbouring crystal planes parallel to that facet is equal to $d = 2.0$ Å. The maximum mirror reflection is observed at a certain accelerating voltage V_0. Determine V_0 if the next maximum mirror reflection is observed when the accelerating voltage is increased $\eta = 2.25$ times.

3.16. A stream of electrons with kinetic energy $T = 180$ eV falls normally on the surface of a Ni single crystal. The reflecting maximum of fourth order is observed in the direction forming an angle $\alpha = 55°$ with the normal of the surface. Calculate the interplanar distance corresponding to that reflection.

3.17. A stream of electrons with kinetic energy $T = 10$ keV passes through a thin polycrystalline foil forming a system of diffraction fringes on a screen located at a distance $l = 10.0$ cm from the foil. Find the interplanar distance that is responsible for the reflection of third order forming a diffraction ring of radius $r = 1.6$ cm.

3.18. A stream of electrons accelerated through the potential difference V falls on a surface of nickel whose inner potential is $V_i = 15$ V. Calculate: **(a)** the refractive index of nickel when $V = 150$ V; **(b)** the values of the ratio V/V_i at which the refractive index differs from unity by not more than 1.0%.

3.19. With allowance made for refraction of electron waves, Bragg's formula takes the form

$$2d\sqrt{n^2 - \cos\vartheta^2} = k\lambda,$$

where d is the interplanar distance, n is the refractive index, ϑ is the grazing angle, k is the reflection order, λ is the wavelength of the electrons.

(a) Derive this formula, assuming the reflecting planes to be parallel to the surface of the single crystal.

(b) Find the inner potential of an Ag single crystal, if a stream of electrons accelerated through a potential difference $V = 85$ V forms a maximum of second order due to the mirror reflection from crystal planes with $d = 2.04$ Å. The grazing angle is $\vartheta = 30°$.

3.20. A particle of mass m moves in a unidimensional square potential well with infinitely high walls. The width of the well is equal to l. Find the allowed energy values of the particle taking into account that only those states are realized for which the whole number of de Broglie half-wavelengths are fitted within the well.

3.21. Describe the Bohr quantum conditions in terms of the wave theory: demonstrate that stationary Bohr orbits are those which accommodate a whole number of de Broglie waves. Find the wavelength of an electron in the nth orbit.

3.22. Assuming that the wave function $\Psi(x, t)$ describing a moving particle represents a superposition of de Broglie waves of equal amplitudes and slightly differing wave numbers $k_0 \pm \Delta k$: **(a)** transform $\Psi(x, t)$ to the form $\Psi(x, t) = A(x, t)\, e^{i(\omega_0 t - k_0 x)}$; find the explicit expression for the function $A(x, t)$; **(b)** derive the expression describing the displacement velocity of the given group of waves, i.e. for the maximum of the function $A(x, t)$.

3.23. Demonstrate that the group velocity of a wave packet is equal to the velocity of a freely moving particle. Consider both non-relativistic and relativistic cases.

3.24. Demonstrate that a narrow slit of width b used in measurements of x coordinates of particles introduces the uncertainty Δp_x in their momenta, such that $\Delta x \cdot p_x \geqslant \hbar$.

3.25. Make sure that the measurement of the x coordinate of a particle by means of a microscope (Fig. 6) introduces the uncertainty Δp_x in its momentum, such that $\Delta x \cdot \Delta p_x \geqslant \hbar$. Remember that the microscope resolution $d = \lambda/\sin \vartheta$, where λ is the wavelength of light used in the measurements.

3.26. A plane stream of particles falls normally on a diaphragm with two narrow slits and forms a diffraction pattern on a screen (Fig. 7). Demonstrate that an attempt to determine through which

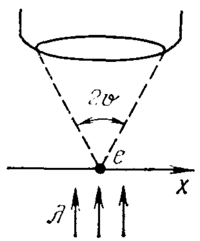

Fig. 6

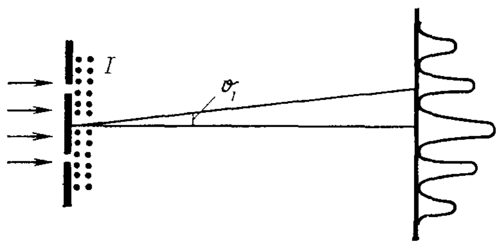

Fig. 7

slit a specified particle passed (for example, by means of an indicator I) results in blurring of the pattern. For simplicity the diffraction angles are assumed to be small.

3.27. Estimate the minimum error of determining the velocity of an electron, proton, and uranium atom, if their coordinates are known with uncertainty 1 μm.

3.28. Evaluate the indeterminancy of the velocity of an electron in a hydrogen atom assuming the size of the atom to be about 10^{-8} cm. Compare the obtained value with the velocity of an electron in the first Bohr orbit.

3.29. Estimate for an electron localized in the region of size l: **(a)** the lowest possible kinetic energy, if $l = 1.0 \cdot 10^{-8}$ cm; **(b)** the relative velocity uncertainty $\Delta v/v$, if its kinetic energy $T \approx 10$ eV and $l = 1.0$ μm.

3.30. A particle is located in a unidimensional potential well of width l with infinitely high walls. Evaluate the pressure exerted by the particle on the walls of the well at the lowest possible value E_{min} of its energy.

3.31. A particle with mass m moves in a unidimensional potential field $U(x) = \varkappa x^2/2$ (a harmonic oscillator with frequency $\omega = \sqrt{\varkappa/m}$). Evaluate the lowest possible energy of this particle.

3.32. On the basis of the uncertainty principle evaluate the electron's binding energy in the ground state of a hydrogen atom and the corresponding distance between the electron and the nucleus.

3.33. Evaluate the lowest possible energy of electrons in a He atom and the corresponding distance between the electrons and the nucleus.

3.34. The kinetic energy of a free moving non-relativistic particle is known with relative uncertainty about $1.6 \cdot 10^{-4}$. Evaluate how much the coordinate uncertainty of such a particle exceeds its de Broglie wavelength.

3.35. A free electron was initially confined within a region with linear dimensions $l = 10^{-8}$ cm. Evaluate the time interval during which the width of the corresponding train of waves increases by a factor of $\eta = 10^2$.

3.36. A parallel stream of hydrogen atoms with velocity $v = 1.2 \cdot 10^3$ m/s falls normally on a diaphragm with a narrow slit behind which a screen is placed at a distance $l = 1.0$ m. Using the uncertainty principle, evaluate the width of the slit at which the width of its image on the screen is minimal.

3.37. Find the density of probability distribution for a particle and effective dimensions of its localization region, if the state of the particle is described by the wave function $\psi(x)$ representing a superposition of de Broglie waves whose amplitudes depend on wave numbers k as follows:

(a) $a_k = \begin{cases} a = \text{const} & \text{(in the interval } k_0 \pm \Delta k,\ \Delta k \ll k_0);\\ 0 & \text{(outside this interval);} \end{cases}$

(b) $a_k = e^{-\alpha^2 (k-k_0)^2}$,

where k_0 and α are constants.

3.38. Find the spectrum of wave numbers k of de Broglie waves whose superposition forms the wave function:

(a) $\psi(x) = \begin{cases} e^{ik_0 x} & \text{(in the interval } l > x > -l);\\ 0 & \text{(outside this interval);} \end{cases}$

(b) $\psi(x) = e^{ik_0 x - \alpha^2 x^2}$,

where k_0 and α are constants.

Estimate the wave number interval in which the amplitude of individual de Broglie waves appreciably differs from zero.

SCHRÖDINGER EQUATION. PENETRATION OF A PARTICLE THROUGH A BARRIER

3.39. What solutions of the Schrödinger time-dependent equation are called stationary? Demonstrate that such solutions are obtained when U depends on time implicitly.

3.40. How will the total wave function $\Psi(x, t)$ describing stationary states change, if the origin of potential energy scale is shifted by the certain value ΔU?

3.41. Solve the Schrödinger time-dependent equation for the case of a free particle moving in the positive direction of the x axis with momentum p.

3.42. Demonstrate that the energy of a free moving particle can be of any magnitude.

3.43. A particle of mass m is located in a unidimensional square potential well with absolutely impenetrable walls $(0 < x < l)$. Find: **(a)** the energy eigenvalues and normalized eigenfunctions of the particle; **(b)** the probability of the particle with the lowest energy staying within the region $l/3 < x < 2l/3$; **(c)** the number of energy levels in the interval $(E, E + dE)$.

3.44. A particle of mass m is located in a two-dimensional square potential well with absolutely impenetrable walls $(0 < x < a, 0 < y < b)$. Find: **(a)** the energy eigenvalues and normalized eigenfunctions of the particle; **(b)** the probability of the particle with the lowest energy staying within the region $0 < x < a/3$, $0 < y < b/3$; **(c)** the energy values of the first four levels, if $a = b = l$; **(d)** the number of states that the particle possesses in the energy interval $(E, E + dE)$.

3.45. A particle of mass m is located in a three-dimensional square potential well with absolutely impenetrable walls $(0 < x < a, 0 < y < b, 0 < z < c)$. Find: **(a)** the energy eigenvalues and normalized eigenfunctions of the particle; **(b)** the energy difference between the third and fourth levels, if $a = b = c = l$; **(c)** the number of states corresponding to the sixth level (the degree of degeneracy), if $a = b = c$; **(d)** the number of states in the energy interval $(E, E + dE)$.

3.46. Demonstrate that at the point, where the potential energy $U(x)$ of a particle has a finite discontinuity, the wave function remains smooth, i.e. its first derivative with respect to coordinate is continuous.

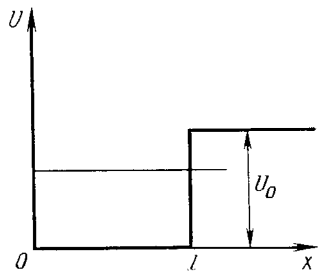

Fig. 8

3.47. A particle of mass m is located in the unidimensional potential field $U(x)$ whose shape is shown in Fig. 8. Find: **(a)** the energy eigenvalues of the particle in the region $E > U_0$; **(b)** the equation describing the energy eigenvalues of the particle in the region $E < U_0$; transform it into the form

$$\sin kl = \pm \sqrt{\hbar^2/2ml^2U_0}\,kl, \quad k = \sqrt{2mE}/\hbar;$$

demonstrate by graphical means that the energy eigenvalues of the particle form a discontinuous spectrum; **(c)** the values of l^2U_0 at which the first and nth discrete levels appear. What is the total number of levels in the well for which $l^2U_0 = 75\hbar^2/m$? **(d)** The value of l^2U_0 at which the energy of the only level is equal to $E = U_0/2$. What are in this case the most probable coordinate of the particle and probability of the particle being outside the classical boundaries of the field? **(e)** The discrete energy levels of the particle, if $l^2U_0 = (25/18)\pi^2\hbar^2/m$.

3.48. A particle of mass m is located in a symmetrical potential field (Fig. 9). Find: **(a)** The equation defining the energy eigenvalue spectrum of the particle in the region $E < U_0$; reduce that equation to the form

$$kl = n\pi - 2 \arcsin \frac{\hbar k}{\sqrt{2mU_0}}; \quad k = \frac{\sqrt{2mE}}{\hbar},$$

where n is an integer. Solving this equation by graphical means, demonstrate that the energy eigenvalues are discrete. **(b)** The value

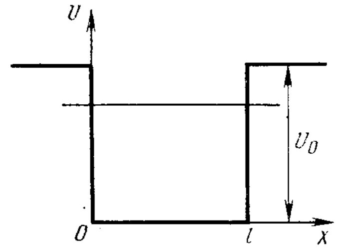

Fig. 9

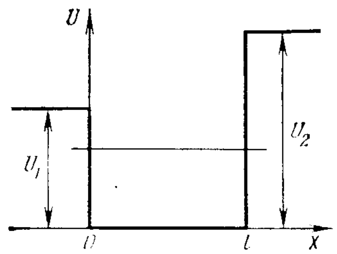

Fig. 10

of l^2U_0 at which the energy of the particle in the ground state is equal to $E = U_0/2$. What is the probability of the particle being outside the well? **(c)** The value of l^2U_0 at which the second level appears. What is the energy of the ground state? **(d)** The value of l^2U_0 at which the nth level appears. How many discrete levels does the given well contain, if $l^2U_0 = 75\hbar^2/m$?

3.49. A particle of mass m is located in a potential field shown in Fig. 10, where $U_1 < U_2$. Find: **(a)** the equation defining the energy eigenvalue spectrum of the particle in the region $E < U_1$; reduce that equation to the form

$$kl = n\pi - \arcsin \frac{\hbar k}{\sqrt{2mU_1}} - \arcsin \frac{\hbar k}{\sqrt{2mU_2}},$$

where $n = 1, 2, \ldots$, $k = \sqrt{2mE}/\hbar$;

(b) the value of U_1 at which the first discrete level appears, if $U_2 = 2U_1$.

3.50. Using the Schrödinger equation, find the energy of a harmonic oscillator of frequency ω in the stationary state **(a)** $\psi(x) = Ae^{-\alpha^2x^2}$; **(b)** $\psi(x) = Bxe^{-\alpha^2x^2}$, where A, B and α are constants.

3.51. The Schrödinger equation for a harmonic oscillator of frequency ω can be reduced to the form $\psi_\xi'' + (\lambda - \xi^2)\psi = 0$, where $\xi = \alpha x$, α is a constant, λ is a parameter whose eigenvalues are equal to $2n + 1$ $(n = 0, 1, 2, \ldots)$. Find the oscillator's energy eigenvalues.

3.52. Making use of the formula given in the introduction to Chapter 3, find for the first three levels of an oscillator of mass m

and frequency ω: **(a)** the eigenfunctions and their normalization coefficients; **(b)** the most probable values of the oscillation coordinate x. Draw the approximate graphs of the probability density function for x values in these states.

3.53. A particle in the ground state is located in the unidimensional potential field $U(x) \propto x^2$. What is the probability of the particle being outside the classical limits of the field?

3.54. Provided the eigenfunctions and energy eigenvalues of a harmonic oscillator are known, find the energy eigenvalues of a particle of mass m moving in the unidimensional potential field $U(x) = \varkappa x^2/2$ at $x > 0$ and $U = \infty$ at $x \leqslant 0$.

3.55. A particle of mass m moves in a three-dimensional potential field $U(x, y, z) = \frac{\varkappa}{2}(x^2 + y^2 + z^2)$, where $\varkappa$ is the quasi-elastic force constant. Determine: **(a)** the particle's energy eigenvalues; **(b)** the degree of degeneracy of the nth energy level.

Instruction. Use the formulas for a unidimensional oscillator.

3.56. A particle of mass m and energy E approaches a square potential barrier (Fig. 11) from the left-hand side. Find: **(a)** the

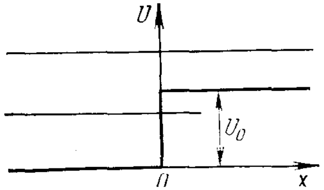

Fig. 11

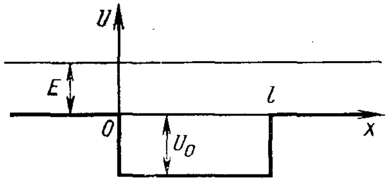

Fig. 12

coefficients of reflection R and transparency D of the barrier for the case $E > U_0$. Make sure that the values of these coefficients do not vary with the direction of incident particles; **(b)** the reflection coefficient R, if $E < U_0$. For this case determine the effective penetration depth x_{eff}, i.e. the distance from the barrier boundary to the point at which the probability of finding a particle decreases e-fold. Calculate x_{eff} for an electron, if $U_0 - E = 1.0$ eV.

3.57. A particle of mass m and energy E approaches a square potential well (Fig. 12). Find: **(a)** the coefficients of transparency D and reflection R; **(b)** the values of E at which the particle would freely pass that well. Demonstrate that it happens when $l = n\lambda/2$, where λ is the particle's wavelength in the well, $n = 0, 1, 2, \ldots$.

3.58. A particle of mass m and energy E tunnels through a square potential barrier (Fig. 13). Find: **(a)** the coefficients of transparency D and reflection R for the case $E > U_0$. See that the expressions obtained coincide with the corresponding formulas of the previous problem provided the sign of U_0 is reversed. Find D for $E \to U_0$; **(b)** the first three values of E at which an electron would freely tunnel through such a barrier, if $U_0 = 10.0$ eV and $l = 5.0 \cdot 10^{-8}$ cm; **(c)** the transparency coefficient D for the case $E < U_0$. Simplify the obtained expression for $D \ll 1$; **(d)** the probability of an electron and proton with $E = 5.0$ eV tunnelling through that barrier, if $U_0 = 10.0$ eV and $l = 1.0 \cdot 10^{-8}$ cm.

3.59. Find the transparency coefficient of a potential barrier shown in Fig. 14 for a particle of mass m and energy E. Consider

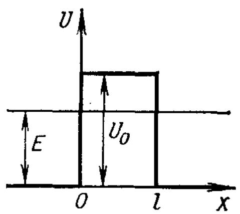

Fig. 13

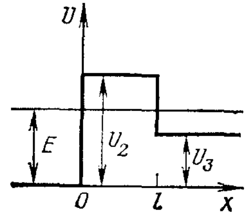

Fig. 14

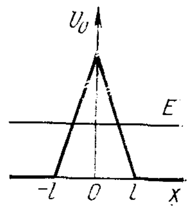

Fig. 15

the two cases: **(a)** $E > U_2$; **(b)** $U_2 > E > U_3$. See that the obtained expressions coincide with the solutions of Problem 3.58, **(a)** and **(c)**, when $U_3 = 0$.

3.60. Using formula (3.5), find the probability of a particle of mass m and energy E tunnelling through the potential barrier shown in **(a)** Fig. 15; **(b)** Fig. 16, where $U(x) = U_0(1 - x^2/l^2)$.

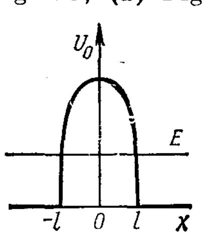

Fig. 16

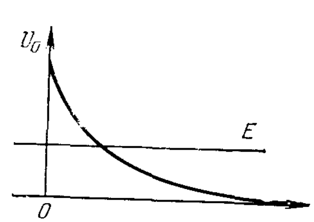

Fig. 17

3.61. A particle of mass m and energy E tunnels through a barrier of the form

$$U(x) = \begin{cases} 0 & \text{at } x < 0; \\ U_0/(1 + x/a) & \text{at } x \geqslant 0, \end{cases}$$

where U_0 is the barrier height (Fig. 17), a is a positive constant. Using formula (3.5), demonstrate that the transparency coefficient of that barrier is equal to

$$D \approx e^{-\varkappa(\pi - 2\varphi_0 - \sin 2\varphi_0)},$$

if $E < U_0$, with $\varkappa = (aU_0/\hbar)\sqrt{2m/E}$, $\varphi_0 = \arcsin\sqrt{E/U_0}$. Simplify the obtained formula for the case $E \ll U_0$.

Instruction. When integrating, introduce the new variable φ according to the formula $\sin^2\varphi = E/U(x)$.

4

FUNDAMENTALS OF QUANTUM MECHANICS*

● Operator $\hat{A}$ is linear if

$$\hat{A}\,(c_1\psi_1+c_2\psi_2)=c_1\hat{A}\psi_1+c_2\hat{A}\psi_2, \tag{4.1}$$

where c_1 and c_2 are constants; ψ_1 and ψ_2 are arbitrary functions.

● Operators $\hat{A}$ and $\hat{B}$ are commutative if their commutator

$$[\hat{A},\,\hat{B}]=\hat{A}\hat{B}-\hat{B}\hat{A}=0. \tag{4.2}$$

● Operator $\hat{A}$ is hermitian (self-conjugate) if

$$\int \psi_1^*\hat{A}\psi_2\,dx=\int \psi_2\hat{A}^*\psi_1^*\,dx, \tag{4.3}$$

where ψ_1 and ψ_2 are arbitrary functions.

● Expansion of the function ψ into eigenfunctions ψ_n which compose the discrete spectrum of a certain operator:

$$\psi(x)=\sum c_n\psi_n(x);\qquad c_n=\int \psi\psi_n^*\,dx. \tag{4.4}$$

● Mean value of a mechanical quantity A in a state ψ:

$$\langle A\rangle \int \psi^* A\psi\,d\tau, \tag{4.5}$$

where $\hat{A}$ is the corresponding operator; ψ is the normalized wave function; $d\tau$ is a unit volume.

● Schrödinger equation in operator form:

$$i\hbar\,\frac{\partial\Psi}{\partial t}=\hat{H}\Psi, \tag{4.6}$$

where $\hat{H}$ is the total energy operator (the Hamiltonian).

● Time derivative of operator $\hat{A}$:

$$\frac{d\hat{A}}{dt}=\frac{\partial\hat{A}}{\partial t}+\frac{i}{\hbar}\,[\hat{H},\,\hat{A}], \tag{4.7}$$

where $[\hat{H},\,\hat{A}]$ is the commutator of operators; $\hat{H}$ is the Hamiltonian.

* In this chapter all operators (except for the coordinates and functions that depend on coordinates only) are marked with the sign "^" over a letter.

● Basic quantum-mechanical operators:

projection and square of momentum $\hat{p}=-i\hbar\,\dfrac{\partial}{\partial x},\ \hat{p}^2=\hat{p}_x^2+\hat{p}_y^2+\hat{p}_z^2;$

total energy (the Hamiltonian) $\hat{H}=\dfrac{\hat{p}^2}{2m}+U=-\dfrac{\hbar^2}{2m}\nabla^2+U;$

projections of angular momentum $\hat{L}_x=y\hat{p}_z-z\hat{p}_y,\ \hat{L}_y=z\hat{p}_x-x\hat{p}_z,$
$\hat{L}_z=x\hat{p}_y-y\hat{p}_x=-i\hbar\,\dfrac{\partial}{\partial\varphi};$

square of angular momentum $\hat{L}^2=\hat{L}_x^2+\hat{L}_y^2+\hat{L}_z^2=-\hbar^2\nabla^2_{\vartheta,\,\varphi},$

where ∇^2 is the Laplace operator taking the following form in spherical coordinates:

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{2}{r}\,\frac{\partial}{\partial r}+\frac{1}{r^2}\nabla^2_{\vartheta,\,\varphi};$$

$$\nabla_{\vartheta,\,\varphi}=\frac{1}{\sin\vartheta}\,\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\,\frac{\partial}{\partial\vartheta}\right)+\frac{1}{\sin^2\vartheta}\,\frac{\partial^2}{\partial\varphi^2},$$

with $\nabla^2_{\vartheta,\,\varphi}$ designating the angle-dependent part of the Laplace operator.

● Eigenvalues and eigenfunctions of the operator $\hat{L}^2$:

$$L^2=l\,(l+1)\,\hbar^2,\qquad l=0,\ 1,\ 2,\ \ldots, \tag{4.8}$$

$$Y_{lm}(\vartheta,\ \varphi)=\Theta_{l|m|}(\vartheta)\,e^{im\varphi},\qquad m=0,\ \pm1,\ \pm2,\ \ldots,\ \pm l. \tag{4.9}$$

The functions $\Theta(\vartheta)$ for s-, p-, and d-states are presented in Table 4.1 (accurate except for normalization factor).

Table 4.1

State	$l,\ m$	$\Theta_{l\,\|m\|}(\vartheta)$
s	0, 0	1
p	1, 0	$\cos\vartheta$
	1, 1	$\sin\vartheta$
d	2, 0	$3\cos^2\vartheta-1$
	2, 1	$\sin\vartheta\cos\vartheta$
	2, 2	$\sin^2\vartheta$

Table 4.2

State	$n,\ l$	$R(\rho),\ \rho=r/r_1$
$1s$	1, 0	$e^{-\rho}$
$2s$	2, 0	$(2-\rho)\,e^{-\rho/2}$
$2p$	2, 1	$\rho e^{-\rho/2}$
$3s$	3, 0	$(21-81\rho+2\rho^2)\,e^{-\rho/3}$
$3p$	3, 1	$\rho\,(6-\rho)\,e^{-\rho/3}$
$3d$	3, 2	$\rho^2e^{-\rho/3}$

● Schrödinger equation for the radial part of wave function $R(r)$ in the central-symmetry field $U(r)$:

$$\frac{\partial^2R}{\partial r^2}+\frac{2}{r}\,\frac{\partial R}{\partial r}+\frac{2m}{\hbar^2}\left(E-U-\frac{L^2}{2mr^2}\right)R=0. \tag{4.10}$$

The functions $R(r)$ for hydrogen-like atoms are given in Table 4.2 (accurate except for normalization factor); the symbol r_1 in the table denotes the first Bohr radius.

4.1. Check the following operator equalities:

(a) $\frac{d}{dx}x = 1 + x\frac{d}{dx}$;

(b) $x^2\frac{d}{dx}\frac{1}{x} = x\frac{d}{dx} - 1$;

(c) $\left(1+\frac{d}{dx}\right)^2 = 1 + 2\frac{d}{dx} + \frac{d^2}{dx^2}$;

(d) $\left(x+\frac{d}{dx}\right)^2 = 1 + x^2 + 2x\frac{d}{dx} + \frac{d^2}{dx^2}$;

(e) $\left(\frac{1}{x}\frac{d}{dx}x\right)^2 = \frac{d^2}{dx^2} + \frac{2}{x}\frac{d}{dx}$;

(f) $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}\right)^2 = \frac{\partial^2}{\partial x^2} + 2\frac{\partial^2}{\partial x\,\partial y} + \frac{\partial^2}{\partial y^2}$.

4.2. Find the result of operation carried out by the operators $\frac{d^2}{dx^2}x^2$ and $\left(\frac{d}{dx}x\right)^2$ on the functions: **(a)** $\cos x$ and **(b)** e^x.

4.3. Find the eigenvalue of the operator $\hat{A}$ corresponding to the eigenfunction ψ_A, if:

(a) $\hat{A} = -\frac{d^2}{dx^2}$, $\psi_A = \sin 2x$;

(b) $\hat{A} = -\frac{d^2}{dx^2} + x^2$, $\psi_A = e^{-x^2/2}$;

(c) $\hat{A} = \frac{d^2}{dx^2} + \frac{2}{x}\frac{d}{dx}$, $\psi_A = \frac{\sin \alpha x}{x}$.

4.4. Find the eigenfunctions ψ and eigenvalues of the following operators:

(a) $-i\frac{d}{dx}$, if $\psi(x) = \psi(x+a)$; a is a constant;

(b) $-\frac{d^2}{dx^2}$, if $\psi = 0$ at $x = 0$ and $x = l$.

4.5. Demonstrate that if the operators $\hat{A}$ and $\hat{B}$ are linear, the operators $\hat{A} + \hat{B}$ and $\hat{A}\hat{B}$ are also linear.

4.6. Prove the following commutative relations:

(a) $[\hat{A}, \sum \hat{B}_i] = \sum [\hat{A}, \hat{B}_i]$; **(b)** $[\hat{A}, \hat{B}\hat{C}] = [\hat{A}, \hat{B}]\,\hat{C} + \hat{B}\,[\hat{A}, \hat{C}]$.

4.7. Prove that if the operators $\hat{A}$ and $\hat{B}$ commute, then

(a) $(\hat{A}+\hat{B})^2 = \hat{A}^2 + 2\hat{A}\hat{B} + \hat{B}^2$; $(\hat{A}+\hat{B})(\hat{A}-\hat{B}) = \hat{A}^2 - \hat{B}^2$;

(b) $[(\hat{A}+\hat{B}), (\hat{A}-\hat{B})] = 0$.

48. Suppose $\hat{A}^2 = \sum \hat{A}_i^2$. Prove that if the operators $\hat{A}_i$ commute with the operator $\hat{B}$, the operators $\hat{A}^2$ and $\hat{B}$ also commute.

4.9. Prove that if the commutator $[\hat{A}, \hat{B}] = 1$, then

(a) $[\hat{A}, \hat{B}^2] = 2\hat{B}$; **(b)** $[\hat{A}, \hat{B}^3] = 3\hat{B}^2$; **(c)** $[\hat{A}^2, \hat{B}^2] = 2(\hat{A}\hat{B} + \hat{B}\hat{A})$.

4.10. Check the following equalities involving commutators:

(a) $[x, \hat{p}_x] = i\hbar$, $[x, \hat{p}_y] = 0$, $[\hat{p}_x, \hat{p}_y] = 0$;

(b) $[f(x), \hat{p}_x] = i\hbar\frac{\partial f}{\partial x}$, $[f(x), \hat{p}_x^2] = 2i\hbar\frac{\partial f}{\partial x}\hat{p}_x + \hbar^2\frac{\partial^2 f}{\partial x^2}$;

(c) $[x^2, [x, \hat{p}_x^2]] = -4\hbar^2 x$.

In the above equalities $f(x)$ denotes an arbitrary function of a coordinate.

4.11. Check the following commutation rules for the Hamiltonian $\hat{H}$ in the potential field $U(x)$:

(a) $[\hat{H}, x] = -\frac{i\hbar}{m}\hat{p}_x$;

(b) $[\hat{H}, \hat{p}_x] = i\hbar\frac{\partial U}{\partial x}$;

(c) $[\hat{H}, \hat{p}_x^2] = 2i\hbar\frac{\partial U}{\partial x}\hat{p}_x + \hbar^2\frac{\partial^2 U}{\partial x^2}$.

4.12. The operator $\hat{A}$ commutes with operators $\hat{B}$ and $\hat{C}$. Can one infer that the operators $\hat{B}$ and $\hat{C}$ are commutative?

4.13. Prove the following theorems: **(a)** if the operators $\hat{A}$ and $\hat{B}$ have their characteristic functions (eigenfunctions), such operators commute; **(b)** if the operators $\hat{A}$ and $\hat{B}$ commute, they have common eigenfunctions. The proof is to be carried out for the case when there is no degeneracy.

4.14. Find the common eigenfunction of the following operators: **(a)** x and $\hat{p}_y$; **(b)** $\hat{p}_x$, $\hat{p}_y$, and $\hat{p}_z$; **(c)** $\hat{p}_x$ and $\hat{p}_x^2$.

4.15. In a certain state ψ_A a system possesses a definite value of the mechanical quantity A. Does the quantity B also possess a definite value in that state provided the corresponding operators $\hat{A}$ and $\hat{B}$ are commutative?

4.16. Prove that if the operator $\hat{A}$ is hermitian, its eigenvalues are real.

4.17. Prove that the following operators are hermitian: **(a)** $\hat{p}_x$; **(b)** $x\hat{p}_x$; **(c)** $\hat{p}_x^2$; **(d)** $\hat{H}$. *Instruction*: note that in the infinity both wave functions and their derivatives turn to zero.

4.18. Find the operator that is conjugate to the operator: **(a)** $x\hat{p}_x$; **(b)** $i\hat{p}_x$.

4.19. Prove that if the operators $\hat{A}$ and $\hat{B}$ are hermitian and commutative, the operator $\hat{A}\hat{B}$ is hermitian.

4.20. Prove that if the operator $\hat{A}$ is hermitian, the operator $\hat{A}^n$ is also hermitian (n is a positive integer).

4.21. Prove that if the operators $\hat{A}$ and $\hat{B}$ are hermitian, the operators $\hat{A}+\hat{B}$ and $\hat{A}\hat{B}+\hat{B}\hat{A}$ are also hermitian.

4.22. Prove that if the operators $\hat{A}$ and $\hat{B}$ are hermitian and non-commutative, the operator **(a)** $[\hat{A}, \hat{B}]$ is non-hermitian; **(b)** $i[\hat{A}, \hat{B}]$ is hermitian.

4.23. Find the eigenvalues and normalized eigenfunctions of the operators: **(a)** $\hat{L}_z$; **(b)** $\hat{L}_z^2$.

4.24. Find the eigenvalue of the operator $\hat{L}^2$ that corresponds to its eigenfunction $Y(\theta, \varphi) = A(\cos\vartheta + 2\sin\vartheta\cos\varphi)$.

4.25. Prove that the operator $\hat{L}_z$ is hermitian. The proof is to be carried out both in **(a)** polar and **(b)** Cartesian coordinates.

4.26. Prove that the operator $\hat{L}^2$ is hermitian, taking into account that the operators $\hat{L}_x$, $\hat{L}_y$, and $\hat{L}_z$ are hermitian.

4.27. Check the following commutation rules:

(a) $[x, \hat{L}_x] = 0$; **(b)** $[y, \hat{L}_x] = -i\hbar z$; **(c)** $[z, \hat{L}_x] = i\hbar y$.

4.28. Prove the following commutation rules:

(a) $[\hat{L}_x, \hat{p}_x] = 0$; **(b)** $[\hat{L}_x, \hat{p}_y] = i\hbar\hat{p}_z$; **(c)** $[\hat{L}_x, \hat{p}_z] = -i\hbar\hat{p}_y$.

4.29. Using the commutation rules of the foregoing problem, demonstrate that:

(a) $[\hat{L}_x, \hat{p}_x^2] = 0$; **(b)** $[\hat{L}_x, \hat{p}^2] = 0$; **(c)** $[\hat{L}_x^2, \hat{p}^2] = 0$.

4.30. Prove that the operator $\hat{L}^2$ and kinetic energy operator $\hat{T}$ commute.

4.31. Check the following commutation rules:

(a) $[\hat{L}_x, \hat{L}_y] = i\hbar\hat{L}_z$; **(b)** $[\hat{L}_y, \hat{L}_z] = i\hbar\hat{L}_x$; **(c)** $[\hat{L}_z, \hat{L}_x] = i\hbar\hat{L}_y$.

4.32. Using the commutation rules of the foregoing problem, demonstrate that: **(a)** the operator $\hat{L}^2$ commutes with the operators $\hat{L}_x$, $\hat{L}_y$, and $\hat{L}_z$; **(b)** $[\hat{L}_+, \hat{L}_-] = 2\hbar\hat{L}_z$, where $\hat{L}_+ = \hat{L}_x + i\hat{L}_y$ and $\hat{L}_- = \hat{L}_x - i\hat{L}_y$.

4.33. A spatial rotator can be pictured as a particle of mass μ moving at a permanent distance r_0 from a centre. Find the energy eigenvalues of such a rotator, assuming the eigenvalues of the operator $\hat{L}^2$ to be known.

MEAN VALUES AND PROBABILITIES

4.34. Prove that if a mechanical quantity A is described by the hermitian operator $\hat{A}$, then: **(a)** its mean value is real; **(b)** the mean value of that quantity squared is $\langle A^2\rangle = \int |\hat{A}\psi|^2\,dx$.

4.35. Demonstrate that in a unidimensional case

$$\langle p_x\rangle = \frac{i\hbar}{2}\int\left(\psi\frac{d\psi^*}{dx} - \psi^*\frac{d\psi}{dx}\right)dx.$$

4.36. Demonstrate that the mean value of the particle's momentum projection equals zero provided its discrete spectrum is stationary. *Instruction.* Use the expression for the operator $\hat{p}_x$ in terms of the commutator of the two operators $\hat{H}$ and $\hat{x}$.

4.37. Find the mean kinetic energy of a particle in a unidimensional square potential well with absolutely impenetrable walls $(0 < x < l)$, if the particle is in the state: **(a)** $\psi(x) = A\sin^2(\pi x/l)$; **(b)** $\psi(x) = Ax(l-x)$.

4.38. Calculate the mean values of kinetic and potential energies of an oscillator with frequency ω in the ground state $\psi(x) = Ae^{-\alpha^2x^2}$, where $\alpha^2 = \varkappa/2\hbar\omega$, $\varkappa$ is the quasi-elastic force constant $(U = \varkappa x^2/2)$.

4.39. Calculate the mean values $\langle(\Delta x)^2\rangle$ and $\langle(\Delta p_x)^2\rangle$ and their product for **(a)** a particle located in the nth level of a unidimensional square potential well with absolutely impenetrable walls $(0 < x < l)$; **(b)** an oscillator in the ground state $\psi(x) = Ae^{-\alpha^2x^2}$; **(c)** a particle in the state $\psi(x) = Ae^{ikx-\alpha^2x^2}$.

4.40. Determine the mean value of a mechanical quantity, described by the operator $\hat{L}_z^2$, in the state $\psi(\varphi) = A\sin^2\varphi$.

4.41. Calculate the mean values $\langle(\Delta\varphi)^2\rangle$ and $\langle(\Delta L_z)^2\rangle$ and their product for a system in the state $\psi(\varphi) = A\sin\varphi$.

4.42. Demonstrate that in the state ψ, where the operator $\hat{L}_z$ has a definite eigenvalue, the mean values $\langle L_x\rangle$ and $\langle L_y\rangle$ are equal to zero. *Instruction.* Make use of the commutative relations given in Problem 4.31.

4.43. Calculate the mean value of the squared angular momentum in the state $\psi(\vartheta, \varphi) = A\sin\vartheta\cos\varphi$.

4.44. The allowed values of projections of angular momentum on an arbitrary axis are equal to $m\hbar$, where $m = l, l-1, \ldots, -l$. Keeping in mind that these projections are equally probable and all axes are equivalent, demonstrate that in the state with definite value of l the mean value of the squared angular momentum is $\langle L^2\rangle = \hbar^2 l(l+1)$.

4.45. Prove that the eigenfunctions ψ_1 and ψ_2 of the hermitian operator $\hat{A}$, that correspond to different eigenvalues A_1 and A_2 of the discrete spectrum, are orthogonal.

4.46. Through direct calculations demonstrate the orthogonality of eigenfunctions of: **(a)** the operator $\hat{H}$ in the case of a particle located in a unidimensional square potential well with absolutely impenetrable walls; **(b)** the operator $\hat{L}_z$.

4.47. A system is in the state described by the normalized wave function $\psi(x)$ that can be expanded into the eigenfunctions of the hermitian operator $\hat{A}$, i.e. $\psi(x) = \sum c_k\psi_k(x)$. Assuming the functions ψ_k to be normalized to unity, **(a)** derive the expression defining the coefficients c_k; **(b)** demonstrate that the mean value of a mechanical quantity is $\langle A \rangle = \sum A_k \mid c_k \mid^2$, where A_k are the eigenvalues of the operator $\hat{A}$. What is the physical meaning of $\mid c_k \mid^2$?

4.48. A unidimensional square potential well with absolutely impenetrable walls $(0 < x < l)$ contains a particle in the state $\psi(x)$. Determine the probability of its staying **(a)** in the ground state, if $\psi(x) = A \sin^2(\pi x/l)$; **(b)** in the nth level, if $\psi(x) = Ax(l-x)$; calculate the probabilities for the first three levels.

4.49. Determine the allowed eigenvalues of the operator $\hat{L}_z$ together with their probabilities for a system in the state: **(a)** $\psi(\varphi) = A \sin^2\varphi$; **(b)** $\psi(\varphi) = A(1+\cos\varphi)^2$.

4.50. Keeping in mind that the eigenfunctions of the wave number operator $\hat{k}$ $(\hat{k} = \hat{p}/\hbar)$ are $\psi_k(x) = (2\pi)^{-\frac{1}{2}} e^{ikx}$, find the probability distribution of values of k: **(a)** for a particle located in the nth level of a unidimensional square potential well of width l with absolutely impenetrable walls; **(b)** for an oscillator in the state $\psi(x) = Ae^{-\alpha^2x^2}$.

VARIATION OF THE STATE AND MECHANICAL VALUES IN THE COURSE OF TIME

4.51. Find out whether a wave function composed as a superposition of stationary states, $\Psi(x, t) = \sum \psi_k(x) e^{i\omega_k t}$, can be a solution of the Schrödinger equation in both time-dependent and time-independent forms.

4.52. A particle is located in a unidimensional square potential well of width l with absolutely impenetrable walls. Find the wave function of the particle at the moment t, if at the initial moment it had the form $\Psi(x, 0) = Ax(l-x)$.

4.53. A system of two rigidly connected particles rotating in a plane about its centre of inertia is referred to as a plane rotator. The energy operator of such a rotator has the form $\hat{H} = \frac{\hbar^2}{2I}\cdot\frac{\partial^2}{\partial\varphi^2}$, where I is the system's moment of inertia. Assuming that the rotator's wave function had the form $\Phi(\varphi, 0) = A\cos^2\varphi$, at the initial moment, determine this function at any moment t.

4.54. Having calculated the time derivative of the mean value of the mechanical quantity A described by the operator $\hat{A}$ by means of the Schrödinger time-dependent equation, demonstrate that

$$\text{(a)}\ \frac{d\hat{A}}{dt} = \frac{\partial\hat{A}}{\partial t} + \frac{i}{\hbar}(\hat{H}\hat{A} - \hat{A}\hat{H}); \quad \text{(b)}\ \frac{d}{dt}\langle A\rangle = \left\langle \frac{dA}{dt} \right\rangle.$$

4.55. Prove the following operator equalities:

$$\text{(a)}\ \frac{d}{dt}(\hat{A}+\hat{B}) = \frac{d\hat{A}}{dt} + \frac{d\hat{B}}{dt}; \quad \text{(b)}\ \frac{d}{dt}(\hat{A}\hat{B}) = \frac{d\hat{A}}{dt}\hat{B} + \hat{A}\frac{d\hat{B}}{dt}.$$

4.56. Prove the validity of the following motion equations in the operator form: **(a)** $dx/dt = \hat{p}_x/m$; **(b)** $d\hat{p}_x/dt = -\partial U/\partial x$.

4.57. According to Ehrenfest's rule the mean values of mechanical quantities obey the laws of classical mechanics. Prove that when a particle moves in the potential field $U(x)$: **(a)** $\langle dx/dt\rangle = \langle p_x\rangle/m$; **(b)** $\langle dp_x/dt\rangle = -\langle \partial U/\partial x\rangle$.

4.58. Prove that in the case of a particle moving in the potential field $U(x)$, the following operator equalities are valid:

$$\text{(a)}\ \frac{d}{dt}(x^2) = \frac{1}{m}(x\hat{p}_x + \hat{p}_x x);$$

$$\text{(b)}\ \frac{d}{dt}(x\hat{p}_x) = \frac{\hat{p}_x^2}{m} - x\frac{\partial U}{\partial x};$$

$$\text{(c)}\ \frac{d}{dt}(\hat{p}_x^2) = -\left(\hat{p}_x\frac{\partial U}{\partial x} + \frac{\partial U}{\partial x}\hat{p}_x\right).$$

4.59. Demonstrate that the time derivative of the operator $\hat{L}_x$ is equal to the operator of projection of the moment of external forces, i.e.

$$\frac{d}{dt}\hat{L}_x = \hat{M}_x = -\left(y\frac{\partial U}{\partial z} - z\frac{\partial U}{\partial y}\right).$$

4.60. A particle is in the state described by the eigenfunction ψ of the operator $\hat{A}$ that depends on time implicitly. Demonstrate that the corresponding eigenvalue A of that operator does not vary with time provided the operator $\hat{A}$ commutes with the Hamiltonian $\hat{H}$.

4.61. What mechanical quantities (the energy E, projections of momentum, projections and the square of angular momentum) retain their values during the motion of a particle: **(a)** in the absence of a field (free motion); **(b)** in the uniform potential field $U(z) = az$, a being a constant; **(c)** in a central-symmetry potential field $U(r)$; **(d)** in a uniform variable field $U(z, t) = a(t)z$?

4.62. A particle is in a certain state $\Psi(x, t)$, with $\Psi(x, t)$ not being an eigenfunction of the operator $\hat{A}$. Knowing that the operator $\hat{A}$ does not depend on time explicitly and commutes with the Hamiltonian $\hat{H}$, demonstrate that: **(a)** the mean value of the mechanical quantity A does not vary with time; **(b)** the probabilities of definite values of the mechanical quantity A are also independent of time.

4.63. Transform the total energy operator for a particle in the central-symmetry field $U(r)$ to the following form:

$$\hat{H} = \hat{T}_r + \frac{\hat{L}^2}{2mr^2} + U(r).$$

What form does the operator $\hat{T}_r$ take?

4.64. A particle of mass μ moves in the central-symmetry potential field $U(r)$. Find: **(a)** the Schrödinger equations for the angle-dependent and radial parts of the wave function $\psi(r, \vartheta, \varphi) = R(r) \cdot Y(\vartheta, \varphi)$. Assuming the eigenvalues of the operator $\hat{L}^2$ to be known, reduce the equation for the function $R(r)$ to the form of Eq. (4.10); **(b)** how the wave function depends on the azimuth φ.

4.65. A particle is located in a central-symmetry potential field in the state $\psi(r, \vartheta, \varphi) = R_l(r) \cdot Y_{lm}(\vartheta, \varphi)$. What is the physical meaning of the function $|Y_{lm}|^2$? Making use of Table 4.1, calculate the normalization coefficients of the functions: **(a)** $Y_{1,0}$; **(b)** $Y_{2,1}$.

4.66. A particle of mass m moves in a spherical-symmetry potential well with absolutely impenetrable walls ($0 < r < r_0$). Throughout the well's interior $U = 0$.

(a) Using the substitution $\psi(r) = \chi(r)/r$, find the energy eigenvalues and normalized eigenfunctions of the particle in s states ($l = 0$).

(b) Calculate the most probable value r_{pr} for the ground state of the particle, and the probability of the particle being in the region $r < r_{pr}$. Draw the approximate graphs of the functions $\psi^2(r)$ and $r^2\psi^2(r)$ in this state. What is the physical meaning of these functions?

(c) Find the radial part $R_1(r)$ of the wave function describing the p state of the particle ($l = 1$). To do this, differentiate the equation defining the function $R_0(r)$ for s states and compare the expression obtained with the equation defining the function $R_1(r)$.

(d) Calculate the energy of the first p level and compare it with the ground state energy. Draw the approximate graph of the function $r^2R_1^2(r)$ for the first p level.

4.67. Using the results obtained in the foregoing problem find: **(a)** the mean values $\langle r \rangle$, $\langle r^2 \rangle$, and $\langle (\Delta r)^2 \rangle$ for a particle located in the nth s level ($l = 0$); **(b)** the mean value of kinetic energy of a particle in the state $\psi(r) = A(r_0^2 - r^2)$; **(c)** the probability distribution of various values of wave number k in the ground state, if the eigenfunctions of the operator $\hat{k}$ are known to have the form $\psi_{\mathbf{k}}(\mathbf{r}) = (2\pi)^{-3/2} e^{i\mathbf{kr}}$.

4.68. A particle of mass m is located in a spherical-symmetry potential field: $U(r) = 0$ at $r < r_0$ and $U(r) = U_0$ at $r > r_0$.

(a) Using the substitution $\psi(r) = \chi(r)/r$, derive the equation defining the energy eigenvalues of the particle in s states ($l = 0$) in the region $E < U_0$; reduce this equation to the form

$$\sin kr_0 = \pm \sqrt{\frac{\hbar^2}{2mr_0^2U_0}}\, kr_0, \quad k = \frac{\sqrt{2mE}}{\hbar}.$$

(b) Make sure that the given well not always has discrete levels (bonded states). Determine the interval of values of $r_0^2U_0$ at which the well possesses only one s level.

(c) Assuming $r_0^2U_0 = 8\pi^2\hbar^2/27m$, calculate the most probable value r_{pr} for the particle in the s state and the probability of the particle being in the region $r > r_0$.

4.69. Reduce the equation describing the radial part of the wave function of an electron in the Coulomb field of a nucleus Z to a non-dimensional form. As units of measurement use the atomic unit of length (the first Bohr radius) and the atomic unit of energy (the binding energy of an electron in a hydrogen atom).

4.70. Using the substitution $R(r) = \chi(r)/r$, find the asymptotic form of the radial part $R(r)$ of the wave function for bonded states of an electron in the Coulomb field of a nucleus: **(a)** at long and **(b)** short distance from the nucleus.

4.71. An electron in a hydrogen atom is in the stationary state described by the spherical-symmetry wave function $\psi(r) = A(1 + ar)e^{\alpha r}$, where A, a, and α are constants. Find: **(a)** (using the Schrödinger equation) the constants a, α, and the energy of the electron; **(b)** the normalization coefficient A.

4.72. For an $1s$ electron in a hydrogen atom find: **(a)** the most probable distance r_{pr} from the nucleus and the probability of the electron being in the region $r < r_{pr}$; **(b)** the probability of its being outside the classical borders of the field.

4.73. For an $1s$ electron in a hydrogen atom calculate the mean values of: **(a)** the distance from the nucleus $\langle r \rangle$, as well as $\langle r^2 \rangle$ and square variation $\langle (\Delta r)^2 \rangle$; **(b)** the interaction force and potential energy; **(c)** the kinetic energy and root mean square velocity.

4.74. For $2p$ and $3d$ electrons in a hydrogen atom calculate: **(a)** the most probable distance from the nucleus; **(b)** the mean square variation $\langle (\Delta r)^2 \rangle$.

4.75. Find the mean electrostatic potential developed by an $1s$ electron at the centre of a hydrogen atom.

4.76. Calculate the mean electrostatic potential at the distance r from the nucleus of a hydrogen atom in the ground state. *Instruction*: to find the potential φ_e developed by an "electron cloud", one should integrate Poisson's equation $\nabla^2\varphi_e = -4\pi\rho$ twice.

4.77. Find the probability distribution of values of wave number k for an electron in a hydrogen atom in the ground state, if the eigenfunctions of the operator $\hat{k}$ are known to have the form $\psi_{\mathbf{k}}(\mathbf{r}) = (2\pi)^{-3/2} e^{i\mathbf{kr}}$.

5

SPECTRA. ELECTRON SHELL OF ATOMS

● Spectral labeling of terms: $\varkappa(L)_J$, where $\varkappa$ is the multiplicity ($\varkappa = 2S + 1$); L, S, and J are quantum numbers (corresponding to orbital, spin, and total moments respectively):

$$L = 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ \ldots$$

$$\text{symbol: } S,\ P,\ D,\ F,\ G,\ H,\ I,\ \ldots$$

● Selection rules for quantum numbers S, L, and J:

$$\Delta S = 0; \quad \Delta L = \pm 1; \quad \Delta J = 0,\ \pm 1; \quad J = 0 \not\to J = 0. \tag{5.1}$$

● Terms of an atom (ion) with one valence electron:

$$T = \frac{RZ_{\text{eff}}^2}{(n-\Delta)^2}, \tag{5.2}$$

where R is the Rydberg constant; Z_{eff}^2 is the effective charge (in e units) of the atomic skeleton (ion) in whose field the outer electron moves; n is the principal quantum number of the valence electron; Δ is the quantum defect. The diagram of levels of such an atom (ion) is shown in Fig. 18 (the fine structure is neglected).

● Dirac's equation for the fine structure of levels in an atom (ion) with one electron:

$$T = \frac{RZ^2}{n^2} + \frac{\alpha^2 RZ^4}{n^3}\left(\frac{2}{j+1/2} - \frac{3}{4n}\right), \tag{5.3}$$

where Z is the charge of nucleus (in e units); α is the fine structure constant; n and j are quantum numbers (the principal quantum number and the number corresponding to the total angular momentum).

● Mechanical moments of an atom (orbital, spin, and total ones respectively):

$$p_L = \hbar\sqrt{L(L+1)}; \quad p_S = \hbar\sqrt{S(S+1)}; \quad p_J = \hbar\sqrt{J(J+1)}. \tag{5.4}$$

● In the problems of this chapter the inter-momentum coupling is assumed to be normal, $L-S$ (spin-orbital coupling).

● Hund rules:

of the terms given by electrons of given electronic configuration, the ones with greatest value of S have the least energy, and of these the one with the greatest L is the lowest;

of the basic (normal) term $J = |L - S|$, if the shell is less than half-filled, and $J = L + S$ in the remaining cases.

● Electrons with equal quantum numbers n and l are referred to as equivalent.

● Boltzmann's distribution law

$$\frac{N_1}{N_2} = \frac{g_1}{g_2} e^{-(E_1 - E_2)/kT}, \tag{5.5}$$

where g_1 and g_2 are the statistical weights (degeneracies) of the levels with energies E_1 and E_2.

● Probabilities of radiation transitions between levels *1* and *2* ($E_2 > E_1$), i.e. the number of transitions per 1 s per one atom (Z/N), for the cases of spontaneous radiation, induced radiation, and absorption:

$$Z_{21}^{\text{sp}}/N_2 = A_{21}; \quad Z_{21}^{\text{ind}}/N_2 = B_{21}u_\omega; \quad Z_{12}^{\text{abs}}/N_1 = B_{12}u_\omega, \tag{5.6}$$

where A_{21}, B_{21}, B_{12} are the Einstein coefficients; u_ω is the volume spectral density of radiation corresponding to frequency ω of transition between the given levels.

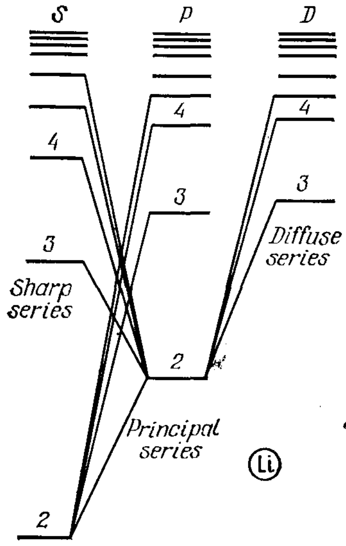

Fig. 18

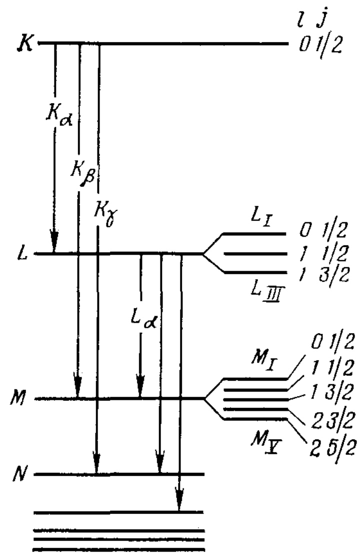

Fig. 19

● Relation between the Einstein coefficients:

$$B_{21} = \frac{g_1}{g_2} B_{12} = \frac{\pi^2 c^3}{\hbar\omega^3} A_{21}. \tag{5.7}$$

● Relation between the mean lifetime τ and the width Γ of a level:

$$\tau \cdot \Gamma \sim \hbar.$$

● X-ray term diagram is shown in Fig. 19.

● Moseley's law for K_α lines:

$$\omega_{K_\alpha} = \frac{3}{4} R^* (Z - \sigma)^2, \tag{5.8}$$

where R^* is the Rydberg constant; Z is the atomic number; σ is the correction constant (to be assumed equal to unity when solving the problems).

THE STATE OF ELECTRONS IN AN ATOM

5.1. Find the ionization potential and first excitation potential of a Na atom in which the quantum defects of the basic term $3S$ and the term $3P$ are equal to 1.37 and 0.88 respectively.

5.2. Calculate the quantum defects of S, P, and D terms for a Li atom, if the binding energy of the valence electron in the ground state is known to be equal to 5.39 eV, the first excitation potential to 1.85 eV, and the wavelength of the main line of the diffuse series to 0.610 μm. Which of the mentioned terms is the closest to the hydrogen-like ones and what is the reason for that?

5.3. Find the binding energy of a valence electron in the ground state of a Li atom, if the wavelength of the main line of the sharp series and the short-wave cut-off wavelength of that series are equal to 0.813 and 0.349 μm respectively.

5.4. How many spectral lines allowed by the selection rules appear in transition of Li atoms to the ground state from the state: **(a)** $4S$; **(b)** $4P$?

5.5. Calculate the quantum defects of S and P terms and the wavelength of the main line of the sharp series in a Be^+ ion, if the wavelengths of the main line of the principal series and its short-wave cut-off are known to be equal to 3210 and 683 Å respectively.

5.6. The terms of atoms and ions with one valence electron can be written in the form $T = R(Z - a)^2/n^2$, where Z is the charge of nucleus (in e units); a is the screening correction; n is the principal quantum number of the valence electron. Using this formula, calculate the correction a and quantum number n of the valence electron in the ground state of a Li atom, if the ionization potentials of Li and Be^+ are known to be equal to 5.39 and 17.0 V, the correction a being the same for both.

5.7. Find the splitting (in eV's) of the level $4P$ in a K atom, if the wavelengths of the resonance line doublet components are known to be equal to 7698.98 and 7664.91 Å. Compare the value obtained with the resonance transition energy.

5.8. The main line of the sharp series of monatomic cesium is a doublet with the wavelengths of 14 695 and 13 588 Å. Find the intervals (in cm^{-1}) between the components of the subsequent lines of this series.

5.9. Write out the spectral designations of electronic terms in a hydrogen atom for $n = 3$. How many fine structure components have the level of a hydrogen atom with the principal quantum number n?

5.10. For a He^+ ion calculate the intervals (in cm^{-1}) between: **(a)** the extreme fine structure components of the levels with $n = 2$, 3, and 4; **(b)** the neighbouring fine structure components of the level with $n = 3$.

5.11. Calculate the difference in wavelengths of doublet components of the line $2P - 1S$ for a hydrogen atom and a He^+ ion.

5.12. What hydrogen-like ion possesses the doublet of the main line of the Lyman series, for which the difference in wave numbers is equal to 29.6 cm^{-1}?

5.13. For He^+ ions determine the number of fine structure components and the interval (in cm^{-1} units and wavelengths) between the extreme components of the main line of: **(a)** the Balmer series; **(b)** the Paschen series.

5.14. What must be the resolving power of the spectroscope capable of observing the fine structure of the main line of the Balmer series in monatomic hydrogen?

5.15. Find allowed values of the total angular momenta of electron shells of atoms in the states ^4P and ^5D.

5.16. Write out allowed terms of atoms possessing besides filled shells: **(a)** two electrons, s and p; **(b)** two electrons, p and d; **(c)** three electrons, s, p, and d.

5.17. How many different types of terms can a two-electron system, consisting of d and f electrons, possess?

5.18. Write out allowed types of terms for the atom possessing, in addition to filled shells, two p electrons with different principal quantum numbers.

5.19. Determine allowed multiplicity of: **(a)** the $D_{3/2}$ term; **(b)** the terms of atoms Li, Be, B, and C, if only the electrons of outermost unfilled subshells get excited.

5.20. Find the greatest possible total angular momentum of an electron shell of an atom in F state, if it is known that five terms of equal multiplicity, but with different values of quantum number J, correspond to that state.

5.21. The number of allowed values of the quantum number J for two different atoms in the P and D states is the same and equal to three. Determine the spin mechanical moment of the atoms in these states.

5.22. Find the angle between the spin and total angular momenta in the vector model of the atom: **(a)** being in the ^3D state with the greatest possible value of the total angular momentum; **(b)** possessing in addition to filled subshells three electrons (p, d, and f) and having the total angular momentum that is the greatest possible for this configuration.

5.23. An atom is in the ^4F state and possesses the greatest possible mechanical moment. Determine the degeneracy of that state in terms of J. What is the physical meaning of the value obtained?

5.24. Determine the greatest possible orbital angular momentum of an atom being in the state whose multiplicity is five and degeneracy (in terms of J) is seven. Indicate the spectral symbol of that state.

5.25. Find the greatest possible angle between the spin and total mechanical moments in the vector model of the atom being in the state whose multiplicity is three and degeneracy (in terms of J) is five.

5.26. Determine the number of allowed states for: **(a)** an atom with the given values of quantum numbers L and S; **(b)** a two-electron system composed of p and d electrons; **(c)** an electron configuration nd^3.

5.27. Find the number of electrons in the atoms whose shells are filled as follows: **(a)** the K and L shells and the $3s$ subshell are filled and the $3p$ subshell is half-filled; **(b)** the K, L, M shells and $4s$, $4p$, and $4d$ subshells are all filled. What are these atoms?

5.28. Find the maximum number of electrons possessing in an atom the following equal quantum numbers: **(a)** n and l; **(b)** n.

5.29. Using the Hund rules, write the electron configurations and find the basic term of the atoms: **(a)** C and N; **(b)** S and Cl. The electron configurations of these atoms correspond to the regular filling of electron shells.

5.30. Making use of the Hund rules, find the basic term of the atom whose unfilled subshell has the electron configuration: **(a)** nd^2; **(b)** nd^3.

5.31. Determine the basic term of the atom whose outer shell is exactly half-filled with five electrons.

5.32. Find the degeneracy of the atom in the ground state, if the electronic configuration of its unfilled subshell if nd^6.

5.33. Find allowed types of terms for an atom whose unfilled subshell has the electronic configuration: **(a)** np^2; **(b)** np^3; **(c)** nd^2.

5.34. There are two electronic configurations, one of which possessing the same number of equivalent electrons that are required to complete the subshell of the other. Using the following examples, demonstrate that such pairs of electronic configurations have the identical sets of allowed types of terms: **(a)** p^1 and p^5; **(b)** p^2 and p^4; **(c)** d^1 and d^9. Explain this fact.

5.35. Write allowed types of terms for the following electronic configurations: **(a)** ns^1, $n'p^2$; **(b)** np^1, $n'p^2$. Here $n \neq n'$.

INTENSITY AND WIDTH OF SPECTRAL LINES

5.36. Find the ratio of the number of atoms of gaseous lithium in the $2P$ state to that in the ground state at a temperature of $T = 3000$ K. The wavelength of the resonance line ($2P - 2S$) is $\lambda = 6708$ Å.

5.37. What fraction of hydrogen atoms is in the state with the principal quantum number $n = 2$ at a temperature of $T = 3000$ K?

5.38. Demonstrate that the number of atoms excited to a certain level diminishes with time as $N = N_0 e^{-t/\tau}$, where τ is the mean lifetime of the atom on that level.

5.39. The intensity of a resonance line diminishes by a factor of $\eta = 65$ over a distance $l = 10$ mm along the beam of atoms moving at a velocity $v = 2.0 \cdot 10^3$ m/s. Calculate the mean lifetime of atoms in the resonance excitation state. Evaluate the level's width.

5.40. Rarefied mercury vapour whose atoms are in the ground state is lighted by a mercury lamp emitting a resonance line of wavelength $\lambda = 2536.5$ Å. As a result, the radiation power of the mercury vapour at that wavelength turns out to be $I = 35$ mW. Find the number of atoms in the resonance excitation state whose mean lifetime is $\tau = 1.5 \cdot 10^{-7}$ s.

5.41. A sample of gaseous lithium containing $N = 3.0 \cdot 10^{16}$ atoms

is heated to a temperature of $T = 1500$ K. In this case, the power emitted at the resonant line's wavelength $\lambda = 6708$ Å ($2P - 2S$) is equal to $I = 0.25$ W. Find the mean lifetime of Li atoms in the resonance excitation state.

5.42. A system of atoms is in thermodynamic equilibrium with its radiation at temperature T. Suppose that the transition between the two atomic energy levels, E_1 and E_2, with statistical weights g_1 and g_2 produces the radiation of frequency ω, the Einstein coefficients being A_{21}, B_{21}, and B_{12}. Recalling that at equilibrium the numbers of direct and reverse transitions ($E_1 \rightleftarrows E_2$) per unit time are equal, find the expression for volume spectral density of thermal radiation energy: **(a)** with allowance made for induced emission; also find the relation between the Einstein coefficients; **(b)** disregarding the induced emission. Under what conditions can it be done?

5.43. Atomic hydrogen is in thermodynamic equilibrium with its radiation. Find: **(a)** the ratio of probabilities of induced and spontaneous radiations of the atoms from the level $2P$ at a temperature of $T = 3000$ K; **(b)** the temperature at which these probabilities become equal.

5.44. A beam of light of frequency ω, equal to the resonant frequency of transition of atoms of gas ($\hbar\omega \gg kT$), passes through that gas heated to temperature T. Taking into account induced radiation, demonstrate that the absorption coefficient of the gas varies as

$$\varkappa(T) = \varkappa_0 (1 - e^{-\hbar\omega/kT}),$$

where $\varkappa_0$ is the absorption coefficient at $T = 0$ K.

5.45. Under what conditions can light passing through matter be amplified? Find the ratio of the populations of levels $^1D_2$ and $^1P_1$ ($E_D > E_P$) in atoms of gas at which a beam of monochromatic light with a frequency equal to the frequency of transition between these levels passes through the gas without attenuation.

5.46. Suppose that a quantum system (Fig. 20) is excited to level *2* and the reverse transition occurs only via level *1*. Demonstrate that in this case light with frequency ω_{21} can be amplified, if the condition $g_1 A_{10} > g_2 A_{21}$ is satisfied, where g_1 and g_2 are the statistical weights of levels *1* and *2* and A_{10} and A_{21} are the Einstein coefficients for the corresponding transitions.

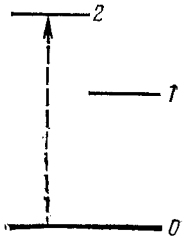

Fig. 20

5.47. Let q be the number of atoms excited to level *2* per unit time (Fig. 20). Find the number of atoms in level *1* after the time interval t following the beginning of excitation. The Einstein coefficients A_{20}, A_{21}, and A_{10} are supposed to be known. The induced transitions are to be ignored.

5.48. A spectral line $\lambda = 5320$ Å appears due to transition in an atom between two excited states whose mean lifetimes are $1.2 \cdot 10^{-8}$ and $2.0 \cdot 10^{-8}$ s. Evaluate the natural width of that line, $\Delta\lambda$.

5.49. The distribution of radiation intensity within a spectral line with natural broadening takes the form

$$J_\omega = J_0 \frac{(\gamma/2)^2}{(\omega-\omega_0)^2+(\gamma/2)^2},$$

where J_0 is the spectral intensity at the line's centre (at $\omega = \omega_0$); γ is the constant which is characteristic for every line (e.g. when an excited state relaxes directly down to the ground state, $\gamma = 1/\tau$, τ being the mean lifetime of the excited state). Using this formula, find: **(a)** the natural linewidth $\delta\omega$, if the value of γ is known; **(b)** the mean lifetime of mercury atoms in the 6^1P state, if the transition to the ground state is known to result in emission of a line $\lambda = 1850$ Å with natural width $\delta\lambda = 1.5 \cdot 10^{-4}$ Å.

Note. The linewidth is the width of the line's contour measured at half its height.

5.50. Making use of the formula of the foregoing problem: **(a)** demonstrate that half the total intensity of a line is confined within its linewidth, that is, within the width of line's contour at the half of its height; **(b)** find the total intensity of a line whose natural width is $\delta\omega$ and spectral intensity at the centre J_0.

5.51. The distribution of radiation intensity in a spectral line with Doppler's broadening takes the form:

$$J_\omega = J_0 e^{-a(\omega-\omega_0)^2/\omega_0^2}; \quad a = mc^2/2kT,$$

where J_0 is the spectral intensity at the line's centre (at $\omega = \omega_0$); m is the mass of the atom; T is the temperature of gas, K.

(a) Derive this formula, using Maxwell's distribution.

(b) Demonstrate that the Doppler width of line λ_0, i.e. the width of line's contour at the half of its height, is equal to

$$\delta\lambda_{\text{Dop}} = 2\lambda_0 \sqrt{(\ln 2)/a}.$$

5.52. The wavelength of the Hg resonance line is $\lambda = 2536.5$ Å. The mean lifetime of the resonance level is $\tau = 1.5 \cdot 10^{-7}$ s. Estimate the ratio of the Doppler broadening of that line to its natural width at a temperature of $T = 300$ K.

5.53. At what temperature is the Doppler broadening of each component of the spectral doublet $2^2P - 1^2S$ of atomic hydrogen equal to the interval between these components?

5.54. To obtain spectral lines without Doppler's broadening, a narrow slightly divergent beam of excited atoms is used, the observation being performed at right angles to the beam. Estimate the beam apex angle in the case of sodium atoms, if the Doppler broadening of the resonance line $\lambda = 5896$ Å is ten times one tenth of its natural width, the velocity of atoms is 1000 m/s, and the mean lifetime of resonance excitation state is $1.6 \cdot 10^{-8}$ s.

5.55. Proceeding from Moseley's law, calculate the wavelengths and energies of photons corresponding to the K_α line in aluminium and cobalt.

5.56. Determine the wavelength of the K_α line of the element of the Periodic Table, beginning from which the appearance of the L series of characteristic X-ray radiation is to be expected.

5.57. Assuming the correction σ in Moseley's law to be equal to unity, find:

(a) to what elements belong the K_α lines with the wavelengths of 1.935, 1.787, 1.656, and 1.434 Å; what is the wavelength of the K_α line of the element omitted in this sequence;

(b) how many elements there are in the sequence between the elements whose K_α line wavelengths are equal to 2.50 and 1.79 Å.

5.58. The correction in Moseley's law differs considerably from unity for heavy elements. Make sure that this is true in the case of tin, cesium, and tungsten, whose K_α line wavelengths are equal to 0.492, 0.402, and 0.210 Å respectively.

5.59. Find the voltage applied to an X-ray tube with nickel anticathode, if the wavelength difference between the K_α line and the short-wave cut-off of the continuous X-ray spectrum is equal to 0.84 Å.

5.60. When the voltage applied to an X-ray tube is increased from 10 to 20 kV, the wavelength interval between the K_α line and the short-wave cut-off of the continuous X-ray spectrum increases threefold. What element is used as the tube anticathode?

5.61. How will the X-ray radiation spectrum vary, if the voltage applied to an X-ray tube increases gradually? Using the tables of the Appendix, calculate the lowest voltage to be applied to X-ray tubes with vanadium and tungsten anticathodes, at which the K_α lines of these elements start to appear.

5.62. What series of the characteristic spectrum are excited in molybdenum and silver by Ag K_α radiation?

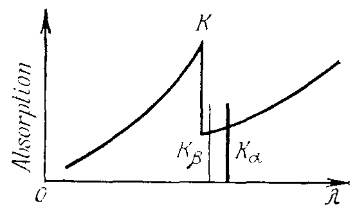

Fig. 21

5.63. Figure 21 shows the K absorption edge of X-ray radiation and the K_α and K_β emission lines.

(a) Explain the nature of the abrupt discontinuity in absorption.

(b) Calculate and plot to scale the diagram of K, L, and M levels of the atom for which $\lambda_{K_\alpha} = 2.75$ Å, $\lambda_{K_\beta} = 2.51$ Å, and $\lambda_K = 2.49$ Å. Of what element is this atom? What is the wavelength of its L_α emission line?

5.64. Knowing the wavelengths of K and L absorption edges in vanadium, calculate (neglecting the fine structure): **(a)** the binding energies of K and L electrons; **(b)** the wavelength of the K_α line in vanadium.

5.65. Find the binding energy of an L electron in titanium, if the wavelength difference between the first line of the K series and its absorption edge is $\Delta\lambda = 0.26$ Å.

5.66. In the first approximation, the X-ray radiation terms can be described in the form $T = R(Z - a)^2/n^2$, where R is the Rydberg constant, Z is the atomic number, a is the screening correction, n is the principal quantum number of a distant electron. Calculate the correction a for the K and L terms of titanium whose K absorption edge has the wavelength $\lambda_K = 2.49$ Å.

5.67. Find the kinetic energy of electrons ejected from the K shell of molybdenum atoms by Ag K_α radiation.

5.68. Carbon subjected to Al K_α radiation emits photoelectrons whose spectrum comprises several monoenergetic groups. Find the binding energy of the electrons ejected from carbon atoms with the kinetic energy of 1.21 keV.

5.69. On irradiation of krypton atoms with monochromatic X-rays of wavelength λ, it was found that in some cases the atoms emit *two* electrons, namely, a photoelectron removed from the K shell and an electron ejected from the L shell due to the Auger effect. The binding energies of the K and L electrons are equal to 14.4 and 2.0 keV respectively. Calculate: **(a)** the kinetic energies of both electrons, if $\lambda = 0.65$ Å; **(b)** the value of λ at which the energies of both electrons are equal.

5.70. **(a)** Demonstrate that the emission spectra of characteristic X-ray radiation consist of doublets.

(b) Why does the K absorption edge consist of a single discontinuity whereas the L absorption edge is triple and M absorption edge consists of five discontinuities?

5.71. **(a)** Indicate the spectral symbol of an X-ray term in an atom with the electron ($l = 1$, $j = 3/2$) removed from one of its closed shells.

(b) Write the spectral designations of allowed X-ray terms of an atom in which one electron is removed from the L shell; from the M shell.

5.72. Determine the number of spectral lines caused by the transitions between the K and L; K and M; L and M shells of an atom.

5.73. Using the tables of the Appendix, calculate: **(a)** the wavelengths of K_α line doublet in tungsten; **(b)** the difference in wavelengths of K_α line doublet in lead.

5.74. Using the tables of the Appendix, calculate the binding energy of $1s$, $2s$, $2p_{1/2}$, and $2p_{3/2}$ electrons in a uranium atom.

6

ATOM IN A MAGNETIC FIELD

● Magnetic moment of an atom:

$$\mu = g\sqrt{J(J+1)}\,\mu_B; \quad g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}, \tag{6.1}$$

where μ_B is the Bohr magneton, g is the Landé splitting factor.

● Zeeman splitting of spectral lines:

$$\Delta\omega = (m_1 g_1 - m_2 g_2)\,\mu_B B/\hbar, \tag{6.2}$$

where m_i and g_i are the magnetic quantum numbers and the Landé factors of the corresponding terms, B is the magnetic induction.

● Zeeman component notation: π-component ($\Delta m = 0$); σ-component ($\Delta m = \pm 1$).

● Selection rules for quantum numbers (in addition to those given in the Introduction to the foregoing chapter):

$$\Delta m_S = 0; \quad \Delta m_L = 0, \pm 1; \quad \Delta m_J = 0, \pm 1; \tag{6.3}$$

if $\Delta J = 0$, the transition $m_J = 0 \nrightarrow m_J = 0$ does not take place.

● Larmor precession frequency:

$$\omega_L = e\mathbf{B}/2mc, \tag{6.4}$$

where e is the elementary charge, m is the electron mass.

● Diamagnetic susceptibility of N isolated atoms:

$$\chi = -\frac{Ne^2}{6mc^2}\sum_{i=1}^{Z}\langle r_i^2\rangle, \tag{6.5}$$

where $\langle r_i^2\rangle$ is the mean squared distance between an ith electron and an atomic nucleus.

● Paramagnetic susceptibility in a weak magnetic field:

$$\chi = \alpha/T; \quad \alpha = N\mu^2/3k, \tag{6.6}$$

where T is the absolute temperature, α is the Curie constant, N is the number of molecules, μ is the magnetic moment of a molecule, k is the Boltzmann constant.

MAGNETIC PROPERTIES OF AN ATOM. ZEEMAN EFFECT

6.1. Taking into account that the magnetic-to-mechanical moment ratio for the spin angular momentum is twice as large as that for the orbital one, derive formula (6.1) by means of the vector model.

6.2. Calculate the Landé factor for the atoms: **(a)** with one valence electron in the S, P, and D states; **(b)** in the ^3P state; **(c)** in the S states; **(d)** in the singlet states.

6.3. Write the spectral symbol of the term with: **(a)** $S = 1/2$, $J = 5/2$, $g = 6/7$; **(b)** $S = 1$, $L = 2$, $g = 4/3$.

6.4. Find the magnetic moment μ and the allowed values of the projection μ_B of an atom in the state: **(a)** ^1F; **(b)** $^2D_{3/2}$.

6.5. The maximum value of projection of the magnetic moment of an atom in the D_2 state is equal to four Bohr magnetons. Determine the multiplicity of that term.

6.6. Determine the allowed values of the magnetic moment of an atom in the ^4P state.

6.7. Calculate the magnetic moment of a hydrogen atom in the ground state.

6.8. Demonstrate that the magnetic moments of atoms in the $^4D_{1/2}$ and $^6G_{3/2}$ states are equal to zero. Interpret this fact on the basis of the vector model of an atom.

6.9. Find the mechanical moments of atoms in the ^5F and ^7H states, if the magnetic moments in these states are known to be equal to zero.

6.10. Using the Hund rules, calculate the magnetic moment of an atom in the ground state, in which the unfilled subshell has the electronic configuration: **(a)** np^5; **(b)** nd^3.

6.11. Using the vector model and the relation $d\mathbf{J}/dt = \mathbf{M}$, where $\mathbf{J}$ is the momentum of an atom and $\mathbf{M}$ is the mechanical moment of external forces, show that the precession angular velocity of the vector $\mathbf{J}$ in the magnetic field B is equal to $\omega = g\mu_B B/\hbar$, g being the Landé factor.

6.12. Find the angular precession velocities of mechanical moments of an atom in a magnetic field of $B = 1000$ G, if the atom is: **(a)** in the ^1P, $^2P_{3/2}$, $^5F_1$ states; **(b)** in the ground state and its unfilled subshell has the electronic configuration np^4. (Use the Hund rule.)

6.13. The mechanical moment of an atom in the ^3F state precesses in a magnetic field of $B = 500$ G at angular velocity $\omega = 5.5 \times \times 10^9$ s^{-1}. Determine the mechanical and magnetic moments of the atom.

6.14. Using the vector model, explain why the mechanical moment of an atom in the $^6F_{1/2}$ state precesses in magnetic field B at angular velocity $\boldsymbol{\omega}$ whose vector is directed oppositely to vector $\boldsymbol{B}$.

6.15. An atom in the $^2P_{1/2}$ state is located on the axis of a circular loop carrying a current $i = 10.0$ A. The radius of the loop is $R = 5.0$ cm, the atom is removed from the centre of the loop by a distance $z = 5.0$ cm. Calculate the maximum value of the interaction force between the atom and the current.

6.16. In the Stern-Gerlach experiment a narrow beam of Ag atoms (in the ground state) passes through a transverse strongly inhomogeneous magnetic field and falls on the screen (Fig. 22). At what value of the gradient of the magnetic field is the distance between the extreme components of the split beam on the screen equal to $\delta = 2.0$ mm if $a = 10$ cm, $b = 20$ cm, and the velocity of the atoms $v = 300$ m/s?

6.17. A narrow beam of atoms passes through a strongly inhomogeneous magnetic field as in the Stern-Gerlach experiment. Determine: **(a)** the maximum values of projections of magnetic moments of the atoms in the ^4F, ^6S, and ^5D states, if the beam splits into 4, 6, and 9 components respectively; **(b)** how many components are observed when in the beam the atoms are in the $^3D_2$ and $^5F_1$ states.

6.18. In the Stern-Gerlach experiment vanadium atoms in the ground $^4F_{3/2}$ state were used. Find the distance between the extreme

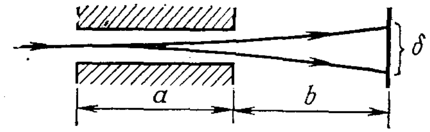

Fig. 22

components of the split beam on the screen (see Fig. 22), if $a = 10$ cm, $b = 20$ cm, $\partial B/\partial z = 23$ G/cm, and the kinetic energy of the atoms is $T = 0.040$ eV.

6.19. An atom is located in the magnetic field $B = 3.00$ kG. Determine: **(a)** the total splitting (in cm^{-1}) of the ^1D term; **(b)** the spectral symbol of the singlet term whose total splitting equals 0.84 cm^{-1}.

6.20. Plot the diagram of allowed transitions in a magnetic field between the following states: **(a)** $^1D \to {}^1P$; **(b)** $^1F \to {}^1D$. How many components are there in the spectral line corresponding to each of these transitions?

6.21. A spectral line $\lambda = 0.612$ μm is caused by the transition between two singlet terms of an atom. Determine the interval $\Delta\lambda$ between the extreme components of that line in magnetic field $B = 10.0$ kG.

6.22. The interval between the extreme components of a spectral line $\lambda = 5250$ Å exhibiting the normal Zeeman effect is equal to $\Delta\lambda = 0.22$ Å. Find the interval (in eV units) between the neighbouring sublevels of the Zeeman splitting of the corresponding terms.

6.23. A spectral instrument with a resolving power $\lambda/\delta\lambda = = 1.0 \cdot 10^5$ is used in observing the components of a spectral line $\lambda = 5360$ Å caused by the transition between the two singlet atomic terms. At what minimum magnitude of magnetic field can the components be resolved, if the observation line is: **(a)** parallel to, **(b)** at right angles to the magnetic field direction?

6.24. In the case of the anomalous Zeeman effect a magnetic field is regarded as weak, if the magnetic splitting of a term is considerably less than the natural multiplet splitting. Find: **(a)** at what value of magnetic field the interval between the neighbouring components of the $3^2P_{1/2}$ and $3^2P_{3/2}$ terms of a Na atom is equal to 1/10 of the natural splitting of the 3^2P state, if the wavelengths of the Na resonance line doublet are 5895.93 Å ($^2P_{1/2} \to {}^2S_{1/2}$) and 5889.96 Å ($^2P_{3/2} \to {}^2S_{1/2}$); **(b)** the same for the $^2P_{1/2}$ and $^2P_{3/2}$ terms of a hydrogen atom in the 2^2P state, recalling that the natural splitting is defined by the Dirac equation for the fine structure; **(c)** the same for the $^2P_{1/2}$ and $^2P_{3/2}$ terms of a He^+ ion in the 2^2P state. Compare the result with that obtained for a hydrogen atom.

6.25. Using the expression for the magnetic moment of an atom, derive the formula for spectral line splitting in the case of the anomalous Zeeman effect in a weak magnetic field.

6.26. What kind of the Zeeman effect, normal or anomalous, is observed in a weak magnetic field in the case of spectral lines:

(a) $^1P \to {}^1S$, $^2D_{5/2} \to {}^2P_{3/2}$; $^3D_1 \to {}^3P_0$, $^5I_5 \to {}^5H_4$;

(b) of the atoms H, He, Li, Be, B, and C?

6.27. Draw the diagram of allowed transition between the $^2P_{3/2}$ and $^2S_{1/2}$ terms in a weak magnetic field. For the corresponding spectral line calculate: **(a)** the shifts of the Zeeman components in $\mu_B B/\hbar$ units; **(b)** the interval (in cm^{-1} units) between the extreme components, if $B = 5.00$ kG.

6.28. Find the minimum resolving power $\lambda/\delta\lambda$ of a spectral instrument capable of resolving the Zeeman structure of the Na spectral line 5890 Å ($^2P_{3/2} \to {}^2S_{1/2}$) in magnetic field $B = 2.0$ kG.

6.29. Draw the diagram of allowed transitions in a weak magnetic field and calculate the displacements (in $\mu_B B/\hbar$ units) of the Zeeman components of the spectral line: **(a)** $^2D_{3/2} \to {}^2P_{3/2}$; **(b)** $^2D_{5/2} \to {}^2P_{3/2}$.

6.30. Calculate the displacements (in $\mu_B B/\hbar$ units) in a weak magnetic field of the Zeeman π components of the spectral line: **(a)** $^3D_3 \to {}^3P_2$; **(b)** $^3D_2 \to {}^3P_2$.

6.31. Using the vector model, demonstrate that in a strong magnetic field, when the $L - S$ coupling breaks up completely, the magnetic interaction energy is equal to $\Delta E_B = (m_L + 2m_S)\,\mu_B B$. Prove that this leads to the normal Zeeman effect.

6.32. At what value of magnetic field will the interval between σ components of the Li resonance line exceed ten-fold the value of natural splitting of that line? The wavelengths of the doublet of that line are equal to 6707.95 and 6707.80 Å.

6.33. Show that the frequency of transition between the neighbouring sublevels of the Zeeman term splitting coincides with the frequency of precession of an angular momentum of atom in a magnetic field.

6.34. The magnetic resonance occurs when a substance consisting of atoms with inherent magnetic moments is exposed to two magnetic fields: the stationary field B and the weak variable field B_ω directed perpendicular to the former one. Demonstrate that the sharp energy absorption maxima are observed when the frequency of the variable field is equal to $\omega = g\mu_B B/\hbar$.

6.35. A gas consisting of atoms in the $^2D_{3/2}$ state is exposed to the joint action of the stationary magnetic field B and the variable field B_ν directed transversely to the stationary one and having a frequency of $2.8 \cdot 10^9$ Hz. At what value of magnetic induction B does the resonance energy absorption occur?

6.36. Find the magnetic moment of Ni atoms in the ^3F state that exhibit the resonance energy absorption under the combined action of the stationary magnetic field $B = 2.00$ kG and the variable field B_ν which is perpendicular to the stationary one and has a frequency $\nu = 3.50 \cdot 10^9$ Hz.

DIA- AND PARAMAGNETISM

6.37. Calculate the magnetic moments of He and Xe atoms in a magnetic field of $B = 10.0$ kG. Their diamagnetic susceptibilities are equal to -1.90 and -43.0 (in 10^{-6} cm^3/mol units) respectively.

6.38. A small sphere of diamagnetic material is slowly moved along the axis of a current-carrying coil from the region where the magnetic field is practically absent to the point at which the field has the value B. Demonstrate that the work performed in the process is equal to $A = -\chi V B^2/2$, where χ is the magnetic susceptibility of a unit volume of the diamagnetic material, V is the volume of the sphere.

6.39. Find the force that the circular loop of radius $R = 5.0$ cm, carrying a current $I = 10$ A, exerts on a Ne atom located on the axis of the loop at a distance $z = 5.0$ cm from its centre. The diamagnetic susceptibility of neon is $\chi = -7.2 \cdot 10^{-6}$ cm^3/mol.

6.40. Using the expression for the Larmor precession frequency, show that the diamagnetic susceptibility of a monatomic gas is $\chi \approx -Ze^2N\langle r^2\rangle/6mc^2$, where Z is the atomic number, N is the number of atoms, $\langle r^2\rangle$ is the mean squared distance between the nucleus and the electrons.

6.41. Calculate the molar diamagnetic susceptibility of atomic hydrogen in the ground state. Its wave function takes the form $\psi(r) = (\pi r_1^3)^{-1/2} e^{-r/r_1}$, where r_1 is the first Bohr radius.

6.42. Recalling that the outer electrons are primarily responsible for diamagnetic properties of an atom (why?), evaluate the radii of outer electron shells in He, Na^+, and Cl^-, whose diamagnetic susceptibilities are equal to -1.9, -6.1, and -24.2 respectively (in 10^{-6} cm^3/mol).

6.43. An atom with spherical-symmetry charge distribution is located in the magnetic field B. Express the magnetic induction B_0 at the atom's centre, caused by the precession of electron shell, via the electrostatic potential V_0 developed by the electron shell at the same point.

6.44. When a paramagnetic gas is located in the magnetic field B at the temperature T, then in the absence of spatial quantization the mean value of projectin of the molecule's magnetic moment

$$\langle \mu_B \rangle = \mu L(a) = \mu \left(\coth a - \frac{1}{a} \right); \quad a = \mu B/kT,$$

where μ is the molecule's magnetic moment, $L(a)$ is Langevin's function.

(a) Derive this expression, using Botzmann's distribution law. Plot the graph $L(a)$.

(b) See how this formula transforms in the case of a weak ($a \ll 1$) and a strong ($a \gg 1$) magnetic field.

6.45. The magnetic moment of a mole of a certain paramagnetic gas in a weak magnetic field $B = 100$ G at $T = 300$ K is equal to $1.5 \cdot 10^{-8}$ J/(G·mol). Determine the Curie constant relating to one mole of gas, and the magnetic moment of a molecule.

6.46. Determine the paramagnetic susceptibility of 1 cm^3 of gas consisting of O_2 molecules with magnetic moments 2.8 μ_B in a weak magnetic field. The gas is under normal pressure and temperature.

6.47. A paramagnetic gas consisting of atoms in the $^2S_{1/2}$ state is in a magnetic field $B = 25$ kG at a temperature of $T = 300$ K. Calculate the ratio $\eta = \Delta N/N$, where ΔN is the difference in the number of atoms with positive and negative projections of magnetic moments on the field direction, N is the total number of atoms. Perform the calculations: **(a)** with allowance made for the spatial quantization; **(b)** in classical terms, i.e., ignoring the spatial quantization.

6.48. Find the magnetic moment of a paramagnetic gas consisting of N atoms in the $^2S_{1/2}$ state at the temperature T in the magnetic field B. Simplify the obtained expression for the case $\mu B \ll kT$.

6.49. A paramagnetic gas is in a magnetic field $B = 20.0$ kG at a temperature of $T = 300$ K. Taking into account the spatial quantization, calculate the ratio $\eta = \Delta N/N$ (see Problem 6.47). Perform the calculation for the cases when the atoms are in the state: **(a)** ^1P; **(b)** $^2P_{3/2}$.

6.50. Demonstrate that in a weak magnetic field the mean projection of the magnetic moment of an atom (with the allowance made for the spatial quantization) $\langle \mu_B \rangle = \mu^2 B/3kT$, where $\mu = g\sqrt{J(J+1)}\,\mu_B$.

6.51. A paramagnetic gas consists of Li atoms in the ground state. Calculate: **(a)** the Curie constant for one mole of that gas; **(b)** the magnetic moment of 1 g of that gas at a temperature of 300 K in a magnetic field $B = 1.00$ kG.

6.52. Calculate the paramagnetic susceptibility of 1 g of monatomic oxygen in a weak magnetic field at the temperature 1600 K. The atoms are in the ground $^3P_2$ state.

7

DIATOMIC MOLECULES

● Rotational energy of a diatomic molecule

$$E_J = \hbar B J (J+1); \quad B = \hbar/2I, \tag{7.1}$$

where B is the rotation constant, I is the molecule's moment of inertia, J is the rotation quantum number, $J = 0, 1, 2, \ldots$

J selection rule: $\Delta J = \pm 1$.

● Vibrational energy of a diatomic molecule

$$E_v = \hbar\omega \left(v + \frac{1}{2} \right) \left[1 - x \left(v + \frac{1}{2} \right) \right], \tag{7.2}$$

where $\omega = \sqrt{\varkappa/\mu}$ is the vibration frequency, $\varkappa$ is the quasielastic force constant,

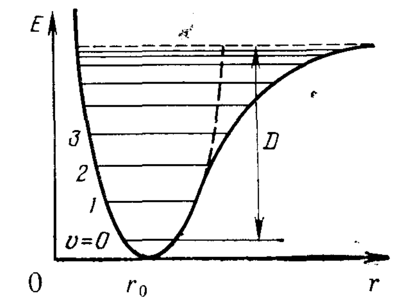

Fig. 23

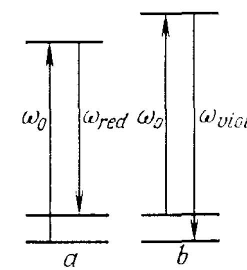

Fig. 24

μ is the reduced mass of a molecule, v is the rotation quantum number, $v = 0, 1, 2, \ldots$, x is the anharmonicity coefficient ($x = 0$ for harmonic oscillator).

v selection rule:

$$\Delta v = \begin{cases} \pm 1, & \text{if } x = 0 \\ \pm 1, & \pm 2, \ldots \text{ in other cases.} \end{cases}$$

● Interaction energy as a function of the distance between the nuclei of a diatomic molecule is shown in Fig. 23, where D is the dissociation energy.

● Mean energy of a quantum harmonic oscillator

$$\langle E \rangle = \frac{\hbar\omega}{2} + \frac{\hbar\omega}{e^{\hbar\omega/kT} - 1}. \tag{7.3}$$

● Fig. 24 illustrates the diagram describing the emergence of red (a) and violet (b) satellites in the Raman scattering of light.

ROTATIONAL AND VIBRATIONAL STATES

7.1. Using the tables of the Appendix, find for H_2 and NO molecules: **(a)** the energy required for their promotion to the first rotational level ($J = 1$); **(b)** the angular rotation velocity in the state with $J = 1$.

7.2. For an HCl molecule find the rotational quantum numbers J for two neighbouring levels whose energies differ by $7.86 \cdot 10^{-3}$ eV.

7.3. Determine the angular momentum of an oxygen molecule in the state with rotational energy $2.16 \cdot 10^{-3}$ eV.

7.4. Find the temperature values at which the mean kinetic energy of translation of H_2 and N_2 molecules is equal to their rotational energy in the state with quantum number $J = 1$.

7.5. Taking into account the degeneracy g of rotational levels ($g = 2J + 1$), calculate the ratio of hydrogen molecules being in true rotational states with $J = 1$ and $J = 2$ at the temperature $T = 300$ K.

7.6. Find the quasielastic force constants for H_2 and CO molecules.

7.7. The potential energy of interaction of atoms in a diatomic molecule can be described approximately by the following formula

$$U(\rho) = U_0 (1 - e^{-a\rho})^2; \quad \rho = \frac{r - r_0}{r_0},$$

where r_0 is the equilibrium nuclear separation, U_0 is the depth of the potential well, a is the intrinsic molecular constant. Calculate the values U_0 and a for a hydrogen molecule.

7.8. For a hydrogen molecule calculate: **(a)** the classical vibration amplitude corresponding to the zero vibrational energy; **(b)** the root-mean-square value of the vibration coordinate x in the ground state which is described by the wave function $\psi(x) \propto e^{-\alpha^2 x^2/2}$, where $\alpha^2 = \mu\omega/\hbar$; μ is the reduced mass, ω is the vibration frequency.

7.9. Find the energy required to promote an H_2 molecule from the ground state to the first vibrational level ($v = 1$). How much higher is this energy as compared to that required to promote the given molecule to the first rotational level ($J = 1$)?

7.10. Determine the temperature at which the mean kinetic energy of translation of Cl_2 molecules is equal to the energy required to promote these molecules from the ground state to the first vibrational level ($v = 1$).

7.11. For an OH molecule find the difference in energies of the states with quantum numbers $v' = 1$, $J' = 0$ and $v = 0$, $J = 5$.

7.12. For an HF molecule calculate the number of rotational levels located between the ground and first vibrational levels.

7.13. Determine the greatest possible vibrational quantum number, the corresponding vibrational energy, and dissociation energy of a diatomic molecule whose natural vibration frequency is ω and anharmonicity coefficient x. Calculate these quantities for an H_2 molecule.

7.14. Calculate the anharmonicity coefficient for a Cl_2 molecule, if its natural vibration frequency and dissociation energy are known.

7.15. Calculate the difference in dissociation energies of heavy and light hydrogen molecules, D_2 and H_2, if the vibration frequency of an H_2 molecule is known.

7.16. Find the ratio of HBr molecules being in purely vibrational states (without rotation) with quantum numbers $v = 2$ and $v = 1$ at $T = 910$ K. At what temperature will that ratio be equal to 1 : 10?

7.17. Taking into account the degeneracy of rotational levels (see Problem 7.5), determine the ratio of the number of hydrogen molecules in the states with quantum numbers $v' = 1$, $J' = 0$ to those with $v = 0$, $J = 5$ at $T = 1500$ K.

7.18. Derive formula (7.3).

7.19. Using formula (7.3), calculate the temperature at which: **(a)** the mean vibrational energy of a Cl_2 molecule is twice the zero vibrational energy; **(b)** the level corresponding to the mean vibrational energy of an O_2 molecule coincides with the fifth rotational level of that molecule ($J = 5$, $v = 0$).

7.20. Proceeding from formula (7.3) derive the expression for molar vibrational heat capacity of diatomic gas at constant volume. Find the approximate forms of that expression for low and high temperatures ($kT \ll \hbar\omega$ and $kT \gg \hbar\omega$).

7.21. Using formula (7.3), calculate the molar vibrational heat capacity at constant volume for a gas consisting of Cl_2 molecules at temperatures of 150, 300, and 450 K. Plot the approximate graph of $C_{vib}(T)$.

MOLECULAR SPECTRA. RAMAN EFFECT

7.22. Demonstrate that the intervals (expressed in terms of wave numbers) between the neighbouring spectral lines of the true rotational spectrum of a diatomic molecule are of the same value. Find the moment of inertia of a CH molecule and the distance between its nuclei, if the intervals between the neighbouring lines of the true rotational spectrum of these molecules are equal to $\Delta\bar{\nu} = 29.0$ cm^{-1}.

7.23. The wavelengths of two neighbouring lines of the true rotational spectrum of HCl molecules are equal to 117 and 156 μm. Determine: **(a)** the rotational constant B' in cm^{-1} units and the moment of inertia of these molecules; **(b)** the rotation quantum numbers of the levels between which occur the transitions corresponding to these lines.

7.24. Determine by what amount the angular momentum of a CO molecule changes on emission of a spectral line $\lambda = 1.29$ mm belonging to the true rotational spectrum.

7.25. How many lines are there in a true rotational spectrum of an OH molecule?

7.26. In the vibration-rotation absorption spectrum of HBr molecules, the wavelengths of the zero lines corresponding to forbidden transitions ($\Delta J = 0$) between the ground level and the closest vibrational levels ($v = 0$ and $v' = 1, 2$) are equal to 2559.3 and 5028.2 cm^{-1}. Determine the vibration frequency and anharmonicity coefficient of these molecules.

7.27. Consider the vibration-rotation band of spectrum of a diatomic molecule for which the selection rule $\Delta J = \pm 1$ is valid. Show that if the rotational constant is the same for the states between which the transition occurs, the spectral line frequencies of the band are

$$\omega = \omega_0 \pm 2Bk; \quad k = 1, 2, 3, \ldots,$$

where ω_0 is the frequency of the zero line forbidden by the J selection rule, B is the rotational constant.

7.28. Calculate the moment of inertia and anharmonicity coefficient of an HF molecule, if the wave numbers of four consecutive spectral lines of rotational structure of vibration band are equal to 3874, 3916, 4000, and 4042 cm^{-1}. These lines are known to correspond to the transitions $\Delta J = \pm 1$ and $v' = 1 \rightarrow v = 0$. The vibration frequency of the given molecule corresponds to the wave number $\bar{\nu} = 4138.5\ cm^{-1}$. The rotational constant is assumed to be equal for all levels.

7.29. Find the fractional isotope shift ($\Delta\lambda/\lambda$) of lines in the true rotational spectrum of the mixture of $H^{35}Cl$ and $H^{37}Cl$ molecules.

7.30. Consider the spectral line caused by the transition $v' = 1$, $J' = 0 \rightarrow v = 0$, $J = 1$ in CO molecules. Calculate in terms of wave numbers the isotope shifts $\Delta\bar{\nu}_{vib}$ and $\Delta\bar{\nu}_{rot}$ of vibrational and rotational components of the line ($\bar{\nu} = \bar{\nu}_{vib} - \bar{\nu}_{rot}$) and their ratio for the mixture of $^{12}C^{16}O$ and $^{12}C^{17}O$ molecules. The anharmonicity is to be neglected.

7.31. Find the vibration frequency and quasielastic force constant of an S_2 molecule, if the wavelengths of the red and violet satellites, closest to the fixed line, in the vibrational spectrum of Raman scattering are equal to 3466 and 3300 Å. The anharmonicity is to be neglected.

7.32. Determine the vibration frequency of an HF molecule, if in the vibrational spectrum of Raman scattering the difference in wavelengths of the red and violet satellites, closest to the fixed line, is equal to $\Delta\lambda = 1540$ Å. The wavelength of the incident light is $\lambda = 4350$ Å. The anharmonicity coefficient of the molecule is $x = 0.0218$.

7.33. Find the ratio of intensities of the violet and red satellites, closest to the fixed line, in the vibrational spectrum of Raman scattering by Cl_2 molecules at a temperature of $T = 300$ K. By what factor will this ratio change, if the temperature is doubled?

7.34. Suppose that for certain molecules the selection rule for the rotational quantum number is $\Delta J = \pm 1$. Demonstrate that the rotational spectrum of Raman scattering of these molecules obeys the selection rule $\Delta J = 0, \pm 2$.

7.35. In the rotational spectrum of Raman scattering the frequencies of red and violet satellites of diatomic molecules (with the selection rule being $\Delta J = 0, \pm 2$) are described by the formula

$$\omega = \omega_0 \pm 2B\,(2k + 1); \quad k = 1, 2, 3, \ldots,$$

where ω_0 is the fixed line frequency and B rotational constant.

(a) Derive this formula.

(b) Determine the moment of inertia and the nuclear separation in an O_2 molecule, if the difference in wave numbers of the two neighbouring red satellites is equal to 5.8 cm^{-1}.

7.36. In the rotational spectrum of Raman scattering of light with wavelength $\lambda_0 = 5461$ Å the difference in wavelengths of the red and violet satellites, closest to the fixed line, equals $\Delta\lambda = 7.2$ Å for N_2 molecules. Bearing in mind the selection rule $\Delta J = 0, \pm 2$, find the rotational constant B' in cm^{-1} units and moment of inertia of these molecules.

8

CRYSTALS

• Period of identity is a distance between the neighbouring identical atoms along a certain direction in a lattice.
• Interplanar distance in a simple cubic lattice

$$d = a/\sqrt{h^2+k^2+l^2}, \tag{8.1}$$

where a is the lattice constant, h, k, l are the Miller indices of the considered system of planes.
• Bragg's equation

$$2d \sin \vartheta = n\lambda, \tag{8.2}$$

where ϑ is the glancing angle, n is the reflection order, λ is the wavelength.
• Conditions under which the reflections of the nth order are possible from the set of planes $(h^*k^*l^*)$, where $h^* = nh$, $k^* = nk$, $l^* = nl$: in the case of space-centered lattice the sum of h^*, k^*, and l^* is even; in the case of face-centered lattice the indices h^*, k^*, and l^* must possess parity.

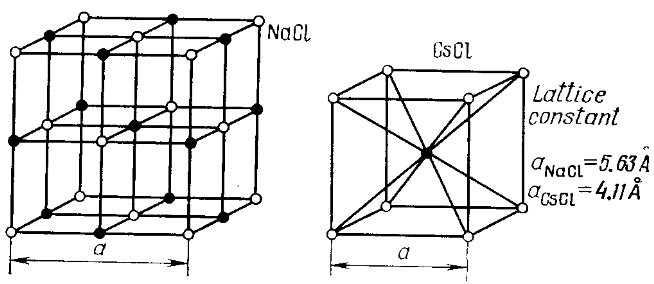

Fig. 25

• Born-Landé formula, defining the binding energy of ionic crystals as related to a pair of ions of opposite charge

$$U = -\alpha\frac{q^2}{r} + \frac{\beta}{r^n}, \qquad \alpha = \begin{cases} 1.748 \text{ for a NaCl type lattice} \\ 1.763 \text{ for a CsCl type lattice,} \end{cases} \tag{8.3}$$

where q is the ionic charge, r is the smallest distance between the ions of opposite charge, α, β, and n are constants (α is the Madelung constant). The structure of NaCl and CsCl crystals is illustrated in Fig. 25.
• Compressibility (due to hydrostatic pressure)

$$K = -\frac{1}{V}\frac{dV}{dp}, \tag{8.4}$$

where V is the volume of a crystal, p is the pressure.

• Debye equation for molar vibrational energy of a crystal:

$$E = 9R\theta\left[\frac{1}{8} + \left(\frac{T}{\theta}\right)^4 \int_0^{\theta/T} \frac{x^3\,dx}{e^x-1}\right]; \qquad \theta = \frac{\hbar\omega_{max}}{k}, \tag{8.5}$$

where R is the universal gas constant, θ is the Debye characteristic temperature, ω_{max} is the maximum vibration frequency calculated from the condition that the total number of vibrations is equal to the number of vibrational degrees of freedom in a crystal.

Molar vibrational heat capacity of a crystal at $T \ll \theta$:

$$C = \frac{12}{5}\pi^4 R\left(\frac{T}{\theta}\right)^3. \tag{8.6}$$

CRYSTALLINE STRUCTURE. X-RAY DIFFRACTION

8.1. Knowing the density and crystal type, determine the lattice constant of sodium and copper.

8.2. Find the density of NaCl and CsCl crystals (see Fig. 25).

8.3. Derive formula (8.1).

8.4. Knowing the lattice constant a, calculate the interplanar distances d_{100}, d_{110}, d_{111} and their ratios for: **(a)** simple; **(b)** space-centered; **(c)** face-centered cubic lattices.

8.5. Calculate the periods of identity along the straight lines [111] and [011] in the crystalline lattice of AgBr whose density is 6.5 g/cm³. The lattice in question is of the cubic NaCl type.

8.6. Determine the ratio of periods of identity along the directions [100], [110], and [111] for the simple, space-centered, and face-centered cubic lattices.

8.7. Determine the structure of an elementary cell of a crystal belonging to the cubic system with 4-fold symmetry axis, if the interplanar distance for the set of planes (100) is known to be equal to $d_1 = 1.58$ Å and for the planes (110) $d_2 = 2.23$ Å. The density of the crystal is 19.3 g/cm³.

8.8. A parallel X-ray beam with the wavelength λ falls in an arbitrary direction on a plane rectangular net with periods a and b. What pattern will be observed on a screen positioned parallel to the plane net? Find the directions to the diffraction maxima.

8.9. A plane X-ray beam falls on a three-dimensional rectangular lattice with periods a, b, c. Find the directions to the diffraction maxima, if the incident beam direction is parallel to the edge a of the elementary cell. For which wavelengths will the maxima be observed?

8.10. A plane X-ray beam falls in an arbitrary direction on a simple cubic lattice with constant a. At which wavelengths are the diffraction maxima observable?

8.11. Using a simple cubic lattice as an example, demonstrate that Bragg's equation follows from Laue's equations.

8.12. Find the lattice constant for AgBr (of a NaCl lattice type), if the K_α line of vanadium is known to form the first-order reflection from the set of planes (100) with glancing angle $\vartheta = 25.9°$.

8.13. Calculate the wavelength of X-rays forming the second-order reflections from the set of planes (100) in a NaCl crystal (see Fig. 25) with glancing angle $\vartheta = 25.0°$. Find also the angle at which these X-rays form the highest order reflections from the given set of planes.

8.14. A NaCl single crystal (see Fig. 25) is photographed by Laue's method along the four-fold axis (z axis). The photoplate is located at a distance $L = 50$ mm from the crystal. Find for the maxima corresponding to reflections from the planes (031) and (221): **(a)** their distances from the centre of Laue's diagram; **(b)** the wavelengths of X-rays.

8.15. A beam of X-rays of wavelength λ falls on a NaCl crystal (see Fig. 25) rotating about the four-fold symmetry axis, with the incident beam direction being at right angles to the rotation axis. Determine the value of λ if the directions to the maxima of the second and the third order formed by the set of planes (100) make an angle $\alpha = 60°$.

8.16. A beam of X-rays with a wavelength $\lambda = 0.71$ Å falls on a rotating single crystal of metal located on the axis of cylindrical

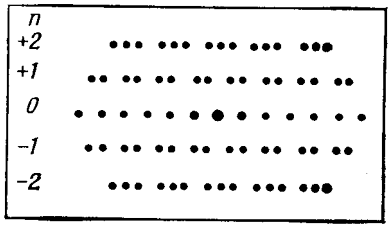

Fig. 26

photographic camera of radius 57.3 mm. The incident beam direction is perpendicular to the rotation axis (the camera's axis). The X-ray pattern comprises the system of maxima distributed over the layer lines (Fig. 26). Determine the type of the metal's cubic lattice (space- or face-centered) and find its constant a, if the distance between the layer lines $n = 2$ and $n = -2$ is equal to 65.0 and 23.5 mm respectively, the rotation being performed about directions [110] and [111].

8.17. What orders of monochromatic X-ray reflection will disappear on transition from the simple cubic lattice to space- and face-centered ones? The lattice constants are assumed to be equal in all three cases. Consider the reflections from the planes (100), (110), and (111).

8.18. Find the values of the Miller indices h, k, l for the planes that provide the reflections forming the first five lines in a Debye powder photograph of the face- and space-centered cubic lattices.

8.19. Calculate the magnitude of diffraction angles 2ϑ for the first five lines in a Debye powder photograph of: **(a)** aluminium and **(b)** vanadium, if $\lambda = 1.54$ Å.

8.20. Determine the reflection indices h^*, k^*, l^* and the corresponding interplanar distances for three lines in Debye powder photograph of aluminium whose diffraction angles (2ϑ) are equal to 17°30′, 33°50′, and 54°20′, when $\lambda = 0.71$ Å.

8.21. A narrow beam of electrons with an energy of 25 keV passes through a thin polycrystalline film and forms a set of diffraction rings on a flat screen fixed at a distance $L = 20.0$ cm from the film. The diameter of the first ring is $D = 13.1$ mm. Calculate the lattice constant. The lattice is known to be a space-centered cubic one.

8.22. In an electron diffraction photograph of a polycrystalline film with cubic lattice, the diameter ratio of the first two diffraction rings is 1 : 1.4. Taking into account that the diameters of these rings are considerably smaller than the distance between the film and the screen, determine the type of the lattice (face- or space-centered).

BINDING ENERGY. HEAT CAPACITY OF CRYSTALS

8.23. Calculate the Madelung constant for a unidimensional crystal, that is, a chain of ions with alternating positive and negative charges. In calculations use the expansion of the function $\ln(1 + x)$ into a series.

8.24. Using formula (8.3), find: **(a)** the expression for the binding energy of ionic crystal at equilibrium; **(b)** the refractive index n of NaCl and CsCl crystals (see Fig. 25) whose binding energies at equilibrium are equal to 765 and 627 kJ/mol respectively.

8.25. A NaCl crystal with compressibility $K = 3.47 \cdot 10^{-11}$ Pa^{-1} was hydrostatically compressed so that its volume diminished by 1.0%. Find: **(a)** the pressure to which the crystal was subjected; **(b)** the increment of the volume density of crystal's binding energy.

8.26. The compressibility of a NaCl crystal at equilibrium (see Fig. 25) is equal to $K = 3.47 \cdot 10^{-11}$ Pa^{-1}. Using formula (8.3), calculate: **(a)** the refractive index n; **(b)** the binding energy of a mole of this crystal at equilibrium.

8.27. Calculate the values of the same quantities as in the foregoing problem for a CsCl crystal (see Fig. 25) whose compressibility is $K = 5.10 \cdot 10^{-11}$ Pa^{-1}.

8.28. A NaCl crystal (see Fig. 25) whose compressibility is equal to $K = 3.47 \cdot 10^{-11}$ Pa^{-1} at equilibrium was subjected to omnidirectional tension. Using formula (8.3), find how the distance between the ions increases when the crystal becomes expanded up to the theoretical tensile strength value (at which the negative pressure reaches the highest magnitude). What is the magnitude of that pressure?

8.29. Along with formula (8.3), another expression for binding energy of ionic crystal is frequently used:

$$U = -\alpha q^2/r + Ae^{-r/\rho},$$

where α and q have the same meaning, A and ρ are certain new constants. Using that formula, find: **(a)** the expression for the binding energy of an ionic crystal at equilibrium; calculate the constant ρ for a NaCl crystal (see Fig. 25) whose binding energy at equilibrium is equal to 765 kJ/mol; **(b)** the expression for compressibility of crystals with the NaCl type of the lattice at equilibrium.

8.30. Determine the vibrational energy and heat capacity of a crystal at the temperature T, treating each atom of the lattice as a quantum harmonic oscillator and assuming the crystal to consist of N identical atoms vibrating independently with the same frequency ω. Simplify the obtained expression for heat capacity for the cases $kT \gg \hbar\omega$ and $kT \ll \hbar\omega$.

8.31. Consider a unidimensional crystal model, a chain of N identical atoms, whose extreme atoms are stationary. Let a be the chain's period, m the mass of an atom, $\varkappa$ the quasielastic force constant. Taking into account only the interaction between the neighbouring atoms, find: **(a)** the oscillation equation of this chain and the spectrum of characteristic values of the wave number k; **(b)** the frequency dependence of the wave number and the total number of allowed vibrations; determine the highest vibration frequency and the corresponding wavelength; **(c)** the phase velocity as a function of the wave number and the ratio of phase velocities corresponding to the longest and shortest wavelengths; **(d)** the number of characteristic vibrations of the chain in the frequency interval $(\omega, \omega + d\omega)$.

8.32. Assuming the propagation velocity of vibrations to be independent of frequency and equal to v, find for a unidimensional crystal, that is, the chain of N identical atoms and length L: **(a)** the number of longitudinal vibrations in the frequency interval $(\omega, \omega + d\omega)$; **(b)** the characteristic temperature θ; **(c)** the molar vibrational energy and molar heat capacity at the temperature T; simplify the expression obtained for heat capacity for the cases when $T \gg \theta$ and $T \ll \theta$.

8.33. Assuming the propagation velocity of transverse and longitudinal vibrations to be the same, independent of frequency, and equal to v, find for a two-dimensional crystal, that is, a square plane net consisting of N atoms and having the area S: **(a)** the number of vibrations in the frequency interval $(\omega, \omega + d\omega)$; **(b)** the characteristic temperature θ; **(c)** the molar vibrational energy and molar heat capacity at the temperature T; simplify the expression obtained for the heat capacity for the cases when $T \gg \theta$ and $T \ll \theta$.

8.34. Find the values of the quantities of the foregoing problem from a three-dimensional crystal, that is, a cubic lattice consisting of N identical atoms. The volume of the lattice is V.

8.35. Assuming the propagation velocities of longitudinal and transverse vibrations to be independent of frequency and equal to v_l and v_t respectively, find the number of vibrations dZ in the frequency interval $(\omega, \omega + d\omega)$ and the characteristic temperature θ: **(a)** of a two-dimensional crystal (a plane net consisting of N identical atoms); the area of the plane net is S; **(b)** of a three-dimensional crystal (a cubic lattice of N identical atoms and volume V).

8.36. Calculate the characteristic temperature for iron in which the propagation velocities of longitudinal and transverse vibrations are equal to 5850 and 3230 m/s respectively.

8.37. Using the Debye equation, calculate: **(a)** the ratio $\Delta E/E_0$, where ΔE is the energy required to heat a crystal from 0 K up to θ; E_0 is the zero vibrational energy; **(b)** the energy required to heat a mole of aluminium crystal from $\theta/2$ up to θ.

8.38. Using the Debye equation, calculate the molar heat capacity of a crystal lattice at temperatures $\theta/2$ and θ. By how many percents does the heat capacity at temperature θ differ from the classical value?

8.39. Calculate the characteristic temperature and zero vibrational energy (in units of J/mol) for silver, if its heat capacity at temperatures 16 and 20 K is known to be equal to 0.87 and 1.70 $J \cdot K^{-1} \times mol^{-1}$ respectively.

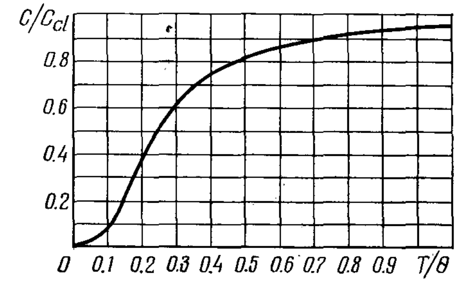

Fig. 27

8.40. Figure 27 illustrates the temperature dependence of the heat capacity of a crystal (according to Debye). C_{cl} is the classical heat capacity, θ is the characteristic temperature. Using this graph, find:

(a) the characteristic temperature for silver, if its molar heat capacity equals 15 $J \cdot K^{-1} \cdot mol^{-1}$ at $T = 65$ K;

(b) the molar heat capacity of aluminium at $T = 100$ K, if it equals 22.5 $J \cdot K^{-1} \cdot mol^{-1}$ at $T = 280$ K;

(c) the highest vibration frequency ω_{max} for copper whose heat capacity at $T = 125$ K differs from the classical value by 25%.

8.41. Evaluate the maximum values of energy and momentum of a phonon (acoustic quantum) in aluminium.

8.42. In a crystal consisting of N identical atoms the number of phonons in the frequency interval $(\omega, \omega + d\omega)$ at the temperature T is equal to

$$n(\omega)\,d\omega = 9N\left(\frac{\hbar}{k\theta}\right)^3 \frac{\omega^2\,d\omega}{e^{\hbar\omega/kT}-1},$$

where θ is the characteristic temperature of the crystal.

(a) Derive this expression, using the formula for dZ_ω obtained in the solution of Problem 8.34.

(b) Determine the most probable values of energy and frequency of phonons at the temperature $\theta/2$.

(c) Find the temperature, beginning from which the most probable frequency of phonons becomes equal to their maximum frequency. The temperature θ is assumed to be known.

(d) Find the character of temperature dependence of the total number of phonons when $T \ll \theta$ and $T \gg \theta$.

8.43. The scattering of light by a transparent solid can be treated as a scattering of photons by phonons, assuming that photons possess the momentum $\hbar\omega/c'$ in a substance, where c' is the velocity of light in that substance. Using the laws of conservation of energy and momentum, demonstrate that the light scattered through angle ϑ contains, in addition to the fixed component, two more components with fractional shift $\Delta\omega/\omega = 2(v/c')\sin(\vartheta/2)$, where ω is the incident light frequency, and v the sonic velocity in the substance.

8.44. At cryogenic temperatures some substances (e.g. paramagnetic salts) possess a heat capacity C_i which exceeds a lattice heat capacity C_{lat} many-fold. The heat capacity C_i has been found depending on internal degrees of freedom, in particular, on the interaction of spins with intracrystalline fields. Assuming that each atom independently orients its spin either parallel or antiparallel to a certain direction and the difference in energies of the atom in these states equals ΔE, find: **(a)** the temperature dependence of C_i; **(b)** the value of the ratio $kT/\Delta E$ at which C_i reaches a maximum; **(c)** the ratio $C_{i\,max}/C_{lat}$ for the case when $C_{i\,max}$ corresponds to a temperature $T = \theta/100$; θ is the characteristic temperature.

9

METALS AND SEMICONDUCTORS

● Concentration of free electrons with energies falling into the interval $(E, E + dE)$

$$n(E)\,dE = f(E)\,g(E)\,dE = \frac{\sqrt{2m^3}}{\pi^2\hbar^3}\,\frac{\sqrt{E}\,dE}{1+e^{(E-E_f)/kT}}, \qquad (9.1)$$

where $f(E) = [1 + e^{(E-E_f)/kT}]^{-1}$ is called the Fermi-Dirac function, $g(E)$ is the density of states, E_f is the Fermi level. For metals

$$E_f = E_{f0}\left[1 - \frac{\pi^2}{12}\left(\frac{kT}{E_{f0}}\right)^2\right]; \quad E_{f0} = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3},$$

where E_{f0} is the Fermi level at 0 K, n is the concentration of free electrons.

In the above formulas the energies E and E_f are counted off the bottom of the conduction band.

● Hall coefficient for a semiconductor

$$R = \frac{E_\perp}{jB} = \frac{1}{ec}\,\frac{n_e b_e^2 - n_h b_h^2}{(n_e b_e + n_h b_h)^2}, \qquad (9.2)$$

where e is the electronic charge, n_e and n_h are the concentrations of electrons and holes, b_e and b_h are their mobilities, B is the magnetic induction.

FERMI DISTRIBUTION

9.1. Using the uncertainty principle, find the number of free electrons whose kinetic energies fall within the interval $(T, T + dT)$ in a metal at 0 K. The metal is shaped as a rectangular parallelepiped of volume V. The number of quantum states is to be determined under the assumption that only those states can be physically distinguished whose electron momentum projections differ at least by $\Delta p_x = 2\pi\hbar/l_x$, l_x being the edge of the parallelepiped (similarly, for Δp_y and Δp_z).

9.2. Using the Fermi distribution, derive an expression for the highest kinetic energy T_{max} of free electrons in a metal at 0 K, if their concentration is equal to n. Calculate T_{max} for silver, assuming one free electron to correspond to each atom.

9.3. Using the Fermi distribution, find at 0 K: **(a)** the mean kinetic energy of free electrons in a metal, if their highest kinetic energy T_{max} is known; **(b)** the total kinetic energy of free electrons in 1 cm³ of gold, assuming one free electron to correspond to each atom.

9.4. What fraction of free electrons in a metal at 0 K has a kinetic energy exceeding half the maximum energy?

9.5. Calculate the temperature of an ideal gas consisting of particles whose mean kinetic energy is equal to that of free electrons in copper at 0 K. Only one free electron is supposed to correspond to each copper atom.

9.6. Calculate the interval (in eV units) between the neighbouring levels of free electrons in a metal at 0 K near the Fermi level, if the volume of the metal is $V = 1.00\ cm^3$ and the concentration of free electrons is $2.0 \cdot 10^{22}\ cm^{-3}$.

9.7. The difference in the values of E_f and E_{f0} is frequently neglected in calculations. Evaluate by how many percents E_f and E_{f0} differ in the case of tungsten at the temperature of its melting. Assume that there are two free electrons per each atom.

9.8. For a metal at 0 K whose free electrons can reach the highest velocity v_m, find the mean values of: (a) the velocity of free electrons; (b) the reciprocal of their velocity, $1/v$.

9.9. Calculate the most probable and mean velocities of free electrons in copper at 0 K, if their concentration is $8.5 \cdot 10^{22}\ cm^{-3}$.

9.10. Using a simple cubic lattice as an example, demonstrate that, if one free electron corresponds to each atom, the shortest de Broglie wavelength of such electrons is approximately double the distance between the neighbouring atoms.

9.11. Derive the function defining the distribution of free electrons over the de Broglie wavelengths in a metal at 0 K. Draw the graph.

9.12. The mean energy of free electrons in a metal at the temperature T is equal to

$$\langle E \rangle = \frac{3}{5} E_{f0} \left[1 + \frac{5\pi^2}{12} \left(\frac{kT}{E_{f0}} \right)^2 \right].$$

Using this formula, find for silver with free electron concentration $6.0 \cdot 10^{22}\ cm^{-3}$: (a) the ratio of heat capacities of an electron gas and a crystal lattice at a temperature of $T = 300$ K; (b) the temperature at which the heat capacity of electron gas equals that of the lattice.

9.13. The free electron concentration in metallic sodium is $n = 2.5 \cdot 10^{22}\ cm^{-3}$. Find the electron gas pressure p; demonstrate that $p = (2/3) E$, where E is the volume density of its kinetic energy.

9.14. The number of free electrons in a metal which fall within velocity interval $(\mathbf{v}, \mathbf{v} + d\mathbf{v})$ is defined as follows:

$$n(\mathbf{v})\, d\mathbf{v} = 2 \left(\frac{m}{2\pi\hbar} \right)^3 \frac{d\mathbf{v}}{1 + e^{(E - E_f)/kT}}; \quad d\mathbf{v} = dv_x\, dv_y\, dv_z.$$

(a) Derive the above expression from formula (9.1).

(b) Find the concentration of electrons whose velocity projections fall within interval $(v_x, v_x + dv_x)$ at 0 K, if the highest velocity of free electrons is equal to v_m.

9.15. Using the formula of the foregoing problem, prove that, when two different metals are in contact, their Fermi levels reach the same height.

9.16. Using the formula of Problem 9.14, show that the number of electrons leaving a metallic surface (due to thermionic emission) of 1 cm² are per 1 s with the velocities falling within interval $(v, v + dv)$ is equal to

$$\nu(v)\, dv = 2\pi (m/2\pi\hbar)^3 e^{-(A + T_e)/kT} v^3\, dv,$$

where T_e is the kinetic energy of the electron, A is the work function. Take into account that $A \gg kT$.

9.17. Using the formula of the foregoing problem, find: (a) the mean kinetic energy of thermionic emission electrons; (b) the thermionic current density; (c) the work function, if the increase in temperature from 1500 K to 2000 K leads to the increase of thermionic current $5.0 \cdot 10^3$ times.

9.18. Having determined the concentrations of free electrons and holes, demonstrate that at sufficiently low temperatures the Fermi level in an impurity-free semiconductor is in the middle of the forbidden band.

9.19. At sufficiently low temperatures the concentration of free electrons in a semiconductor of n type is

$$n_e = \sqrt{2n_0}\, (mkT/2\pi\hbar^2)^{3/4} e^{-\Delta E/2kT},$$

where n_0 is the concentration of donor atoms and ΔE their activation energy.

(a) Derive this expression using the Fermi distribution.

(b) Find the location of the Fermi level.

PROPERTIES OF METALS AND SEMICONDUCTORS

9.20. The electric conductance of a metal $\sigma = ne^2\tau/m$, where n is the free electron concentration; e and m are the electronic charge and mass; τ is the relaxation time which is related to the electron's mean free path as $\langle\lambda\rangle = \tau \langle v \rangle$; $\langle v \rangle$ is the mean velocity of electrons. Calculate τ, $\langle\lambda\rangle$ and the free electron mobility, if $n = 8.5 \cdot 10^{22}\ cm^{-3}$ and resistivity $\rho = 1.60 \cdot 10^{-6}\ \Omega \cdot cm$. Compare the obtained value of $\langle\lambda\rangle$ with the interatomic distance in copper.

9.21. Find the refractive index of metallic sodium for electrons with kinetic energy $T = 135$ eV. Only one free electron is assumed to correspond to each sodium atom.

9.22. Suppose that due to a certain reason the free electrons shift by the distance x at right angles to the surface of a flat metallic layer. As a result, a surface charge appears together with restoring force which brings about the so-called plasma oscillations. Determine the frequency of these oscillations in copper whose free electron concentration is $n = 8.5 \cdot 10^{22}\ cm^{-3}$. How high is the energy of plasma waves in copper?

9.23. Experiments show that alkali metals are transparent to ultraviolet radiation. Using the model of free electrons, find the threshold wavelength of light beginning from which that phenomenon is observed in the case of metallic sodium (whose free electron concentration is $n = 2.5 \cdot 10^{22}$ cm^{-3}).

9.24. The alkali metals exhibit temperature-independent paramagnetic properties which can be explained as follows. On application of the external magnetic field **B**, the free electrons with spins oriented oppositely to the vector **B** start reorienting along it and, in accordance with the Pauli principle, promoting to higher non-occupied levels. This process will proceed until the decrease in the magnetic energy of electrons equalizes the increase in their kinetic energy. From this condition find the paramagnetic susceptibility of a metal of 1 cm^3 volume in a weak magnetic field, if the free electron concentration is $n = 2.0 \cdot 10^{22}$ cm^{-3}.

9.25. The photoconduction limit in impurity-free germanium is equal to $\lambda_0 \approx 1.7$ µm at very low temperatures. Calculate the temperature coefficient of resistance of this semiconductor at $T = 300$ K.

9.26. Find the lowest energy of electron-hole pair formation in pure tellurium at 0 K whose electric conductance increases $\eta = 5.2$ times, when the temperature is raised from $T_1 = 300$ K to $T_2 = 400$ K.

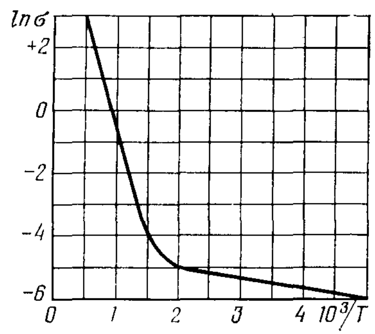

Fig. 28

9.27. Figure 28 illustrates the logarithmic electric conductance as a function of reciprocal temperature (T in Kelvins) for boron-doped silicon (n-type semiconductor). Explain the shape of the graph. By means of the graph find the width of the forbidden band in silicon and activation energy of boron atoms.

9.28. A sample of impurity-free germanium, whose forbidden band width is 0.72 eV and electron and hole mobilities are 3600 and 1800 cm^2/(V·s), is exposed to an electromagnetic radiation field at 300 K. Under these conditions the sample's resistivity equal 43 Ω·cm. Determine the fraction of the electric conductance caused by photoconduction. *Instruction.* Make use of the solution of Problem 9.18.

9.29. The resistivity of impurity-free germanium at room temperature $\rho = 50$ Ω·cm. It becomes equal to $\rho_1 = 40$ Ω·cm, when the semiconductor is illuminated with light, and $t = 8$ ms after switching off the light source, the resistivity becomes equal to $\rho_2 = 45$ Ω·cm. Find the mean lifetime of conduction electrons and holes.

9.30. Using the formula cited in Problem 9.19, calculate the activation energy of donor atoms in an n-type semiconductor, if the electron mobility is known to be equal to 500 cm^2/(V·s), concentration of donor atoms to $5.0 \cdot 10^{17}$ cm^{-3}, and resistivity at 50 K to 1.5 kΩ.

9.31. A sample of n-type germanium has a resistivity $\rho = 1.70$ Ω·cm and a Hall coefficient $R = 7.0 \cdot 10^{-17}$ CGSE at the temperature $T = 300$ K. Find: **(a)** the concentration and mobility of conduction electrons; **(b)** their mean free path.

9.32. In the Hall effect measurements a plate of width $d = 1.0$ cm and length $l = 5.0$ cm made of p-type semiconductor was placed in the magnetic field $B = 5.0$ kG. A potential difference $U = 10.0$ V was applied across the edges of the plate. In this case the Hall field is $V = 0.050$ V and resistivity $\rho = 2.5$ Ω·cm. Determine the Hall coefficient, concentration of holes and hole mobility.

9.33. Having considered the motion characteristics of electrons and holes in a semiconductor carrying a current and placed in an external magnetic field, find the Hall coefficient as a function of concentration and mobility of charge carriers.

9.34. Calculate the difference in the mobilities of conduction electrons and holes in impurity-free germanium, if in a magnetic field $B = 3.0$ kG the ratio of the transverse electric field strength $E_\perp$ to the longitudinal one, E, is known to be equal to 0.060.

9.35. The Hall effect could not be observed in a germanium sample whose conduction electron mobility is 2.1 times that of holes. For this sample find: **(a)** the ratio of conduction electron and hole concentrations; **(b)** what fraction of electric conductance is effected by electrons.

10

PRINCIPAL CHARACTERISTICS OF NUCLEI

- Radius of nucleus with mass number A:

$$R = 1.4A^{1/3} \cdot 10^{-13} \text{ cm}. \qquad (10.1)$$

- Binding energy of nucleus (in mass units):

$$E_b = Zm_H + (A - Z)\,m_n - M, \qquad (10.2)$$

where Z is the atomic number of nucleus, A is the mass number, m_H, m_n, and M are the masses of hydrogen, neutron, and the given atom. In calculations one can use the more convenient formula

$$E_b = Z\Delta_H + (A - Z)\,\Delta_n - \Delta, \qquad (10.3)$$

where Δ_H, Δ_n, and Δ are the mass surpluses of a hydrogen atom, a neutron, and an atom corresponding to the given nucleus.

- Semi-empirical formula for the binding energy of a nucleus:

$$E\,(\text{MeV}) = 14.0A - 13.0A^{2/3} - 0.584\frac{Z^2}{A^{1/3}} - 19.3\frac{(A-2Z)^2}{A} + \frac{33.5}{A^{3/4}}\delta; \qquad (10.4)$$

$$\delta = \begin{cases} +1 & \text{when } A \text{ and } Z \text{ are even,} \\ 0 & \text{when } A \text{ is odd (for any } Z), \\ -1 & \text{when } A \text{ is even and } Z \text{ is odd.} \end{cases}$$

- Total angular momentum of an atom:

$$\mathbf{F} = \mathbf{J} + \mathbf{I},$$

$$F = J + I,\ J + I - 1,\ \ldots,\ |J - I|, \qquad (10.5)$$

where $\mathbf{J}$ is the angular momentum of the atom's electron shell, $\mathbf{I}$ is the spin of the nucleus. For allowed transitions

$$\Delta F = 0,\ \pm 1; \qquad F = 0 \not\to F = 0.$$

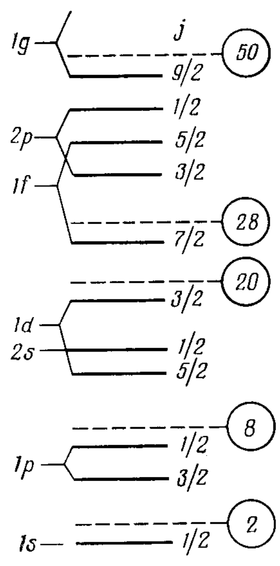

Fig. 29

- Magnetic moment of a nucleus (or rather its maximum projection):

$$\mu = gI\mu_N, \qquad (10.6)$$

where g is the gyromagnetic factor, I is the spin of a nucleus, μ_N is the nuclear magneton.

- Model of nuclear shells (Fig. 29). Here j is the quantum number of the nucleon's total angular momentum; the encircled numbers indicate the number of nucleons of one sort (either protons or neutrons) which fill up all the levels lying below the corresponding dotted line, a shell's boundary. The protons and neutrons fill up the levels independently and in accordance with the Pauli principle.

RADIUS, MASS, AND BINDING ENERGY OF NUCLEI

10.1. Evaluate the density of nuclear matter, nucleon concentration, and volume density of electric charge in a nucleus.

10.2. The scattering of protons by a thin lead foil obeys the Rutherford formula provided the values $B\rho$ for protons do not exceed 450 kG·cm. Evaluate the radius of a lead nuclei.

10.3. The results of experiments on scattering fast electrons by nuclei agree well enough with the distribution of electric charge volume density in a nucleus:

$$\rho(r) \propto [1 + e^{(r-r_0)/\delta}]^{-1},$$

where $r_0 = 1.08A^{1/3} \cdot 10^{-13}$ cm, $\delta = 0.545 \cdot 10^{-13}$ cm. Find the most probable radius of distribution of electric charge in an Ag nucleus. Compare the result obtained with the nucleus' radius.

10.4. In the modern system of atomic masses a unit mass is adopted to be 1/12 of the mass of a ^{12}C atom (to replace the old unit mass equal to 1/16 of the mass of an ^{16}O atom). Find the relation between the new and the old unit masses. How did the numerical values of atomic masses change on adoption of the new unit?

10.5. Find the percentage (atomic and mass) of the ^{13}C isotope in natural carbon which consists of ^{12}C and ^{13}C isotopes. The atomic mass of natural carbon and of both isotopes are supposed to be known.

10.6. Find the atomic masses of ^1H, ^2H, and ^{16}O isotopes, if the differences in masses of the three fundamental doublets are known (in a.m.u.'s): $^1H_2 - {}^2H = 0.001548$; $^2H_3 - {}^1\!/_2{}^{12}C = 0.042306$; $^{12}C^1H_4 - {}^{16}O = 0.036386$.

10.7. Using formula (10.3), find:

(a) the binding energy of a nucleus possessing an equal number of protons and neutrons and a radius 2/3 of that of ^{27}Al nucleus;

(b) the binding energy per nucleon in ^6Li, ^{40}Ar, ^{107}Ag, and ^{208}Pb nuclei.

10.8. Determine: **(a)** the binding energy of a neutron and α-particle in a ^{21}Ne nucleus; **(b)** the energy required to split an ^{16}O nucleus into four identical particles.

10.9. Find the excitation energy of a ^{207}Pb nucleus appearing on the capture of a slow neutron by a ^{206}Pb nucleus.

10.10. Calculate the binding energy of a neutron in a ^{14}N nucleus, if the binding energies of ^{14}N and ^{13}N nuclei are known to be equal to 104.66 and 94.10 MeV.

10.11. Find the energy required to split an ^{16}O nucleus into an α-particle and ^{12}C nucleus, if the binding energies of ^{16}O, ^{12}C, and ^4He nuclei are known to be equal to 127.62, 92.16, and 28.30 MeV.

10.12. Find the energy liberated on the formation of two α-particles as a result of fusion of ^2H and ^6Li nuclei, if the binding energy per nucleon in ^2H, ^4He, and ^6Li nuclei are known to be equal to 1.11, 7.08, and 5.33 MeV respectively.

10.13. Demonstrate that in the case of uniform distribution of charge over the volume of a nucleus, the energy of the Coulomb repulsion of protons is equal to $U_C = 0.6Z^2e^2/R$, where Z and R are the charge and the radius of the nucleus.

10.14. Calculate the difference in binding energies of mirror nuclei ^{33}S and ^{33}Cl, if the mass of ^{33}Cl is known to exceed the mass of ^{33}S atom by 0.00599 a.m.u. Compare the obtained value with the difference in energies of the Coulomb repulsion of protons in these nuclei (see the formula in the foregoing problem). Explain the coincidence of the results.

10.15. Assuming the difference in binding energies of mirror nuclei ^{23}Na and ^{23}Mg to be determined only by the difference in energies of the Coulomb repulsion in these nuclei (see the formula in Problem 10.13), calculate their radii. Compare the obtained result with the result calculated from the formula for nucleus' radius.

10.16. Using the semi-empirical formula, calculate: **(a)** the binding energies of ^{40}Ca and ^{107}Ag nuclei; **(b)** the binding energies per nucleon in ^{50}V and ^{200}Hg nuclei; **(c)** the masses of ^{45}Sc and ^{70}Zn atoms.

10.17. Using formula (10.4) determine the charge of a nucleus whose mass is the smallest among nuclei with the same odd value of mass number A. Using the obtained formula, predict the character of activity (either electron or positron) of the following β-active nuclei: ^{103}Ag, ^{127}Sn, and ^{141}Cs.

SPIN AND MAGNETIC MOMENT OF A NUCLEUS

10.18. Determine the number of hyperfine structure components in the basic term of the following atoms: ^3H ($^2S_{1/2}$), ^6Li ($^2S_{1/2}$), ^9Be ($^1S_0$), ^{15}N ($^4S_{3/2}$), and ^{35}Cl ($^2P_{3/2}$). The basic term of electron shell is indicated in parentheses.

10.19. Determine the spin of a ^{59}Co nucleus whose basic atomic term $^4F_{9/2}$ possesses eight components of hyperfine splitting.

10.20. Find the number of components of hyperfine spitting of the spectral lines $^2P_{1/2} \rightarrow {}^2S_{1/2}$ and $^2P_{3/2} \rightarrow {}^2S_{1/2}$ for ^{39}K atoms. The nuclear spin is supposed to be known.

10.21. Two terms of an atom have different values of the quantum number J (J_1 and J_2). What quantum number (J or I) can be determined from the number N of the components of hyperfine splitting of each term in the case when the numbers of the components of both terms are: **(a)** equal ($N_1 = N_2$); **(b)** different ($N_1 \neq N_2$)?

10.22. The intensities of hyperfine components of the spectral line $^2P_{1/2} \rightarrow {}^2S_{1/2}$ in sodium relate approximately as 10 : 6. Taking into account that the hyperfine structure emerges due to splitting of the $^2S_{1/2}$ term (the splitting of the $^2P_{1/2}$ term is negligible), find the spin of ^{23}Na nucleus.

10.23. The electron shell of an atom produces at the nucleus' site the magnetic field B_0 whose direction coincides with that of the angular momentum **J** of the electron shell. An additional energy of interaction of the nucleus' magnetic moment with that field depends on orientation of the angular momenta **J** and **I** which is specified by the spatial quantization rules. Proceeding from these concepts demonstrate that the intervals between the neighbouring sublevels defined by the quantum numbers F, $F+1$, $F+2$, ... relate as $(F+1):(F+2): \ldots$

10.24. The $^2D_{3/2}$ term of a ^{209}Bi atom has four components of hyperfine splitting with the ratio of the intervals between the neighbouring components being equal to 4 : 5 : 6. By means of the rule of the intervals (see the foregoing problem) find the nuclear spin and the number of the components of hyperfine splitting for the line $^2S_{1/2} \rightarrow {}^2D_{3/2}$.

10.25. The hyperfine structure sublevels of the $^2P_{3/2}$ term in a ^{35}Cl atom experience splitting in a *weak* magnetic field. Find the total number of the Zeeman components.

10.26. In a strong magnetic field each sublevel of the $^2S_{1/2}$ term in ^{42}K and ^{85}Rb splits into five and six components respectively. Find the nuclear spins of these atoms.

10.27. Calculate the angular precession velocities of an electron, proton, and neutron in a magnetic field $B = 1000$ G.

10.28. In studies of magnetic properties of ^{25}Mg atoms in the ground $^2S_0$ state by the magnetic resonance method, the resonance energy absorption is observed at a constant magnetic field $B = 5.4$ kG and a frequency of a.c. magnetic field $\nu_0 = 1.40$ MHz. Determine the gyromagnetic ratio and nuclear magnetic moment. (The spin is supposed to be known.)

10.29. The magnetic resonance method was used to study the magnetic properties of $^7Li^{19}F$ molecules whose electron shells possess the zero angular momentum. At constant magnetic field $B = 5000$ G, two resonance peaks were observed at frequencies $\nu_1 = 8.30$ MHz and $\nu_2 = 2.00$ MHz of an a.c. magnetic field. The control experiments showed that the peaks belong to lithium and fluorine atoms respectively. Find the magnetic moments of these nuclei. The spins of the nuclei are supposed to be known.

10.30. According to the gas model of a nucleus, the nucleons form a gas filling up the volume of the nucleus and obeying the Fermi-Dirac statistics. On the basis of that assumption, evaluate the highest kinetic energy of nucleons inside a nucleus. The gas is supposed to be completely degenerate, and the number of protons and neutrons in a nucleus to be equal.

10.31. Using the nuclear shell model, write the electronic configurations of ^7Li, ^{13}C, and ^{25}Mg nuclei in the ground state.

10.32. Using the nuclear shell model, determine the spins and parities of the following nuclei in the ground state: ^{17}O, ^{29}Si, ^{39}K, ^{45}Sc, and ^{63}Cu.

10.33. Using the vector model, demonstrate that the gyromagnetic ratio for a nucleon in the state l, j is

$$g_j = g_l \pm \frac{g_s - g_l}{2l+1},$$

where the plus sign is to be taken for $j = l + 1/2$ and the minus sign for $j = l - 1/2$, g_s and g_l are the spin and orbital gyromagnetic ratios.

10.34. Using the formula of the foregoing problem, calculate the magnetic moment in the states $s_{1/2}$, $p_{1/2}$, and $p_{3/2}$ of: **(a)** a neutron ($g_l = 0$); **(b)** a proton ($g_l = 1$).

10.35. Using the formula of Problem 10.33, determine the quantum number j for a proton in the f state, if its magnetic moment (in that state) is equal to $\mu = 5.79\ \mu_N$.

10.36. Using the nuclear shell model, determine the magnetic moments of the nuclei: **(a)** ^{3}H and ^{3}He; **(b)** ^{17}O and ^{39}K in the ground state.

10.37. Contrary to the assumption of uniform filling of nuclear shells, the spin of a ^{19}F nucleus equals 5/2, and not 1/2. Supposing that the magnetic moment, equal to $2.63\ \mu_N$, is defined by an unpaired proton, determine the level occupied by that proton. The proton's gyromagnetic ratios are $g_s = 5.58$ and $g_l = 1$.

11

RADIOACTIVITY

- Fundamental law of radioactive decay:

$$N = N_0 e^{-\lambda t}; \quad \lambda = \frac{1}{\tau} = \frac{\ln 2}{T}, \tag{11.1}$$

where λ is the decay constant, τ is the mean lifetime of radioactive nuclei, T is their half-life.

- Specific activity is the activity of a unit mass of a substance.
- Poisson distribution law

$$p(n) = \frac{\langle n \rangle^n e^{-\langle n \rangle}}{n!}, \tag{11.2}$$

where $p(n)$ is the probability that n random events will occur in a certain period of time, $\langle n \rangle$ is the average number of times the event occurs during this period.

- Gaussian (normal) distribution

$$p(\varepsilon) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\varepsilon^2/2\sigma^2}, \tag{11.3}$$

where $\varepsilon = |n - \langle n \rangle|$ is the deviation from the mean, σ is the standard error of a single measurement,

$$\sigma = \sqrt{\langle n \rangle} \approx \sqrt{n}.$$

- Standard error of the sum or difference of independent measurements

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 + \ldots}, \tag{11.4}$$

where σ_i are the standard errors of independent measurements.

RADIOACTIVE DECAY LAWS

11.1. Find the decay probability for a nucleus during the time interval t, if its decay constant is equal to λ.

11.2. Demonstrate that the mean lifetime of radioactive nuclei equal $\tau = 1/\lambda$, where λ is their decay constant.

11.3. What fraction of the original number of ^{90}Sr nuclei: **(a)** will remain after 10 and 100 years? **(b)** will disintegrate during one day; during 15 years?

11.4. There is a stream of neutrons with a kinetic energy of 0.025 eV. What fraction of neutrons decays at a distance of 2.0 m?

11.5. Calculate the decay constant, mean lifetime, and half-life of a radionuclide whose activity diminishes by 6.6 per cent during 100 days.

11.6. Determine the age of ancient wooden items, if it is known that the specific activity of ^{14}C nuclide amounts to 3/5 of that in lately felled trees.

11.7. A fresh preparation contains 1.4 μg of ^{24}Na radionuclide. What will its activity be after one day?

11.8. Determine the number of radioactive nuclei in a fresh specimen of ^{82}Br, if after one day its activity became equal to 0.20 Ci.

11.9. Calculate the specific activity of pure ^{239}Pu.

11.10. How many milligrams of beta-active ^{89}Sr should be added to 1 mg of inactive strontium to make the specific activity of the preparation equal to 1370 Ci/g?

11.11. In the bloodstream of a man a small amount of solution was injected, containing a ^{24}Na radionuclide of activity $A = 2.0 \cdot 10^3\ s^{-1}$. The activity of 1 cm³ of blood sample taken after $t = 5.0$ hours turned out to be $a = 16\ min^{-1} \cdot cm^{-3}$. Find the total blood volume of the man.

11.12. A preparation contains two beta-active components with different half-lives. The measurements resulted in the following dependence of the count rate N (s^{-1}) on time t expressed in hours:

t	0	1	2	3	5	10	20	30
N	60.0	34.3	21.1	14.4	8.65	5.00	2.48	1.25

Find the half-lives of both components and the ratio of radioactive nuclei of these components at the moment $t = 0$.

11.13. A radionuclide A_1 with decay constant λ_1 transforms into a radionuclide A_2 with decay constant λ_2. Assuming that at the initial moment the preparation consisted of only N_{10} nuclei of radionuclide A_1, find: **(a)** the number of nuclei of radionuclide A_2 after a time interval t; **(b)** the time interval after which the number of nuclei of radionuclide A_2 reaches the maximum value; **(c)** under what condition the transitional equilibrium state can evolve, so that the ratio of the amounts of the radionuclides remains constant. What is the magnitude of that ratio?

11.14. The decay product of ^{238}U is ^{226}Ra which is contained in the former substance in the proportion of one atom per $2.80 \cdot 10^6$ uranium atoms. Find the half-life of ^{238}U, if it is known to be much longer than that of ^{226}Ra (equal to 1620 years).

11.15. Via beta-decay a ^{112}Pd radionuclide transforms into a beta-active ^{112}Ag radionuclide. Their half-lives are equal to 21 and 3.2 hours respectively. Find the ratio of the *highest* activity of the second nuclide to the initial activity of the preparation, if at the initial moment the preparation consisted of the first nuclide only.

11.16. A ^{118}Cd radionuclide goes through the transformation chain

$$^{118}\mathrm{Cd} \xrightarrow[30\ \mathrm{min}]{} {}^{118}\mathrm{In} \xrightarrow[4.5\ \mathrm{min}]{} {}^{118}\mathrm{Sn}\ \text{(stable)}$$

(the corresponding half-lives are indicated under the arrows). Assuming that at the moment $t = 0$ the preparation consisted of Cd only, find: **(a)** the fraction of nuclei transformed into stable ones after 60 min; **(b)** in what proportion the activity of the preparation diminishes after 60 min.

11.17. A radionuclide A_1 decays via the chain: $A_1 \xrightarrow[\lambda_1]{} A_2 \xrightarrow[\lambda_2]{} A_3 \xrightarrow[\lambda_3]{} \ldots$ (the corresponding decay constants are indicated under the arrows). Assuming that at the initial moment the preparation consisted of N_{10} nuclei of radionuclide A_1, derive the expression for the law of accumulation of A_3 nuclide.

11.18. Find the mass of lead formed from 1.0 kg of ^{238}U during the period equal to the age of the Earth ($2.5 \cdot 10^9$ years).

11.19. A preparation contains 10 μg of ^{226}Ra and its decay products which are in the state of transitional equilibrium with radon. Using the tables of the Appendix, determine: **(a)** the α-activity of ^{222}Rn and β-activity of ^{210}Pb of the preparation; **(b)** the total α-activity of the preparation.

11.20. A ^{27}Mg radionuclide is produced at a constant rate of $q = 5.0 \cdot 10^{10}$ nuclei per second. Determine the number of ^{27}Mg nuclei that would accumulate in the preparation over the time interval: **(a)** exceeding considerably its half-life; **(b)** equal to its half-life.

11.21. A ^{124}Sb radionuclide is produced at a constant rate of $q = 1.0 \cdot 10^9$ nuclei per second. Having the half-life $T = 60$ days it decays into the stable ^{124}Te. Find: **(a)** how soon after the beginning of production the activity of ^{124}Sb radionuclide becomes equal to $A = 10$ μCi; **(b)** what mass of ^{124}Te will be accumulated in the preparation four months after the beginning of its production.

11.22. An A_1 radionuclide produced at the constant rate q nuclei per second goes through the following transformation chain:

$$A_1 \xrightarrow[\lambda_1]{} A_2 \xrightarrow[\lambda_2]{} A_3\ \text{(stable)}$$

(the decay constants are indicated under the arrows). Find the law describing the accumulation of nuclei A_1, A_2, and A_3 in the course of time, assuming that at the initial moment the preparation did not contain any of them.

11.23. A ^{138}Xe radionuclide produced at a constant rate of $q = 1.0 \cdot 10^{10}$ nuclei per second, goes through the following transformation chain

$$^{138}\mathrm{Xe} \xrightarrow[17\ \mathrm{min}]{} {}^{138}\mathrm{Cs} \xrightarrow[32\ \mathrm{min}]{} {}^{138}\mathrm{Ba}\ \text{(stable)}$$

(the half-lives are indicated under the arrows). Calculate the combined activity of the given preparation 60 min after the beginning of accumulation.

11.24. A ^{99}Mo radionuclide with a half-life of 67 hours transforms into a stable ^{99}Tc nuclide through β-decay. As this takes place, 75% of β-transformations goes through the isomer $^{99}Tc^m$ whose half-life is 6.04 hours. Determine: **(a)** the fraction of stable nuclei in the preparation 5.00 hours after, assuming that at the initial moment it contained only ^{99}Mo; **(b)** the number of stable ^{99}Tc nuclei in the preparation 20 hours after the beginning of accumulation, assuming ^{99}Mo to be produced at a constant rate of $1.0 \cdot 10^{10}$ nuclei per second.

α- AND β-DECAY

11.25. A stationary ^{213}Po nucleus emits an α-particle with kinetic energy $T_\alpha = 8.34$ MeV. Provided that a daughter nucleus is produced in the ground state, find the total energy released in this process. What fraction of that energy is accounted for by the kinetic energy of the daughter nucleus? What is the recoil velocity of the daughter nucleus?

11.26. ^{210}Po nuclei emit α-particles with kinetic energy $T = 5.30$ MeV, with practically all daughter nuclei being formed immediately in the ground state. Determine: **(a)** the amount of heat released by 10.0 mg of ^{210}Po preparation during the mean lifetime period of these nuclei; **(b)** the initial activity of the ^{210}Po preparation, if during its half-life period it releases 2.2 kJ of heat.

11.27. The decay of ^{210}Po nuclei (in the ground state) is accompanied by emission of two groups of α-particles: the principal one with an energy of 5.30 MeV and low-intensity one with an energy of 4.50 MeV. Find the energy of α-decay of the initial nuclei and that of gamma-quanta emitted by the daughter nuclei.

11.28. ^{226}Th nuclei decay from the ground state with emission of α-particles with energies of 6.33, 6.23, 6.10, and 6.03 MeV. Calculate and plot the diagram of the levels of the daughter nucleus.

11.29. The decay of ^{212}Po nuclei is accompanied by emission of four groups of α-particles: the principal one with an energy of 8.780 MeV and long-range ones with energies of 9.492, 10.422, and 10.543 MeV. Calculate and plot the diagram of the levels of a ^{212}Po nucleus, if the daughter nuclei are known to be produced in the ground state.

11.30. Evaluate the height of the Coulomb barrier for α-particles emitted by ^{222}Rn nuclei (the rounded top of the barrier is to be ignored). What is the barrier width (tunnelling distance) of these nuclei for α-particles ejected with a kinetic energy of 5.5 MeV?

11.31. Determine the ratio of the height of the centrifugal barrier to that of the Coulomb barrier for α-particles emitted by ^{209}Po nuclei and having the orbital moment $l = 2$. The rounded top of the Coulomb barrier is to be ignored.

11.32. A nucleus emits an α-particle whose kinetic energy T is considerably less than the Coulomb barrier height. In this case the coefficient of transparency of the barrier is equal to

$$D = e^{-\varkappa/\sqrt{T}}; \qquad \varkappa = 2\pi Z e^2 \sqrt{2m}/\hbar,$$

where Ze is the charge of the daughter nucleus, m is the mass of α-particle.

(a) Derive this formula from the general expression for D (3.5).

(b) Calculate the transparency ratio for α-particles emitted by ^{226}Th nuclei with energies of 6.33 and 6.22 MeV.

11.33. ^{212}Po nuclei in the first excited state decay through two competing processes: the direct emission of α-particles (long-range group) and emission of α-particle after transition of the excited nucleus to the ground state (principal group of α-particles). 35 long-range α-particles are emitted for each $1.0 \cdot 10^6$ α-particles of the principal group. Find the decay constant of the given excited level in terms of emission of long-range α-particles, if the mean lifetime of that level is $\tau = 1.8 \cdot 10^{-12}$ s.

11.34. Find the width of the first excited level of ^{214}Po in terms of emission of gamma-quanta, if the decay of the excited nuclei involves $4.3 \cdot 10^{-7}$ long-range α-particles and 0.286 γ-quanta for each α-particle of the principal group. The decay constant in terms of emission of long-range α-particles is equal to $2.0 \cdot 10^5$ s^{-1}.

11.35. Calculate the total kinetic energy of particles emerging on β-decay of a stationary neutron.

11.36. How does one determine the amount of energy released in β^--decay, β^+-decay, and K-capture, if the masses of the parent and daughter nuclei, and the electron mass are known.

11.37. Knowing the mass of the daughter atom and β-decay energy Q, find the atomic mass of: **(a)** ^{6}He which undergoes a β^--decay with an energy of $Q = 3.50$ MeV; **(b)** ^{22}Na, undergoing a β^+-decay with an energy of $Q = 1.83$ MeV.

11.38. Determine whether the following processes are possible: **(a)** β^--decay of ^{51}V nuclei (—0.05602); **(b)** β^+-decay of ^{39}Ca nuclei (—0.02929); **(c)** the K-capture in ^{63}Zn atoms (—0.06679). The excess of atomic mass is indicated in parentheses, $M - A$ (in a.m.u.'s).

11.39. A ^{32}P nucleus undergoes β-decay to produce a daughter nucleus directly in the ground state. Determine the highest kinetic energy of β-particles and the corresponding kinetic energy of the daughter nucleus.

11.40. Calculate the maximum magnitude of momentum for electrons emitted by ^{10}Be nuclei, if the daughter nuclei are produced directly in the ground state.

11.41. A ^{11}C nucleus undergoes a positronic decay to produce a daughter nucleus directly in the ground state. Calculate: **(a)** the highest kinetic energy of positrons and the corresponding kinetic energy of the daughter nucleus; **(b)** the energies of the positron and neutrino in the case when the daughter nucleus does not recoil.

11.42. A ^{6}He nucleus undergoes β^--decay to produce a daughter nucleus directly in the ground state. The decay energy is $Q = 3.50$ MeV. An electron with the kinetic energy $T = 0.60$ MeV escapes at right angles to the direction of motion of the recoil nucleus. At what angle to the direction at which the electron escapes is the antineutrino emitted?

11.43. Calculate the energy of γ-quanta released in β-decay of ^{28}Al nuclei (Fig. 30).

11.44. Determine the number of γ-quanta per one β-decay of ^{38}Cl nuclei (Fig. 31), if the relative number of β-decays with the given partial spectrum of β-particles is equal to: 31% (β_1), 16% (β_2), and 53% (β_3).

11.45. β-decay of ^{56}Mn nuclei in the ground state is accompanied by the emission of three partial spectra of β-particles with maximum kinetic energies of 0.72, 1.05, and 2.86 MeV. The concurrent γ-quanta

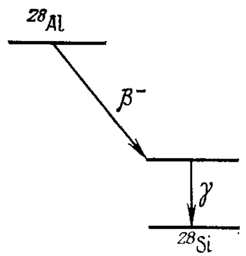

Fig. 30

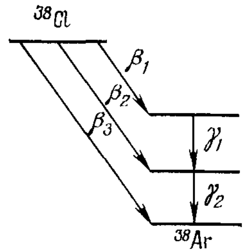

Fig. 31

have the energies of 0.84, 1.81, 2.14, 2.65, and 2.98 MeV. Calculate and draw the diagram of levels of the daughter nucleus.

11.46. ^{37}Ar nuclei experience the K-capture after which the daughter nuclei are formed directly in the ground state. Neglecting the binding energy of the K-electron, determine the kinetic energy and velocity of the daughter nucleus.

11.47. Find the energy of a neutrino in the K-capture in ^{131}Cs atoms, if the total energy released in this process equals 355 keV, and the binding energy of the K-electron in the daughter atom is 35 keV, with the daughter nucleus being formed directly in the ground state.

11.48. The K-capture in ^{7}Be results occasionally in the formation of an excited daughter nucleus emitting a γ-quantum with an energy of 479 keV. Determine the kinetic energy of the daughter nucleus after emission of γ-quantum, if the neutrino and the recoil nucleus move at right angles to each other. What is the magnitude of that energy in the case when the daughter nucleus is formed directly in the ground state?

γ-RADIATION: INTERNAL CONVERSION, MÖSSBAUER EFFECT

11.49. An isomeric nucleus ^{81}Se^m with an excitation energy of 103 keV passes to the ground state, emitting either a γ-quantum or conversion electron from the K-shell of the atom (the binding energy of the K-electron being equal to 12.7 keV). Find the velocity of the recoil nucleus in both cases.

11.50. Passing to the ground state, an isomeric ^{109}Ag^m nucleus emits either a γ-quantum with an energy of 87 keV or a conversion

K-electron with $B\rho$ = 860 G·cm. Calculate the binding energy of the K-electron.

11.51. ^{203}Tl atoms resulting from β-decay of ^{203}Hg atoms emit 4 groups of conversion electrons with kinetic energies of 266.3, 264.2, 263.6, and 193.3 keV. To what shell of a Tl atom, K, L_1, L_2, L_3, does each group correspond? The electron binding energy in the shells is 87.7, 15.4, 14.8 and 12.7 keV respectively. Calculate the energies of γ-quanta concurrent with that decay.

11.52. Excited ^{141}Pr nuclei resulting from β-decay of ^{141}Ce nuclei pass to the ground state by emission of γ-quanta or conversion electrons. Determine the excitation energy of a ^{141}Pr nucleus, if for the conversion K-electrons $B\rho$ = 1135 G·cm and the binding energy of the K-electrons is equal to 42 keV.

11.53. Excited ^{117}Sn nuclei resulting from β-decay of ^{117}In nuclei pass to the ground state, emitting two consecutive γ-quanta. This process is followed by emission of conversion K-electrons for which $B\rho$ is equal to 3050 and 1300 G·cm. The binding energy of K-electrons equals 29 keV. Determine the energy of the γ-quanta.

11.54. Find the number of conversion electrons emitted per second by a ^{59}Fe preparation with an activity of 1.0 mCi. The diagram of β-decay of ^{59}Fe nuclei is shown in Fig. 32. The internal conversion coefficients for γ-quanta are equal to $1.8\cdot10^{-4}$ (γ_1), $1.4\cdot10^{-4}$ (γ_2), and $7\cdot10^{-3}$ (γ_3). The probabilities of γ_2 and γ_3 emission relate as 1 : 15. *Note*: the internal conversion coefficient is the ratio of the probability of conversion electron emission to that of γ-quantum emission.

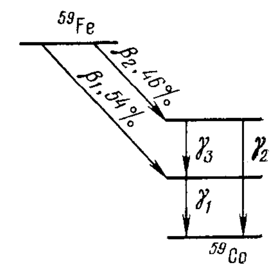

Fig. 32

11.55. A free nucleus ^{191}Ir with an excitation energy of E = 129 keV passes to the ground state, emitting a γ-quantum. Find the fractional change of energy of the given γ-quantum due to recoil of the nucleus.

11.56. A free nucleus ^{119}Sn with an excitation energy of E = 23.8 keV passes to the ground state, emitting a γ-quantum. The given level has a width $\Gamma = 2.4\cdot10^{-8}$ eV. Determine whether the resonance absorption of such a γ-quantum by another free ^{119}Sn nucleus is possible, if initially both nuclei were stationary.

11.57. What must be the relative velocity of a source and an absorber consisting of free ^{191}Ir nuclei to observe the maximum absorption of γ-quanta with an energy of 129 keV?

11.58. As it was shown by Mössbauer, each γ-line of the spectrum emitted by the excited nuclei of a solid has two components: a very narrow one with energy equal to the transition energy in the nuclei, and a much broader one which is displaced relative to the former. For a ^{57}Fe γ-line corresponding to the energy of 14.4 keV the fractional shift of the displaced component is $\Delta\lambda/\lambda = 1.35 \cdot 10^{-7}$. Demonstrate that that component is caused by the recoil nuclei occurring in the process of γ-emission.

11.59. Figure 33 illustrates the absorption of the Mössbauer γ-line with an energy of 129 keV as a function of the relative velocity of a source and an absorber (^{191}Ir). Taking into account that the emission of the given line is caused by the transition of excited nuclei directly to the ground state, find the width and lifetime of the corresponding excited level.

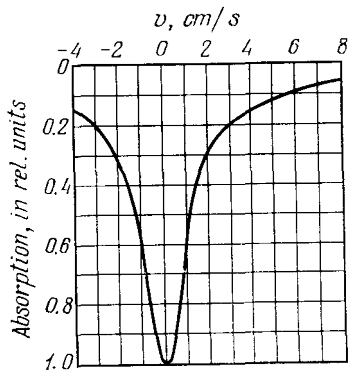

Fig. 33

11.60. A gamma source is placed 20.0 m above an absorber. With what velocity should the source be displaced upward to counterbalance completely the gravitational variation of the γ-quanta energy due to the Earth's gravity at the point where the absorber is located?

11.61. The relative widths of the Mössbauer γ-lines in ^{57}Fe and ^{67}Zn are equal to $3.0 \cdot 10^{-13}$ and $5.0 \cdot 10^{-16}$ respectively. To what height above the Earth surface has one to raise an absorber (^{57}Fe and ^{67}Zn) to make the gravitational displacement of the Mössbauer line exceed the width of the lines when being registered on the Earth surface?

11.62. In the process of emission of γ-quanta corresponding to a Mössbauer line the recoil momentum is taken by a crystal as

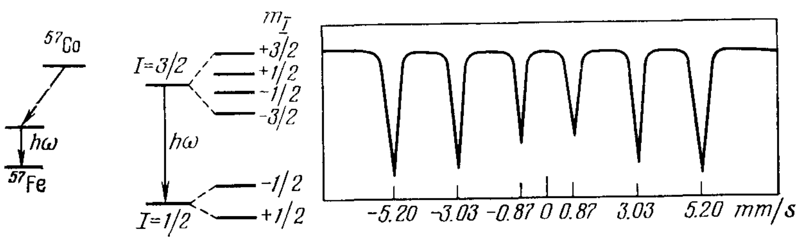

Fig. 34

a whole, so that the momentum of the emitting atom remains constant. The mean kinetic energy of such an atom, however, increases due to a decrease in its mass following the radiation. As a result, the Mössbauer line frequency turns out to be less than the transition frequency ω_0, that is, $\omega = \omega_0 (1 - \langle v^2 \rangle / 2c^2)$, where $\langle v^2 \rangle$ is the square of the root-mean-square velocity of the atom.

(a) Derive this expression from energy considerations.

(b) By how many Kelvins must the temperature of the source

exceed that of the absorber, so that the temperature shift of the Mössbauer γ-line in ^{57}Fe counterbalances completely the gravitational shift? The source is placed at a distance of $l = 20$ m above the absorber. The mean kinetic energy of atoms in the crystal is assumed to be equal to $3kT/2$.

11.63. Figure 34 illustrates the Mössbauer absorption velocity spectrum in the case when the emission line of γ-quanta with an energy of $\hbar\omega = 14.4$ keV is not split (a ^{57}Co source is inserted into non-magnetic steel), while a plate of natural iron serves as an absorber. The positive velocity values signify the motion of the source toward the absorber. Using the level diagram of Fig. 34, find the magnetic moment of a ^{57}Fe nucleus in the excited state and induction of the magnetic field acting on the nucleus in iron. The magnetic moment of a ^{57}Fe nucleus in the ground state is equal to $\mu = 0.090\ \mu_N$.

REGISTRATION STATISTICS OF NUCLEAR RADIATION. COUNTERS

11.64. In measuring the activity of a certain preparation a counter registers 6 pulses per minute on the average. Using formula (11.2) evaluate the probability of the count rate having a value between 9 and 11 pulses per minute.

11.65. 2000 measurements of activity of a preparation are to be performed during equal time intervals. The mean number of pulses registered during each measurement is equal to 10.0. Assuming the total measurement time to be small in comparison with the half-life of the radionuclide being investigated, determine the number of measurements in which one should expect registering 10 and 5 pulses exactly.

11.66. The mean count rate value registered in the studies of a radionuclide with a long half-life is 100.0 pulses per minute. Determine the probability of obtaining 105.0 pulses per minute, and the probability of absolute deviation from the mean value exceeding 5.0 pulses per minute.

11.67. Calculate the probability of obtaining the absolute error of measurement exceeding: **(a)** σ and **(b)** 2σ, where σ is the standard error.

11.68. A counter placed in a radiation field to be investigated registered 3600 pulses during 10 minutes. Find: **(a)** the standard error in the count rate, pulses per minute; **(b)** the duration of measurement sufficient to determine the count rate with an error of 1.00%.

11.69. While measuring the intensity of radiation (including the background), a counter registered 1700 pulses during 10.0 minutes. The separate background measurement yielded 1800 pulses during 15.0 minutes. Find the count rate (pulses per minute) caused by the radiation investigated and its standard error.

11.70. Demonstrate that in the presence of a background whose intensity is equal to that of the investigated radiation one has to register 6 times as many pulses to provide the same measurement accuracy as in the case when the investigated radiation is not accompanied with any background radiation.

11.71. The count rate of the background pulses is equal to $n_b = 15$ pulses per minute, and the count rate of a preparation studied in the presence of the background is $n_{pb} = 60$ pulses per minute. Let t_b and t_{pb} be the times of measurement of the background and that of the preparation in the presence of the background. Find the optimum ratio t_b/t_{pb} at which the preparation count rate is determined with the highest accuracy for the given total time of measurement $(t_b + t_{pb})$.

11.72. Using the data of the foregoing problem, find the minimum values of t_b and t_{pb} at which the preparation count rate can be determined with accuracy $\eta = 0.050$.

11.73. A Geiger-Müller counter with time resolution $\tau = 2.0 \cdot 10^{-4}$ s registered $n = 3.0 \cdot 10^4$ pulses per minute. Determine the real number N of particles crossing the counter during one minute.

11.74. What fraction of particles crossing a counter with time resolution $\tau = 1.0 \cdot 10^{-6}$ s will be missed at the count rates of $n = 100$ and $1.0 \cdot 10^5$ pulses per second?

11.75. In measuring the activity of a preparation a Geiger-Müller counter with time resolution of $2.0 \cdot 10^{-4}$ s registered 1000 pulses per second in the presence of the background. The separate background measurement by means of the same counter yielded 600 pulses per second. Determine the number of particles from the preparation crossing the counter during 1 s.

11.76. Two radioactive sources are placed near a counter. With alternate screening of the sources the counter registers n_1 and n_2 pulses per second. Both sources simultaneously yield n_{12} pulses per second. Find the time resolution of the given counter.

11.77. The number of particles crossing a counter per unit time is equal to N. Find the number of pulses recorded per unit time, if the time resolution of the counter is known to be equal to τ_1 and that of the recording facility to τ_2. Consider the cases: **(a)** $\tau_1 > \tau_2$; **(b)** $\tau_1 < \tau_2$.

11.78. In a scintillation counter with a photomultiplier tube, the de-excitation time of the scintillator is equal to $\tau_1 = 0.6 \cdot 10^{-8}$ s and the time resolution of the photomultiplier tube to $\tau_2 = 3.0 \cdot 10^{-8}$ s. Determine the number of electrons falling on the scintillator during one second, if the photomultiplier tube yields $n = 5.0 \cdot 10^6$ pulses per second.

11.79. An electromechanical reader with the time resolution τ is incorporated at the output of an amplifier (without a scaler). Find how the number of pulses n registered per unit time depends on the mean number of particles N crossing a Geiger-Müller counter per unit time. *Instruction*: it should be taken into account that if at the moment when the electromechanical reader has not yet completed a cycle of pulse registration another pulse comes in, the latter will not be registered. The non-registered pulse, however, will increase the reader's dead time caused by the first pulse.

11.80. The pulses from a Geiger-Müller counter are amplified and fed directly to an electromechanical reader. Determine the time resolution of the electromechanical reader, if on bringing a radioactive preparation closer to the counter the number of registered pulses exceeds a maximum value $n_{max} = 46$ pulses per second.

11.81. Two identical counters incorporated into a coincidence circuit are exposed to cosmic radiation. Determine the number of spurious coincidences Δn, if the number of pulses coming to the input of the coincidence circuit from one counter is equal to n_1 and from the other to n_2, the time resolution of the circuit being τ.

11.82. A radioactive preparation is placed symmetrically in front of two identical counters incorporated in a coincidence circuit. The time resolution of the circuit is $\tau = 1.0 \cdot 10^{-7}$ s. The registration efficiency of each counter is 25%. Determine the number of particles falling on each counter during one second, if the count rate of the coincidence circuit $\Delta n = 2.0 \cdot 10^3$ pulses per second.

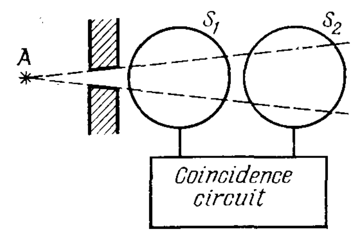

Fig. 35

11.83. A radioactive preparation A is placed in front of two identical counters S_1 and S_2 as shown in Fig. 35. The counters are incorporated in a coincidence circuit with the time resolution $\tau = 1.0 \cdot 10^{-8}$ s. To determine the preparation activity, the count rates of background radiation Δn_b and of preparation in the presence of the background n_{pb} are measured. Both measurements are taken during equal time intervals t. Find the magnitude of t at which the preparation count rate is determined accurate to 5.0%, if the number of pulses produced by each counter is equal to $1.00 \cdot 10^5$ pulses per second when the background radiation is measured and to 100 pulses per second when only the preparation is measured.

11.84. A radioactive preparation is placed between two identical γ-quanta counters incorporated into a coincidence circuit. The preparation's β-decay involves the emission of two quanta γ_1 and γ_2. Under experimental conditions the given γ-quanta can be registered by the counters with probabilities $\eta_1 = 5 \cdot 10^{-4}$ and $\eta_2 = 7 \cdot 10^{-4}$. Determine the number of counts registered by the coincidence circuit (as a percentage of the number of pulses registered by one of the counters within the same time), neglecting the correlation between the directions of motion of the outgoing γ_1 and γ_2 quanta.

12

INTERACTION OF RADIATION WITH MATTER

● Specific ionization loss of energy of a heavy charged particle moving in a matter:

$$-\left(\frac{\partial E}{\partial x}\right)_{\text{ion}}=\frac{4\pi n_e q^2 e^2}{m_e v^2}\left[\ln\frac{2m_e v^2}{I(1-\beta^2)}-\beta^2\right], \quad (12.1)$$

where q and v are the charge and velocity of the particle, $\beta = v/c$, n_e is the electronic concentration in the matter, $I \simeq 13.5Z$ eV is the mean ionization energy of an atom of the matter, Z is the atomic number.

● Empirical formulas for the mean path of particles with kinetic energy T, MeV:

an α-particle in air at NTP

$$R_\alpha = 0.31T^{3/2}\text{ cm};\quad 4 < T < 7\text{ MeV}; \quad (12.2)$$

an α-particle in a substance with mass number A

$$R_\alpha = 0.56R_\alpha\text{ (cm) }A^{1/3}\text{ mg/cm}^2, \quad (12.3)$$

where R_α (cm) is the range of the particle of the same energy in air;

a proton in air at NTP

$$R_p(T) = R_\alpha(4T) - 0.2\text{ cm};\quad T > 0.5\text{ MeV}, \quad (12.4)$$

where R_α is the mean path of α-particle with kinetic energy $4T$ in air.

● Specific radiation and ionization losses of energy for an electron

$$-\left(\frac{\partial E}{\partial x}\right)_{\text{rad}}=\frac{4r_e^2}{137}nTZ^2\ln\frac{183}{Z^{1/3}};\quad \frac{(\partial E/\partial x)_{\text{ion}}}{(\partial E/\partial x)_{\text{rad}}}\approx\frac{800}{ZT_{\text{MeV}}}, \quad (12.5)$$

where T is the kinetic energy of an electron, r_e is the classical electronic radius n is the concentration of atoms in a substance, Z is the atomic number of a substance.

● If an electron loses its energy primarily due to radiation, its kinetic energy decreases, as it moves in a substance, according to the law

$$T = T_0 e^{-x/l_{\text{rad}}}, \quad (12.6)$$

where l_{rad} is the *radiation length*, that is, the distance over which the electron's energy decreases e-fold.

● Mean path of electrons with kinetic energy T, MeV, in aluminium

$$R\,(\text{g/cm}^2)=\begin{cases}0.407T^{1.38}, & 0.15 < T < 0.8\text{ MeV},\\ 0.542T-0.133, & 0.8 < T < 3\text{ MeV}.\end{cases} \quad (12.7)$$

These formulas give the path values in any substance with sufficient accuracy provided the energy losses of the electrons are due primarily to ionization.

● Absorption law for β-particles

$$J = J_0 e^{-\mu d}, \quad (12.8)$$

where J is the flux of β-particles, μ is the linear absorption coefficient, d is the layer thickness of a substance. The mass absorption coefficient

$$\mu/\rho = 22/T_{\beta\max}^{4/3};\quad 0.5 < T_{\beta\max} < 6\text{ MeV}, \quad (12.9)$$

where $T_{\beta\max}$ is the cut-off energy in the spectrum of β-particles, MeV.

● Attenuation law for a narrow beam of monochromatic γ-radiation:

$$J = J_0 e^{-\mu x};\quad \mu = \tau + \sigma, \quad (12.10)$$

where μ, τ, σ are the linear coefficients of attenuation, absorption, and scattering.

● Units of dose (roentgen, rad, and rem) and tolerance rates are given in Table 15 of Appendix.

PROPAGATION OF CHARGED PARTICLES THROUGH MATTER

12.1. Find the maximum possible angle through which an α-particle can be scattered due to collision with a stationary free electron.

12.2. An α-particle with a kinetic energy of 25 MeV moved past a stationary free electron with an aiming parameter of $2.0\cdot10^{-9}$ cm. Find the kinetic energy of the recoil electron, assuming the trajectory of the α-particle to be rectilinear and the electron to be stationary during the flyby.

12.3. A fast α-particle moves through a medium containing n electrons per 1 cm³ with the velocity v. Determine the energy lost by the α-particle per unit length due to interaction with electrons relative to which its aiming parameter falls within interval $(b, b + db)$.

12.4. Calculate the specific ionization loss of energy for a deuteron with an energy of 4.0 MeV in nitrogen at NTP.

12.5. Find the ratio of specific ionization losses: **(a)** for an α-particle and proton with an energy of 5.0 MeV in neon; **(b)** for an α-particle with an energy of 10.0 MeV in copper and aluminium.

12.6. A point source of α-particles with an energy of 5.3 MeV is located at the centre of a spherical ionization chamber of radius 14.0 cm. At what values of air pressure in the chamber will the saturation current be independent of pressure?

12.7. Using the empirical formulas, find: **(a)** the number of ion pairs produced by an α-particle with an initial energy of 5.5 MeV over the first centimeter of its path in air (if the energy required to produce one ion pair is assumed to be equal to 34 eV); **(b)** the fraction of ion pairs produced by a proton with an initial energy of 2.5 MeV over the first half of its mean path in air.

12.8. A radioactive ^{238}Pu preparation emitting α-particles with an energy of 5.5 MeV is electroplated on a thick metallic base. Find the minimum thickness of the layer at which the further addition of ^{238}Pu will not produce any increase in intensity of α-particles emitted by that preparation.

12.9. Find the kinetic energy of α-particles whose mean path in iron equals 11.0 μm.

12.10. Determine the range of an α-particle in lead, if its energy is known to correspond to a range of 17 μm in aluminium.

12.11. α-particles with an energy of 13.7 MeV fall on an aluminium foil. At what thickness of the foil is the energy of passed-through particles equal to 7.0 MeV?

12.12. An aluminium foil is located at a distance of 5.0 cm from a radioactive preparation emitting α-particles with an energy of 9.0 MeV. Of what minimum thickness must the foil be to screen all the α-particles? The ambient medium is air.

12.13. Using formula (12.1), find how the mean ranges of a proton and a deuteron in a matter are related provided their velocities are equal. Calculate the range of a deuteron with an energy of 2.0 MeV in air.

12.14. Find the mean range of protons with an energy of 3.0 MeV in lead.

12.15. A fast heavy particle with charge q and velocity v moves in a substance with electronic concentration n and produces δ-electrons on its way. Assuming the process of their production to involve the elastic scattering of the primary particle by free electrons, determine: **(a)** the cross-section $d\sigma$ of production of δ-electrons with kinetic energies falling within interval $(T, T + dT)$; **(b)** the total number of δ-electrons produced by the primary particle per unit length of its trajectory; the minimum value of kinetic energy T_{th}, that an electron is to possess to form a visible trace, is supposed to be known.

12.16. When a fast heavy charged particle moves through photographic emulsion, it forms

$$N_\delta = \frac{2\pi n q^2 e^2}{m_e v^2}\left(\frac{1}{T_{th}} - \frac{1}{2m_e v^2}\right)$$

δ-electrons per unit length of its trajectory; n is the electronic concentration, q and v are the charge and velocity of the primary particle, T_{th} is the threshold kinetic energy of an electron required to form a visible trace in emulsion, m_e is the electronic mass. Using this formula, determine: **(a)** the lowest energy of the α-particle sufficient to produce δ-electrons in photographic emulsion for which $T_{th} = 11.0$ keV; **(b)** the energy of the α-particle that produces a maximum number of δ-electrons per unit length in the photographic emulsion with $n = 6.0 \cdot 10^{23}$ cm^{-3} and $T_{th} = 17.5$ keV; calculate the maximum number of δ-electrons produced over 1/10 mm of the α-particle's trajectory; **(c)** the charge of the primary particle if the maximum density of δ-electrons produced by it is known to be one fourth of that produced by an α-particle (in the same emulsion).

12.17. Calculate the specific radiation loss of energy in aluminium for an electron with a kinetic energy of 20 MeV. By what factor does the specific radiation loss of energy of an electron in lead exceed that in aluminium?

12.18. Evaluate the kinetic energies of electrons at which the specific bremsstrahlung loss of energy is equal to the specific radiation loss in nitrogen (at NTP), aluminium, and lead.

12.19. Evaluate the kinetic energy of electrons at which the specific radiation loss of energy in aluminium amounts to 1/4 of the total specific loss of energy.

12.20. Evaluate the total specific loss of energy in aluminium for an electron with a kinetic energy of 27 MeV.

12.21. Find how the radiation length l_{rad} of an electron depends on the atomic number Z of a substance. Calculate l_{rad} for an electron in nitrogen (at NTP), aluminium, and lead.

12.22. Fast electrons that passed through a layer of some substance 0.40 cm thick diminished their energy by 25% on the average. Find the radiation length of the electron if its energy loss is known to be primarily due to radiation.

12.23. Evaluate the initial energy of electrons, if on passing through a lead plate 5.0 mm thick their energy is equal to 42 MeV on the average.

12.24. When electrons of sufficiently high energies decelerate in the field of a nucleus, the cross-section of gamma-quanta emission within the frequency interval $(\omega, \omega + d\omega)$ in the vicinity of the maximum frequency of bremsstrahlung is defined by the formula $d\sigma = \frac{1}{n l_{rad}} \frac{d\omega}{\omega}$, where n is the number of nuclei in unit volume. Find the probability that an electron will lose over 90% of its initial energy on passing through a zinc plate of thickness $l = 1.0$ mm.

12.25. Using the empirical formulas, calculate the kinetic energy of electrons whose mean path in aluminium is equal to 100 mg/cm^2.

12.26. Find the mean path of relativistic electrons whose $B\rho =$ $= 5.0$ kG·cm in graphite.

12.27. A beam of electrons with a kinetic energy of 0.50 MeV falls normally on an aluminium foil 50 mg/cm^2 thick. Using the empirical formulas, evaluate the mean path of the electrons, passed through this foil, in air.

12.28. Evaluate the minimum mass thickness of a β-radioactive ^{204}Tl preparation beginning from which the further increase of its thickness does not increase the intensity of a stream of β-particles emitted by the preparation.

12.29. What fraction of β-particles emitted by ^{32}P is absorbed by an aluminium foil 20 mg/cm^2 thick?

12.30. The increase in the thickness of the window of a Geiger-Müller counter by 60 mg/cm^2 reduces the count rate of β-particles by 50 per cent. What is the highest energy of β-particles of the radioactive source studied?

12.31. Find the half-value thickness for β-particles emitted by a radioactive ^{32}P preparation for air, aluminium, and lead.

12.32. A charged particles, moving uniformly in a medium with the refractive index n, emits light if its velocity v exceeds the phase velocity of light c' in that medium (Cherenkov radiation). Using the laws of conservation of energy and momentum, demonstrate that the angle at which light is emitted is defined by the expression $\cos \vartheta = c'/v$. Recall that the momentum of a photon in a medium is equal to $\hbar\omega/c'$.

12.33. Calculate the threshold kinetic energies of an electron and a proton at which the Cherenkov radiation occurs in a medium with a refractive index of $n = 1.60$. What particles have the threshold kinetic energy in that medium equal to 29.6 MeV?

12.34. Find the kinetic energy of electrons that, moving through a medium with a refractive index of $n = 1.50$, emit light at the angle 30° to the direction of their motion.

PROPAGATION OF GAMMA-RADIATION THROUGH MATTER

12.35. The increase in the thickness of a lead plate by 2.0 mm reduces the intensity of a narrow beam of monochromatic X-rays having passed through that plate by a factor of 8.4. Find the energy of photons, using the tables of the Appendix.

12.36. What is the thickness of an aluminium plate that attenuates a narrow beam of X-ray radiation with an energy of 200 keV to the same degree as a lead plate 1.0 mm in thickness?

12.37. The attenuations of narrow beams of X-ray radiation with energies of 200 and 400 keV passing through a lead plate differ by a factor of four. Find the plate's thickness and the attenuation of the 200 keV beam.

12.38. Calculate the half-value thickness for a narrow beam of X-rays with a wavelength of $6.2 \cdot 10^{-2}$ Å in lead, water, and air.

12.39. How many layers of half-value thickness are there in a plate attenuating a narrow beam of monochromatic X-rays 1000-fold?

12.40. Plot $(\mu/\rho)^{1/3}$ versus X-ray radiation wavelength dependence in the case of copper, using the following data:

λ, Å	0.40	0.80	1.20	1.60	2.00	2.40	2.80
$d_{1/2}$, μm	78.0	11.0	3.34	12.7	7.21	4.55	3.00

($d_{1/2}$ is the half-value thickness).

12.41. Using the tables of the Appendix, select a metal foil that, being transparent to the K_α radiation, attenuates considerably the K_β radiation of: **(a)** cobalt ($\lambda_{K_\alpha} = 1.79$ Å, $\lambda_{K_\beta} = 1.62$ Å); **(b)** nickel ($\lambda_{K_\alpha} = 1.66$ Å, $\lambda_{K_\beta} = 1.50$ Å).

12.42. Calculate the thickness of copper foil at which the attenuation of the K_β radiation of zinc ($\lambda_{K_\beta} = 1.29$ Å) is 10 times that of the K_α radiation ($\lambda_{K_\beta} = 1.43$ Å). Make use of the plot obtained in Problem 12.40.

12.43. In the case of soft X-ray radiation, the differential cross-section of a photon scattering by a free electron is described by the formula

$$\frac{d\sigma}{d\Omega} = \frac{r_e^2}{2}(1 + \cos^2 \vartheta),$$

where r_e is the classical electronic radius, ϑ is the angle of photon scattering. Using this formula, find: **(a)** the total cross-section of scattering; **(b)** the fraction of photons scattered through the angles $\vartheta < 60°$; **(c)** the fraction of the recoil electrons outgoing within the range of angles from 45° to 90°.

12.44. Calculate the mass and linear scattering coefficients for soft X-rays in neon and oxygen at NTP. Recall that the atomic scattering coefficient is defined by the Thomson formula

$$\sigma_a = \frac{8\pi}{3} \cdot \frac{Ze^4}{m^2c^4} \text{ cm}^2 \text{ per atom.}$$

12.45. The mass absorption coefficient of X-ray radiation with $\lambda = 0.209$ Å in iron is equal to 1.26 cm²/g. Calculate the corresponding atomic scattering coefficient.

12.46. Taking into account that the atomic absorption coefficient $\tau_a = CZ^4\lambda^3$ at $\lambda < \lambda_K$, where λ_K is the wavelength of K absorption edge, C is the constant equal for all substances, determine: **(a)** the mass absorption coefficient τ/ρ for X-ray radiation with $\lambda = 1.00$ Å in vanadium, if in aluminium $\tau/\rho = 40$ cm²/g for $\lambda = 1.44$ Å; **(b)** the ratio of mass absorption coefficients of X-ray radiation in bones and tissues of human body; the bones are known to consist of $Ca_3(PO_4)_2$, and the absorption in tissues is mainly due to water.

12.47. After passing through an aluminium plate 2.9 cm thick a monochromatic beam of γ-quanta attenuates by a factor of 2.6. Using the tables of the Appendix, find the corresponding mass scattering coefficient.

12.48. A point source of γ-quanta with an energy of 0.80 MeV is placed in the centre of a spherical layer of lead whose thickness is equal to $\Delta r = 3.0$ cm and outer radius to $r = 5.0$ cm. Find the flux density of non-scattered γ-quanta at the external surface of the layer, if the source activity $A = 1.00$ mCi and each disintegration produces one quantum.

12.49. A narrow beam of γ-quanta composed of equal number of quanta with energies 0.40 and 0.60 MeV falls normally on a lead plate 1.00 cm in thickness. Find the ratio of intensities of both components of the beam after its passing through that plate.

12.50. A narrow beam of γ-radiation composed of quanta of all energies in the range from 0.60 to 0.80 MeV falls normally on an aluminium plate 2.0 cm thick. Find the attenuation of the beam's intensity after passing through the plate, if the attenuation coefficient is a linear function of energy of quanta in this interval and the spectral intensity of incident radiation is independent of frequency.

12.51. Using the table of the Appendix, determine the interaction cross-section (b/atom) of γ-quanta with an energy of 1.00 MeV in aluminium.

12.52. A narrow beam of γ-quanta with an energy of 0.15 MeV attenuates by a factor of four after passing through a silver plate 2.0 mm thick. Find the interaction cross-section (b/atom) of these γ-quanta in silver.

12.53. Using the tables of the Appendix, calculate the mean free path of γ-quanta with an energy of 1.00 MeV in air, water, and aluminium.

12.54. Calculate the mean free path of γ-quanta in a medium whose half-value thickness is equal to 4.50 cm.

12.55. Making use of the plots of the Appendix, find the mean free path of γ-quanta with an energy of 2.0 MeV in lead, as well as the mean paths of these quanta in the case of Compton scattering, photoelectric effect, and electron-positron pair production. How are these paths interrelated?

12.56. Using the plots of the Appendix, find the photoabsorption probability for a γ-quantum with an energy of 2.0 MeV in a lead plate 2.0 mm thick.

12.57. A beam of monochromatic γ-radiation attenuates by a factor of six after passing through a lead plate 3.2 cm thick. Using the plots of the Appendix, calculate the mass Compton scattering coefficient of that radiation in lead.

12.58. The total cross-section of Compton scattering of γ-quantum by a free electron is described by the formula

$$\sigma_{\text{Comp}} = \frac{3}{4}\sigma_T\left[\frac{\varepsilon^2-2\varepsilon-2}{2\varepsilon^3}\ln(1+2\varepsilon)+\frac{\varepsilon^3+9\varepsilon^2+8\varepsilon+2}{\varepsilon^2(1+2\varepsilon)^2}\right],$$

where $\varepsilon = \hbar\omega/mc^2$ is the energy of a γ-quantum expressed in units of electron rest mass, σ_T is the Thomson scattering cross-section.

(a) Simplify this formula for the cases $\varepsilon \ll 1$ and $\varepsilon \gg 1$.

(b) Calculate the linear Compton scattering coefficient for γ-quanta with energy $\varepsilon = 3.0$ in beryllium.

(c) Find the mass Compton scattering coefficient for γ-quanta with energy $\varepsilon = 2.0$ in light-element substances.

12.59. Using the plots of the Appendix, calculate the cross-section of electron-positron pair production for a γ-quantum with an energy of 6.0 MeV in a lead plate whose thickness is equal to the half-value thickness.

12.60. At what thickness of a lead plate is the probability of a γ-quantum with an energy of 7.0 MeV to produce an electron-positron pair equal to 0.10?

12.61. A thin lead plate was irradiated in the Wilson cloud chamber with γ-quanta with energies of 3.0 MeV. In the process, the number of electron tracks was found to exceed the number of positron tracks by a factor of $\eta = 3.7$. Find the ratio of the probability of electron-positron pair production to the total probability of all other processes proceeding in this case.

12.62. Derive the expression for the threshold energy of γ-quantum required to produce a pair in a field of a nucleus with mass M.

12.63. Demonstrate that a γ-quantum cannot produce a pair outside the field of a nucleus, even when such a process is allowed in terms of energy.

12.64. Determine the total kinetic energy of an electron-positron pair produced by a γ-quantum with the threshold value of energy in the field of a stationary proton.

12.65. Calculate the energy of a γ-quantum that produced an electron-positron pair in the field of a heavy nucleus, if for each particle of the pair $B\rho = 3.0$ kG·cm.

RADIATION DOSIMETRY

12.66. The saturation current of an ionization chamber placed in a uniform γ-radiation field is equal to $1.0 \cdot 10^{-9}$ A. The chamber has the volume of 50 cm^3 and is filled with air under a pressure of $2.0 \cdot 10^5$ Pa and at 27 °C. Find the γ-radiation dose rate.

12.67. Determine the radiation dose rate (mR/h) and the absorbed dose rate (mrad/h) in air and in water at the points where the flux density of γ-quanta with an energy of 2.0 MeV is equal to $1.30 \cdot 10^4$ quanta per cm^2 per second.

12.68. At a certain distance from the radioactive source with a half-life of 26 hours, the γ-radiation dose rate amounts to 1.0 R/h at the initial moment. Determine: **(a)** the radiation dose accumulated for 6.0 h; **(b)** the time interval during which the absorbed dose becomes equal to 1.0 rad.

12.69. Disregarding the absorption in air, determine the γ-radiation dose rate (μR/s) at a distance of 2.0 m from a point source with an activity of 100 mCi. The energy of γ-quanta is 1.0 MeV. The γ-quanta yield equals 0.50 quanta per disintegration.

12.70. A point radioactive source with an activity of 18 mCi emits two γ-quanta with energies of 0.80 and 1.00 MeV per disintegration. Ignoring the absorption in air, find the minimum distance from the source at which the radiation dose rate is equal to the tolerance dose rate for a 36-hour working week.

12.71. For radionuclides: **(a)** ^{24}Na, **(b)** ^{42}K, and **(c)** ^{38}Cl calculate the γ-constants (K_γ), i.e. the radiation dose rate (R/h) at a distance of 1 cm from a point source with activity of 1 mCi. The disintegration schemes of these radionuclides are shown in Fig. 36.

12.72. A source of γ-quanta with energy $E = 1.00$ MeV is uniformly distributed along a straight line. The length of the source is

$l = 10.0$ cm and intensity $J = 1.00 \cdot 10^6$ quanta per second. Calculate the radiation dose rate at the point, located at the perpendicular drawn through the midpoint of the source, at a distance $R = 5.0$ cm from the source.

12.73. A source of γ-quanta with energy $E = 2.0$ MeV is uniformly distributed over the surface of a round disc of radius $R = 3.0$ cm.

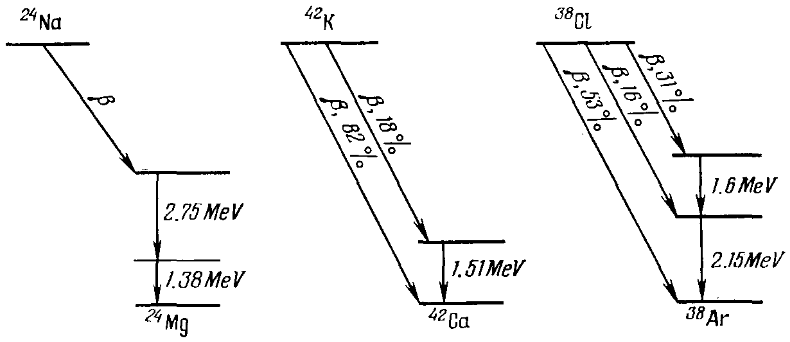

Fig. 36

The source's activity is $A = 100$ mCi/cm², the γ-quanta yield is equal to unity. Find the radiation dose rate at the point removed by a distance $h = 6.0$ cm from the source's centre and located at the axis of the disc.

12.74. At a point through which passes a narrow beam of γ-quanta with an energy of 1.00 MeV, the radiation dose rate amounts to 3.8 μR/s. Determine the thickness of a lead screen reducing the dose rate at that point to the tolerance dose rate for a 36-hour working week.

12.75. A point γ-source with activity $A = 100$ μCi is located at the centre of a spherical lead container with an outside radius $r = 10.0$ cm. Find the minimum thickness of the container's walls at which the dose rate outside the container would not exceed 2.8 mR/h. The energy of γ-quanta is $E = 2.00$ MeV, and the yield $\eta = 0.50$ quanta per disintegration.

12.76. A narrow beam of γ-quanta with an energy of 2.00 MeV falls normally on a lead screen of thickness $l = 5.0$ cm. Determine the absorbed dose rate in lead in the vicinity of the point where the beam leaves the screen, provided the dose rate at the point where it enters the screen equals $P_0 = 1.0$ R/s.

12.77. At what distance from a small isotropic source of fast neutrons with a power of $4.0 \cdot 10^7$ neutrons per second will the neutron radiation dose rate be equal to the tolerance dose rate for an 18-hour working week?

12.78. A stream of neutrons with kinetic energy $T = 0.33$ MeV and density $J = 1.4 \cdot 10^5$ neutrons/(cm²·s) penetrates a thin graphite plate. Calculate the dose absorbed by graphite during $t = 1.0$ h if the elastic scattering cross-section of neutrons $\sigma = 4.8$ b/nucleus. The mean fraction of energy transferred by the neutron to a nucleus with the mass number A during collision $f = 2A/(1 + A)^2$.

12.79. What number of α-particles with an energy of 4.4 MeV absorbed by 1 g of biological tissue corresponds to an absorbed dose of 50 rem? The Q.F. for α-particles is equal to 10.

12.80. $1.6 \cdot 10^4$ α-particles with an energy of 5 MeV fall normally on the skin surface of 1 cm² area. Determine the mean absorbed dose (rad and rem) in the layer equal to penetration depth of α-particles in biological tissue. The range of α-particles in biological tissue is 1/815 of that in air; the Q.F. for α-particles is equal to 10.

12.81. A beam of β-particles from a radioactive ^{90}Sr source falls normally on the surface of water. The flux density $J = 1.0 \cdot 10^4$ particles/(cm²·s). Determine the dose (rad) absorbed by water at its surface during an interval $t = 1.0$ h. The mean energy of β-particles is assumed to be equal to $T_{\beta max}/3$.

13

NUCLEAR REACTIONS

● Energy diagram of a nuclear reaction

$$m + M \to M^* \to m' + M' + Q,$$

proceeding via the compound nucleus M^* is shown in Fig. 37, where $m + M$ and $m' + M'$ are the sums of rest masses of particles before and after the reaction, $\tilde{T}$ and $\tilde{T}'$ are the total kinetic energies of particles before and after the reaction (in the C frame), E^* is the excitation energy of the compound nucleus,

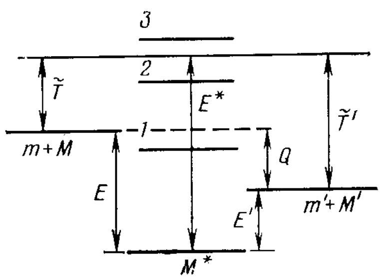

Fig. 37

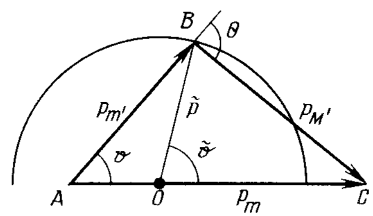

Fig. 38

Q is the *energy of the reaction*, E and E' are the binding energies of the particles m and m' in the compound nucleus. The figure also illustrates the energy levels *1*, *2*, and *3* of the compound nucleus.

● Threshold kinetic energy of an incoming particle (in a laboratory frame L) at which an endoergic nuclear reaction becomes possible

$$T_{\text{th}} = \frac{m+M}{M} \mid Q \mid, \tag{13.1}$$

here m and M are the masses of the incoming particle and the target nucleus.

● Vector diagram of momenta* for particles involved in the reaction $M\,(m,\ m')\,M'$ is shown in Fig. 38. Here p_m, $p_{m'}$, and $p_{M'}$ are the momenta of the incoming particle and particles generated as a result of the reaction (in the L frame), O is the centre of a circle whose radius equals the momentum $\tilde{p}$ of generated particles (in the C frame):

$$\tilde{p} = \sqrt{2\mu' (\tilde{T} + Q)}, \tag{13.2}$$

where μ' is the reduced mass of generated particles, Q is the reaction's energy, $\tilde{T}$ is the total kinetic energy of particles prior to the reaction (in the C frame), the point O divides the section AC into two parts in the ratio $AO : OC = m' : M'$,

* The similar diagram for elastic scattering is given on page 16.

$\tilde{\vartheta}$ is the angle at which the outgoing particle m' moves in the C frame, θ is the angle of divergence of generated particles in the L frame.

● Detailed balancing principle: for the reaction

$$m_1\,(s_1) + M_1\,(I_1) \rightleftarrows m_2\,(s_2) + M_2\,(I_2)$$

the cross-sections of a direct σ_{12} and a reverse process σ_{21} are related as

$$(2s_1 + 1)\,(2I_1 + 1)\,\sigma_{12}\tilde{p}_1^2 = (2s_2 + 1)\,(2I_2 + 1)\,\sigma_{21}\tilde{p}_2^2, \tag{13.3}$$

if both processes proceed at the same magnitude of the total energy of interacting particles in the C frame. Here s_i and I_i are the particles' spins, $\tilde{p}_1$ and $\tilde{p}_2$ are the momenta of particles in the C frame.

CONSERVATION LAWS IN NUCLEAR REACTIONS

13.1. An α-particle with kinetic energy $T_0 = 1.0$ MeV is scattered elastically by an initially stationary ^{6}Li nucleus. Find the kinetic energy of the recoil nucleus ejected at an angle $\vartheta = 30°$ to the initial direction of the α-particle's motion.

13.2. Find the kinetic energy of an incoming α-particle, if after its elastic scattering by a deuteron: **(a)** $B\rho$ of each particle turns out to be 60 kG·cm; **(b)** the angle of divergence of two particles $\vartheta = 120°$ and the amount of energy acquired by the deuteron $T_d = 0.40$ MeV.

13.3. A non-relativistic deuteron is elastically scattered through an angle of 30° by a stationary nucleus. The recoil nucleus is ejected at the same angle to the direction of motion of the incoming deuteron. To what atom does that nucleus belong?

13.4. Plot the vector diagrams of momenta for elastic scattering of a non-relativistic α-particle by a stationary nucleus: **(a)** ^{6}Li, **(b)** ^{4}He, **(c)** ^{2}H, if the angle of scattering of the α-particle in the C frame is equal to 60°. In what case is the relation between the energy of scattered α-particle and its angle of scattering described by a non-single-valued function? Find the greatest possible angle of scattering of the α-particle for each of these three cases.

13.5. Find the fraction of the kinetic energy lost by a non-relativistic α-particle due to elastic scattering at an angle $\tilde{\vartheta} = 60°$ (in the C frame) by a stationary ^{12}C nucleus.

13.6. A proton with a kinetic energy of 0.90 MeV sustains an elastic head-on collision with a stationary deuteron. Find the proton's kinetic energy after the collision.

13.7. A non-relativistic neutron is scattered elastically through the angle ϑ_n by a stationary ^{4}He nucleus so that the latter is ejected at an angle of 60° to the direction of motion of the incoming neutron. Determine the angle ϑ_n.

13.8. A non-relativistic α-particle is elastically scattered by a ^{6}Li. Determine the angle of scattering of the α-particle: **(a)** in the L frame provided that in the C frame $\tilde{\vartheta}_\alpha = 30°$; **(b)** in the C frame provided that in the L frame $\vartheta_\alpha = 45°$.

13.9. Deuterons with a kinetic energy of 0.30 MeV are elastically scattered by protons. Find the kinetic energy of the deuterons scattered through the greatest possible angle in the L frame. What is the magnitude of the angle?

13.10. Find the energy of the reaction $^7Li(p, \alpha)^4He$ if the mean binding energies per nucleon in ^7Li and ^4He nuclei are known to be equal to 5.60 and 7.06 MeV respectively.

13.11. Determine the energies of the following reactions: **(a)** $^3H(p, \gamma)^4He$; **(b)** $^{14}N(\alpha, d)^{16}O$; **(c)** $^{12}C(\alpha, d)^{14}N$; **(d)** $^6Li(d, n\alpha)^3He$.

13.12. Using the tables, calculate the mass of ^{17}N atom, if the energy of the reaction $^{17}O(n, p)^{17}N$ is known to be $Q = -7.89$ MeV.

13.13. Find the velocity with which the products of the reaction $^{10}B(n, \alpha)^7Li$ come apart; the reaction proceeds due to interaction of slow neutrons with stationary boron nuclei.

13.14. Find the energy of neutrons produced due to photodisintegration of beryllium according to the reaction $^9Be(\gamma, n)^8Be$ by γ-quanta with an energy of $\hbar\omega = 1.78$ MeV. The energy of the reaction is $Q = -1.65$ MeV.

13.15. A deuterium target irradiated by γ-quanta with an energy of $\hbar\omega = 2.62$ MeV emits photoprotons for which $B\rho = 63.7$ kG·cm. Ignoring the difference in the masses of a neutron and a proton, find the binding energy of a deuteron.

13.16. Calculate the energies of the following reactions: **(a)** $^2H(d, p)^3H$, if the energy of the incoming deuterons $T_d = 1.20$ MeV and the proton, outgoing at right angles to the direction of the deuteron's motion, has an energy $T_p = 3.30$ MeV; **(b)** $^{14}N(\alpha, p)^{17}O$, if the energy of the incoming α-particles $T_\alpha = 4.00$ MeV and the proton, outgoing at an angle $\vartheta = 60°$ to the direction of motion of α-particles, has an energy of $T_p = 2.08$ MeV.

13.17. Determine the kinetic energy of protons activating the reaction $^9Be(p, \alpha)^6Li + 2.13$ MeV, if the range of α-particles, outgoing at right angles to the direction of motion of the protons, is equal to 2.5 cm in air at NTP.

13.18. Deuterons with a kinetic energy of $T_d = 10.0$ MeV collide with carbon nuclei and initiate the reaction $^{13}C(d, \alpha)^{11}B$, $Q = +5.16$ MeV. Determine the angle between the directions in which the products of the reaction are ejected, if: **(a)** the produced nuclei diverge in a symmetric pattern; **(b)** the α-particle is ejected at right angles to the deuteron beam.

13.19. Derive formula (13.1).

13.20. Calculate the threshold kinetic energies of α-particles and neutrons in the following reactions:

(a) $\alpha + {}^7Li \rightarrow {}^{10}B + n$; **(b)** $\alpha + {}^{12}C \rightarrow {}^{14}N + d$;

(c) $n + {}^{12}C \rightarrow {}^9Be + \alpha$; **(d)** $n + {}^{17}O \rightarrow {}^{14}C + \alpha$.

13.21. Calculate the threshold kinetic energy of an incoming particle in the reaction $p + {}^3H \rightarrow {}^3He + n$, for the cases when that particle is: **(a)** a proton; **(b)** a tritium nucleus.

13.22. Determine the kinetic energies of ^7Be and ^{15}O nuclei produced in the reactions:

(a) $p + {}^7Li \rightarrow {}^7Be + n$, $\quad Q = -1.65$ MeV;

(b) $n + {}^{19}F \rightarrow {}^{15}O + p + 4n$, $\quad Q = -35.8$ MeV

for the threshold value of energy of the proton and neutron.

13.23. A lithium target is irradiated with a beam of protons whose kinetic energy exceeds the threshold value 1.50 times. Find the energy of neutrons ejected as a result of the reaction $^7Li(p, n)^7Be - 1.65$ MeV at an angle of 90° to the proton beam.

13.24. Evaluate the lowest kinetic energy an incoming α-particle requires to overcome the Coulomb potential barrier of a ^7Li nucleus. Will this amount of energy be sufficient for the α-particle to activate the reaction $^7Li(\alpha, n)^{10}B$?

13.25. Neutrons with the kinetic energy $T = 10.0$ MeV activate the reaction $^{10}B(n, d)^9Be$ for which $T_{th} = 4.8$ MeV. Find the kinetic energy of deuterons for the reverse reaction under assumption that the total energies of interacting particles are equal for both processes in the C frame.

13.26. Derive the expression for the momentum $\tilde{p}$ of particles produced by the reaction $M\ (m, m')\ M' + Q$ in the C frame, if the kinetic energy of an incoming particle in the L frame is equal to T_m.

13.27. Determine the kinetic energy of oxygen nuclei ejected following the reaction $^{14}N(p, n)^{14}O - 5.9$ MeV at an angle of 30° to the direction of motion of the striking protons whose kinetic energy is 10.0 MeV. Obtain the solution, using the vector diagram of momenta drawn to scale.

13.28. Find the highest kinetic energy of α-particles produced by the reaction $^{16}O(d, \alpha)^{14}N + 3.1$ MeV, if the energy of the striking deuterons is 2.0 MeV.

13.29. Find the width of the energy spectrum of neutrons produced by the reaction $^{11}B(\alpha, n)^{14}N + 0.30$ MeV, if the kinetic energy of striking α-particles is equal to 5.0 MeV.

13.30. A lithium target is bombarded with α-particles with the kinetic energy T_α. As a result of the reaction $^7Li(\alpha, n)^{10}B$, $Q = -2.79$ MeV, the target emits neutrons. Find:

(a) the kinetic energies of neutrons ejected at the angles 0, 90, and 180° to the direction of motion of the striking α-particles, if $T_\alpha = 10.0$ MeV;

(b) at what values of T_α the neutrons will be emitted into the front hemisphere only ($\vartheta \leqslant 90°$).

13.31. To obtain high-intensity fluxes of fast neutrons, lithium deuteride LiD is placed into a reactor, so that slow neutrons activate the reaction $^6Li(n, \alpha)^3H + 4.80$ MeV. The generated tritium nuclei in its turn activate the reactions: **(a)** $D(t, n)^4He + 17.6$ MeV and **(b)** $^7Li(t, n)^9Be + 10.4$ MeV, providing fast neutrons. Find the highest energies of these neutrons.

13.32. Neutrons with an energy of 1.50 MeV strike a target possessing the nuclides ^6Li and ^2H. Using the vector diagram of momenta, determine the width of energy spectrum of neutrons appearing after the following successive transformations:

$$n + {}^6Li \to {}^4He + {}^3H; \quad {}^3H + {}^2H \to {}^4He + n.$$

13.33. Find the greatest possible angles (in the L frame) at which the products of the following reactions move:

(a) $^9Be(p, n)^9B - 1.84$ MeV, if $T_p = 4.00$ MeV;

(b) $^4He(n, d)^3H - 17.5$ MeV, if $T_n = 24.0$ MeV.

Here T is the kinetic energy of a striking particle.

13.34. A beam of neutrons with an energy of 7.5 MeV activates the reaction $^{12}C(n, \alpha)^9Be$ — 5.70 MeV in a carbon target. Find: **(a)** the fraction of α-particles ejected into the front hemisphere ($\vartheta_\alpha \leqslant 90°$), assuming the angular distribution of the reaction products to be isotropic in the C frame; **(b)** the angle at which the α-particle is ejected in the C frame, if the corresponding angle in the L frame is equal to $\vartheta_\alpha = 30°$.

13.35. Find the threshold energy of a γ-quantum sufficient to activate the endoergic photodisintegration of a stationary nucleus of mass M, if the reaction yield is equal to Q.

13.36. Calculate the kinetic energies of neutrons in the following disintegration reactions: **(a)** $\gamma + d \to n + p$; **(b)** $\gamma + {}^7Li \to n + {}^6Li$, if the γ-quanta possess the threshold values of energy.

13.37. Demonstrate that in a nuclear photodisintegration reaction $\gamma + M \to m_1 + m_2$, when the products of the reaction are nonrelativistic, the momenta of generated particles in the C frame are described by the formula $p \approx \sqrt{2\mu' (Q + \hbar\omega)}$, where μ' is the reduced mass of the generated particles, Q is the energy of the reaction, $\hbar\omega$ is the energy of the γ-quantum.

13.38. γ-quanta with an energy of 6.40 MeV interacting with tritium nuclei activate the reaction $^3H(\gamma, n)^2H$, $Q = -6.26$ MeV. Assuming the angular distribution of neutrons in the C frame to be isotropic, find the probability of a deuteron being ejected into the front hemisphere ($\vartheta_d \leqslant 90°$) in the L frame.

13.39. A beryllium target is irradiated with a narrow beam of deuterons with an energy of $T_d = 190$ MeV. Beyond the target, a beam of neutrons is observed (in the direction of the primary deuteron beam) with an angular width of $\Delta\theta = 16°$. Making use of the assumption concerning the mechanism of stripping reaction, find the energy spread of the neutrons.

13.40. Find the possible spin value of a ^{17}O nucleus in the ground state appearing due to stripping reaction involving the interaction of deuterons with ^{16}O nuclei, if the orbital moment of captured neutrons equals $l_n = 2$. Compare the result with the spin value given by the nuclear shell model.

13.41. Consider the two following reaction branches proceeding via a compound $^8Be^*$ nucleus:

$$p + {}^7Li \to {}^8Be^* \begin{cases} \nearrow 2\,{}^4He & (1) \\ \searrow {}^8Be + \gamma. & (2) \end{cases}$$

The spin and parity of ^7Li and ^8Be nuclei in the ground state are equal to $3/2^-$ and 0^+ respectively, the spin of α-particle is 0, the internal parity of proton is to be assumed positive. Using the laws of conservation of angular momentum and parity, find for the cases when the orbital moment of proton l is equal to 0 and 1: **(a)** the possible values of spin I and parity P of the compound nucleus; **(b)** the states (spin and parity) of the compound nucleus in both reaction branches.

ENERGY LEVELS IN A NUCLEUS. REACTION CROSS-SECTIONS AND YIELDS

13.42. Find the excitation energy of a stationary nucleus of mass M which it acquires on the capture of γ-quantum of energy $\hbar\omega$.

13.43. Determine the excitation energy of a ^4He nucleus appearing after the capture of a proton with a kinetic energy of 2.0 MeV by a stationary ^3H nucleus.

13.44. What is the lowest kinetic energy of a neutron capable, after inelastic scattering by a ^9Be nucleus, to transfer to the latter an excitation energy of 2.40 MeV?

13.45. A ^7Li target is bombarded with a beam of neutrons with energy $T = 1.00$ MeV. Determine the excitation energy of nuclei generated due to inelastic scattering of neutrons, if the energy of neutrons scattered inelastically at right angles to the incident beam is $T = 0.33$ MeV.

13.46. Calculate the energies of protons scattered inelastically at right angles by stationary ^{20}Ne nuclei. The lower levels of ^{20}Ne nucleus are known to correspond to excitation energies of 1.5, 2.2, and 4.2 MeV. The energy of the striking protons $T_0 = 4.3$ MeV.

13.47. Find the kinetic energies of neutrons providing the maximum interaction cross-sections for ^{16}O nuclei, if the lower levels of the compound nucleus correspond to the following excitation energies: 0.87, 3.00, 3.80, 4.54, 5.07, and 5.36 MeV.

13.48. Deuterons bombarding a carbon target activate the nuclear reaction $^{13}C(d, n)^{14}N$ whose maximum yield is observed for the following values of energy of deuterons: 0.60, 0.90, 1.55, and 1.80 MeV. Find the corresponding levels of the compound nucleus through which the given reaction proceeds.

13.49. A boron target is irradiated with a beam of deuterons with an energy of 1.50 MeV. It is found that due to the reaction (d, p) in ^{10}B nuclei the target emits protons with energies of 7.64, 5.51, and 4.98 MeV at right angles to the beam of deuterons. Find the levels of excited ^{11}B nuclei corresponding to these energies.

13.50. Find the ratio of intensities of monochromatic groups of neutrons that are inelastically scattered at right angles to the incident beam by the ^{27}Al nuclei whose lower levels correspond to excitation energies of 0.84, 1.02, and 1.85 MeV. The energy of the striking neutrons is equal to 1.40 MeV. The cross-section of inelastic scattering of neutrons in the vicinity of the threshold is known to be proportional to the velocity of inelastically scattered neutrons.

13.51. Find the expression for the cross-section of the reaction $A\ (a,\ b)\ B$, if the cross-section of the compound nucleus formation σ_a and the widths of its level, Γ and Γ_b, through which the reaction proceeds, are known. Here Γ is the total width of the level, and Γ_b is the partial width corresponding to the emission of particle b.

13.52. Determine the mean lifetime of excited nuclei appearing after the capture of neutrons with an energy of 250 keV by ^6Li nuclei, if the mean lifetimes of these nuclei are known with respect to emission of neutrons and α-particles: $\tau_n = 1.1 \cdot 10^{-20}$ s, $\tau_\alpha = 2.2 \times \times 10^{-20}$ s (no other processes are involved).

13.53. The rate of a nuclear reaction can be characterized by the mean duration τ of bombardment of a given nucleus prior to the moment of its activation. Find τ for the reaction $^{60}Ni(\alpha,\ n)^{63}Zn$, if the current density of α-particles is $J = 16\ \mu A/cm^2$ and the reaction cross-section $\sigma = 0.5$ b.

13.54. Find the flux density of neutrons at a distance of 10 cm from a small (Po-Be) source containing 0.17 Ci of ^{210}Po, if the yield of the reaction $^9Be(\alpha,\ n)^{12}C$ is equal to $0.8 \cdot 10^{-4}$.

13.55. A beryllium target becomes an intensive neutron source due to irradiation with deuterons accelerated to an energy of 10 MeV. Find the number of neutrons emitted per 1 s per 100 μA of deuteron current, if the yield of the reaction $^9Be(d,\ n)^{10}B$ is equal to $5 \cdot 10^{-3}$. What amount of radium must a (Ra-Be) source possess to have the same activity? The yield of that source is assumed to be equal to $2.0 \cdot 10^7$ neutrons per second per one gram of Ra.

13.56. A BF_3 gas having the volume $V = 10\ cm^3$ at NTP is irradiated with thermal neutrons whose flux density $J = 1.0 \cdot 10^{10}$ neutrons/(s$\cdot$cm^2). Find: **(a)** the number of nuclear reactions $(n,\ \alpha)$ involving boron nuclei occurring within the given volume during one second; **(b)** the thermal power liberated in this volume as a result of the reaction $(n,\ \alpha)$ involving boron nuclei.

13.57. The irradiation of a thin target of heavy ice with 1 MeV deuterons activates the reaction $^2H(d,\ n)^3He$ whose yield and cross-section are equal to $0.8 \cdot 10^{-5}$ and 0.020 b respectively. Determine the cross-section of this reaction for the deuterons' energy of 2 MeV, if at this energy the yield amounts to $4.0 \cdot 10^{-5}$.

13.58. The yield of the reaction $(\gamma,\ n)$ on the exposure of a copper plate of thickness $d = 1.0$ mm to γ-quanta with an energy of 17 MeV is $w = 4.2 \cdot 10^{-4}$. Find the cross-section of the given reaction.

13.59. A narrow beam of monochromatic neutrons (0.025 eV) with an intensity of $2.0 \cdot 10^8$ neutrons/s passes through a chamber containing a nitrogen gas at NTP. Find the cross-section of the reaction $(n,\ p)$, if it is known that 95 protons are produced during 5.0 ms over 1.0 cm of the beam's length.

13.60. A thin plate of ^{113}Cd is irradiated with thermal neutrons whose flux density is $1.0 \cdot 10^{12}$ neutrons/(s$\cdot$cm^2). Find the cross-section of the reaction $(n,\ \gamma)$, if the content of ^{113}Cd nuclide is known to diminish by 1.0% after six days of irradiation.

13.61. A thin plate made of boron of natural isotopic content is irradiated for a year with thermal neutrons of intensity $J = 2.00 \times \times 10^{12}$ neutrons/s. The reaction involving ^{10}B nuclei reduces their content to 16.4% by the end of irradiation. Determine the cross-section of the given reaction.

13.62. Determine the yield of the reaction $(n,\ \alpha)$ activated in a target 0.50 cm thick, made of lithium of natural isotopic content, by a beam of thermal neutrons.

13.63. An iron target is irradiated with a beam of protons with an energy of 22 MeV. As a result of the nuclear reaction $(p,\ n)$ whose yield $w = 1.2 \cdot 10^{-3}$ a ^{56}Co radionuclide is produced with a half-life of 77.2 days. Determine the activity of the target $\tau = 2.5$ h after the beginning of irradiation, if the proton's current $J = 21\ \mu A$.

13.64. A target of metallic sodium was irradiated with a beam of deuterons with an energy of 14 MeV and a current of 10 μA for a long period of time. Find the yield of the reaction $(d,\ p)$ producing a ^{24}Na radionuclide, if the activity of the target 10 h after the end of irradiation is 1.6 Ci.

13.65. A thin phosphorus plate of thickness 1.0 g/cm^2 was irradiated for $\tau = 4.0$ h with a neutron flux of $2.0 \cdot 10^{10}$ neutrons/s with a kinetic energy of 2 MeV. One hour after the end of irradiation, the activity of the plate turned out to be 105 μCi. The activity is known to result from ^{31}Si nuclide produced by the reaction $(n,\ p)$. Determine the cross-section of the given reaction.

13.66. A thick* aluminium target irradiated with a beam of α-particles with an energy of 7.0 MeV emits $1.60 \cdot 10^9$ neutrons/s resulting from the reaction $(\alpha,\ n)$. Find the yield and mean cross-section of the given reaction, if the current of α-particles is equal to 50 μA.

13.67. A thick* beryllium target is bombarded with α-particles with an energy of 7.0 MeV. Determine the mean cross-section of the reaction $(\alpha,\ n)$, if its yield amounts to $2.50 \cdot 10^{-4}$.

13.68. A thick* target made of ^7Li nuclide is bombarded with α-particles with an energy of 7.0 MeV. Find the mean cross-section of the reaction $^7Li(\alpha,\ n)^{10}B - 4.4$ MeV, if its yield $w = 2.8 \cdot 10^{-5}$.

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* A target is referred to as "thick" when its thickness exceeds the range of a striking particle in the target's material.

13.69. A beam of α-particles with an energy of 7.8 MeV enters a chamber filled with air at NTP. The length of the chamber along the beam exceeds the range of α-particles of the given energy. Find the mean cross-section of the reaction $^{14}N(\alpha, p)^{17}O - 1.20$ MeV, if the yield of the reaction is $2.0 \cdot 10^{-6}$. The nitrogen content in air is 78% by volume.

13.70. A beam of neutrons with an energy of 14 MeV falls normally on the surface of a beryllium plate. Evaluate the thickness of the plate sufficient for the 10% reproduction of neutrons by means of the reaction $(n, 2n)$ whose cross-section $\sigma = 0.50$ b for the given energy of neutrons. Other processes are assumed non-existent, the secondary neutrons are not to be absorbed in the plate.

13.71. A thick target containing n_0 nuclei/cm^3 is irradiated with heavy charged particles. Find how the cross-section of a nuclear reaction depends on the kinetic energy T of striking particles, if the reaction yield as a function of the particles' energy, $w(T)$, and the expression for ionization loss of energy of these particles, $dT/dx = f(T)$, are known.

13.72. When a deuterium target is irradiated with deuterons, the following reaction occurs: $d + d \rightarrow {}^3He + n$, $Q = +3.26$ MeV. Making use of the detailed balancing principle, find the spin of a ^{3}He if the cross-section of this process equals σ_1 for energy of deuterons $T = 10.0$ MeV, while the cross-section of the reverse process for the corresponding energy of striking neutrons $\sigma_2 = 1.8\sigma_1$. The spins of a neutron and a deuteron are supposed to be known.

13.73. Using the detailed balancing principle, find the cross-section σ_1 of the reaction $\alpha + {}^6Li \rightarrow {}^9Be + p - 2.13$ MeV, if the energy of striking α-particles is $T = 3.70$ MeV and the cross-section of the reverse reaction with the corresponding energy of protons is $\sigma_2 = 0.050$ mb.

13.74. Using the detailed balancing principle, demonstrate that the cross-section of an endoergic reaction $A(p, n)B$ activated due to irradiation of a target with protons of energy T_p is proportional to $\sqrt{T_p - T_{p\,th}}$ in the vicinity of the threshold, if in the case of slow neutrons the cross-section of the reverse reaction is proportional to $1/v_n$, v_n being the velocity of the neutrons.

13.75. The cross-section of the deuteron photodisintegration reaction $\gamma + d \rightarrow n + p$, $Q = -2.22$ MeV is $\sigma_1 = 0.150$ mb for an energy of γ-quanta $\hbar\omega = 2.70$ MeV. Using the detailed balancing principle, find the cross-section σ_2 of the reverse process for the corresponding energy T_n of striking neutrons. Calculate this value of T_n.

14

NEUTRON PHYSICS

● Aiming parameter of a neutron

$$b = \lambda\!\!\!^{-} \sqrt{l(l+1)}, \qquad (14.1)$$

where $\lambda\!\!\!^{-} = \lambda/2\pi$ is its wavelength, l is the orbital quantum number.

● Breit-Wigner formula for an individual level gives the cross-section of formation of compound nucleus by slow s-neutrons ($l = 0$):

$$\sigma_a = \pi \lambda\!\!\!^{-2} g \frac{\Gamma\Gamma_n}{(T-T_0)^2 + (\Gamma/2)^2}; \quad g = \frac{2J+1}{2(2I+1)}, \qquad (14.2)$$

where $\lambda\!\!\!^{-}$ and T are the wavelength and kinetic energy of an incoming neutron, T_0 is the kinetic energy of a neutron corresponding to the given level of the compound nucleus M^* (Fig. 39), g is the statistical weight, I is the spin of the target nucleus, J is the spin of the given level of the compound nucleus, Γ and Γ_n is the total and neutron width of the level, Γ_n depends on the wavelength of the incoming neutron, $\lambda\!\!\!^{-}\Gamma_n = \lambda\!\!\!^{-}_0\Gamma_{n0}$, $\lambda\!\!\!^{-}_0$ and Γ_{n0} are the neutron's wavelength and neutron width of the level at $T = T_0$.

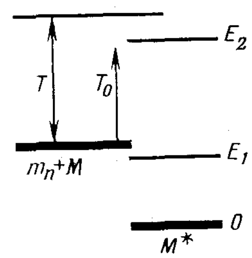


Fig. 39

● Rate of nuclear reaction:

$$R = \langle \Sigma \rangle \Phi \text{ reaction/(cm}^3\cdot\text{s)}, \qquad (14.3)$$

where $\langle \Sigma \rangle = N \langle \sigma \rangle$ is the mean macroscopic cross-section of reaction, N is the concentration of nuclei, $\Phi = n\langle v \rangle$ is the flux density of neutrons, n is the concentration of neutrons, and $\langle v \rangle$ is their mean velocity.

● Mean value of the cosine of the angle at which neutrons are scattered due to elastic collisions with stationary nuclei of mass number A:

$$\langle \cos\vartheta \rangle = \frac{2}{3A}. \qquad (14.4)$$

● Logarithmic loss of energy is $\ln(T_0/T)$, where T_0 and T are the initial and final kinetic energies of a neutron.

● Mean logarithmic loss of energy of a neutron undergoing a single elastic collision with a nucleus:

$$\xi = 1 + \frac{\alpha}{1-\alpha}\ln\alpha; \qquad \alpha = \left(\frac{A-1}{A+1}\right)^2, \qquad (14.5)$$

where A is the mass number of the nucleus.

● Age of neutrons moderated from energy T_0 down to T:

$$\tau = \int_{T}^{T_0} \frac{1}{3\xi\Sigma_s\Sigma_{tr}} \frac{dT}{T} \text{ cm}^2;$$
$$\Sigma_{tr} = \Sigma_s (1 - \langle\cos\vartheta\rangle), \quad (14.6)$$

where ξ is the mean logarithmic loss of energy, Σ_s and Σ_{tr} are the macroscopic scattering and transport cross-sections.

● Moderation density $q(E)$ is the number of neutrons in 1 cm³ crossing a given energy level E per one second in the process of moderation. For a point source of fast monoergic neutrons in an infinite homogeneous moderating medium

$$q_E(r) = \frac{n}{4\pi\tau} e^{-r^2/4\tau}, \quad (14.7)$$

where n is the source intensity, neutrons/s, τ is the neutron age, cm², r is the distance from the source, cm.

● Neutron diffusion equation for a medium without multiplication:

$$\frac{dn}{dt} = D\nabla^2\Phi - \Sigma_a\Phi;$$
$$D = \frac{1}{3\Sigma_{tr}}; \quad L_{dif} = \sqrt{D/\Sigma_a}, \quad (14.8)$$

where n is the concentration of neutrons, D is the diffusion coefficient, ∇^2 is the Laplace operator, Φ is the flux density of neutrons, Σ_a is the macroscopic absorption cross-section, L_{dif} is the diffusion length.

● Neutron albedo β is the probability of neutrons being reflected after multiple scattering in a medium.

NEUTRON SPECTROSCOPY

14.1. One of the first designs for mechanical selection of neutrons consists of two discs fixed to an axle rotating at a speed of n rps. The distance between discs is L. Each disc has a radial slit displaced relative to each other by the angle α. Find the energy of neutrons filtered through such a selector, if $n = 100$ rps, $L = 54$ cm, and $\alpha = 8°$.

14.2. In a mechanical neutron selector, constructed as a stack of alternating aluminium plates of a thickness of 0.75 mm and thin cadmium layers, the total length of the stack is equal to 50 mm. What must be the speed of rotation of the stack to arrest neutrons with energies below 0.015 eV? What is the neutron pulse duration in this case?

14.3. A mechanical time-of-flight neutron selector has a resolution $\Delta\tau/L$ µs/m. Find the energy resolution $\Delta T/T$ of that selector as a function of neutron energy T, eV. Assuming $\Delta\tau/L = 1.0$ µs/m, find $\Delta T/T$ for $T = 5.0$ eV, and the highest value of T at which $\Delta T/T$ is better than 10%.

14.4. Is a mechanical neutron selector with time-of-flight resolution of 0.50 µs/m acceptable to study the shape of a resonance curve in silver for an energy of 5.0 eV and half-width of 0.20 eV?

14.5. In a pulsed cyclotron installation the total width of a neutron pulse and a channel of a time analyzer is equal to 1.0 µs. Evaluate the distance from a moderator to the time analyzer of this installation sufficient to resolve two resonances lying in the vicinity of 50 eV and separated by the interval of 0.50 eV.

14.6. Calculate the energy of neutrons reflected from a set of planes of NaCl crystal with $d = 3.25$ Å through a glancing angle of 4.0°. The incident beam consists of neutrons with energies below 3.0 eV.

14.7. In a beryllium crystal monochromator, the neutron reflection of the first order from a set of planes with $d = 0.75$ Å is used. Evaluate the energy resolution $(\Delta T/T)$ of this monochromator for neutrons with energy about $T = 0.30$ eV, if the incident neutron beam has an angular spread of $\Delta\vartheta = 0.5°$.

14.8. A LiF crystal monochromator that employs neutron reflection of the first order from a set of planes with $d = 2.32$ Å is used to resolve two groups of resonant neutrons with kinetic energies of 0.49 and 0.51 eV. At what angular divergence of the incident neutron beam can it be done?

14.9. When a thermal neutron beam passes through a thick chunk of pressed crystalline powder, the neutrons of sufficiently long wavelength penetrate the whole length of the chunk without reflections from crystalline planes. Find the kinetic energy of neutrons passing through a thick chunk of graphite. The maximum interplanar distance of graphite is $d = 3.35$ Å.

14.10. Figure 40 illustrates a scintillation spectrometer of fast neutrons. A neutron beam to be investigated falls on a stylbene

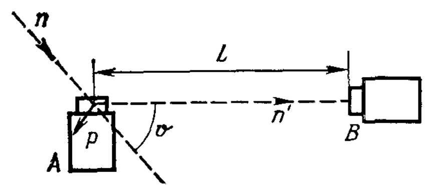


Fig. 40

scintillator of a photomultiplier A. After elastic collision with protons of stylbene, the neutrons scattered through the angle ϑ are registered by a photomultiplier B while the recoil protons by the photomultiplier A. The output pulses from either photomultiplier are fed to coincidence circuit, the pulses from the photomultiplier A being delayed to account for the time taken by the scattered neutron to travel the distance L. Find: **(a)** the kinetic energy of primary neutrons, if the delay time required to observe the highest number of pulse coincidences is $2.20 \cdot 10^{-8}$ s, $L = 50$ cm, and $\vartheta = 45°$; **(b)** the accuracy of determination of neutron energy, if the diameter of the crystal of the photomultiplier B is equal to 3.0 cm.

14.11. On the basis of quasiclassical concepts derive the expression for the aiming parameter b of a striking neutron. Calculate the first three allowed values of b for neutrons with a kinetic energy of 1.00 MeV.

14.12. Find the maximum value of aiming parameter for neutrons with a kinetic energy of 5.0 MeV interacting with Ag nuclei.

14.13. Demonstrate that for neutrons with the wavelength $\lambda\!\!\!^{-}$ the geometric nuclear cross-section $S \approx \pi (R + \lambda\!\!\!^{-})^2$, where R is the radius of the nucleus. Estimate this value for the case of a neutron with a kinetic energy of 10 MeV interacting with an Au nucleus.

14.14. Evaluate the maximum centrifugal barrier height for neutrons with a kinetic energy of 7.0 MeV interacting with Sn nuclei.

14.15. Evaluate the angle ϑ within which the neutrons are scattered after elastic diffraction by lead nuclei. The neutron energy is 50 MeV.

14.16. Find the probability that after interaction of slow neutrons ($l = 0$) with nuclei whose spin $I = 1$, the compound nuclei are formed in the state with spin $J = 3/2$. The spins of neutrons and nuclei are assumed to be randomly oriented relative to one another.

14.17. On the basis of the Breit-Wigner formula for the cross-section of formation of a compound nucleus σ_a, derive the expressions for the cross-sections of elastic scattering and radiative capture of a neutron.

14.18. Using the Breit-Wigner formula, derive the radiative neutron capture cross-section $\sigma_{n\gamma}$ as a function of the kinetic energy of a neutron T, if the cross-section of this process is known for $T = T_0$; values of T_0 and Γ are also known.

14.19. Calculate the cross-section of the reaction $^{115}In(n, \gamma)^{116}In$ for a neutron energy of 0.50 eV, if under the conditions of resonance $\sigma_0 = 2.76 \cdot 10^4$ b, $T_0 = 1.44$ eV, and $\Gamma = 0.085$ eV. The neutron width Γ_n is known to be much smaller than the radiation width Γ_γ.

14.20. When thermal neutrons with an energy of 0.025 eV interact with ^{113}Cd nuclei, the scattering cross-section amounts to 0.22% of the radiative capture cross-section. Determine the ratio of probability of a compound nucleus decaying with emission of neutrons to that with emission of γ-quanta, if the resonant value of neutron energy $T_0 = 0.178$ eV.

14.21. Using the Breit-Wigner formula, find: **(a)** the values of the kinetic energy of a neutron (T_{max} and T_{min}) at which the radiative capture cross-section $\sigma_{n\gamma}$ reaches its maximum and minimum (T_0 and Γ are assumed to be known); find under what conditions $T_{max} \approx T_0$; **(b)** by how many percents the cross-section σ_0 of the process (n, γ) at $T = T_0$ differs from the resonant value σ_{max} of that process, if $\Gamma = T_0$; **(c)** the values of the ratio Γ/T_0 at which radiative capture of neutrons does not exhibit its selective nature.

14.22. Applying the Breit-Wigner formula for radiative capture of neutrons, find the ratio σ_{min}/σ_0, where σ_{min} is the minimum cross-section of the process (n, γ) in the region $T < T_0$; σ_0 is the cross-section of this process at $T = T_0$, if $\Gamma \ll T_0$.

14.23. Using the Breit-Wigner formula, determine the width Γ of a level in a compound nucleus appearing after the capture of a neutron by a ^{113}Cd nucleus, if the radiative capture cross-section for neutron energy $T = 2T_0$ is 1/15 of that for $T = T_0$, where $T_0 = 0.178$ eV. Γ is supposed to be independent of the neutron energy.

14.24. Using the Breit-Wigner formula, show that if the half-width ΔT of a resonant peak of the curve $\sigma_{n\gamma}(T)$ is small ($\Delta T \ll T_0$), then $\Delta T \approx \Gamma$.

14.25. The resonance energy of neutrons interacting with ^{59}Co nuclei is $T_0 = 132$ eV, the corresponding neutron width $\Gamma_{n0} = 0.9\Gamma$, and $\Gamma \ll T_0$. Using the Breit-Wigner formula, find: **(a)** the resonance cross-section of elastic scattering of neutrons; **(b)** the spin of the state of the compound nucleus through which the process proceeds, if the total resonance cross-section $\sigma_0 = 1.0 \cdot 10^4$ b.

14.26. Find the ratio of the resonance cross-section of elastic scattering of neutrons by ^{55}Mn nuclei to the geometric cross-section of these nuclei, if $T_0 = 337$ eV, $\Gamma_{n0} \approx \Gamma \ll T_0$, and the spin of the transitional nucleus, through which the process proceeds, is $J = 2$.

14.27. The cross-section of radiative capture of neutrons by ^{149}Sm nuclei under conditions of resonance ($T_0 = 0.097$ eV) is $\sigma_0 = 1.2 \cdot 10^5$ b. Find the neutron width Γ_{n0} for the resonance energy of neutrons, if $\Gamma_{n0} \ll \Gamma = 0.064$ eV and the spin of ^{149}Sm nucleus $I = 7/2$.

14.28. Evaluate the lifetime of a compound nucleus appearing on capture of a neutron by a ^{103}Rh nucleus, if at the resonance energy of neutrons $T_0 = 1.26$ eV the cross-section of the process (n, γ) is $\sigma_0 = 2700$ b, $\Gamma_\gamma \gg \Gamma_{n0} = 7.8 \cdot 10^{-4}$ eV, and $g = 1/4$.

14.29. Using the Breit-Wigner formula, find the cross-section of radiative capture of slow neutrons as a function of their kinetic energy, when the compound nucleus has a "negative" energy level, $T_0 < 0$ (the level E_1 in Fig. 39). Find out how $\sigma_{n\gamma}$ behaves as T varies in the regions $T \ll |T_0|$ and $T \gg |T_0|$.

14.30. Using the Breit-Wigner formula, identify the conditions under which the cross-section of radiative capture of neutrons obeys the $1/v$ law.

14.31. On irradiation of a magnesium target with neutrons (2.5 MeV), it was observed that in addition to elastically scattered neutrons there is a group of inelastically scattered neutrons whose energy corresponds to a certain excited level of transitional nuclei (1.3 MeV). Determine the relative width of the given level for inelastic scattering, if it is known that the total cross-section of the process $\sigma_{tot} = 2.2$ b and the elastic scattering cross-section $\sigma_{el} = 1.6$ b, 44% of which represents potential scattering.

14.32. What must the thickness of cadmium plate be to reduce the flux of thermal neutrons 100-fold after passing through it?

14.33. How many times will a narrow beam of thermal neutrons be attenuated after passing through a layer of heavy water 1.0 cm in thickness?

14.34. Evaluate in what proportion a narrow beam of fast neutrons with an energy of 10 MeV attenuates on passing through a lead plate 4.0 cm in thickness. The effective nuclear cross-section is assumed to be $\sigma = 2\pi (R + \lambda\!\!\!^{-})^2$; R is the radius of the nucleus; $\lambda\!\!\!^{-}$ is the neutron wavelength.

14.35. In the centre of a spherical graphite layer whose inside and outside radii are $r_1 = 1.0$ cm and $r_2 = 10.0$ cm a point source of monochromatic neutrons is located, emitting $I_0 = 2.0 \cdot 10^4$ neutrons/s with an energy of 2.0 MeV. The interaction of neutrons of such an energy with carbon nuclei is characterized by a cross-section $\sigma = 1.6$ b. Determine the neutron flux density at the outside surface of the layer, counting only neutrons that penetrated the layer without collisions.

14.36. The intensity of a narrow beam of slow monochromatic neutrons diminishes 20-fold on passing through a plate of natural boron with a mass thickness of 1.0 g/cm^2. Determine the energy of neutrons, taking into account that the $1/v$ law is valid in this case.

14.37. A narrow beam of neutrons with an energy of 10.0 eV passes a distance $l = 15.0$ cm along the axis of a counter filled up with BF_3 at NTP (natural boron is used). Determine the counter efficiency provided that the cross-section of the reaction (n, α) is known to obey the $1/v$ law.

14.38. In a neutron counter with LiI crystal sensor the reaction (n, α) in ^{6}Li nuclei is used. Determine the efficiency of the counter for a thermal neutron beam, if the thickness of the crystal is known to be 2.0 cm and density 4.0 g/cm^3 (natural lithium is used). The scattering of neutrons is to be neglected.

14.39. Find the decrease in efficiency (%) of a neutron detector, a thin ^{10}B layer, that was irradiated for a week by a plane flux of thermal neutrons with a density $J = 1.00 \cdot 10^{13}$ neutrons/(cm^2·s).

14.40. A non-monochromatic beam of slow neutrons falls on a thin target activating a nuclear reaction whose cross-section is $\sigma \propto 1/v$. Demonstrate that in this case the mean cross-section of the reaction (averaged over all neutron velocities) $\langle \sigma (v) \rangle = \sigma (\langle v \rangle)$.

14.41. A beam of neutrons with energies falling within the interval, in which the cross-section of the reaction (n, α) is proportional to $1/v$, passes through a thin ^{6}Li foil 10 mg/cm^2 in thickness. What is the mean velocity of the neutrons, if the yield of the reaction (n, α) is known to be 0.40 in this case?

14.42. A neutron counter with a volume of 100 cm^3 filled up with BF_3 gas at NTP is placed in the uniform field of slow neutrons (boron of natural isotopic composition is used). Assuming that the reaction cross-section $\sigma_{n\alpha} \propto 1/v$, determine: (a) the volume density of neutrons if $1.0 \cdot 10^{12}$ reactions occur in the counter per one second; (b) the number of reactions occurring in the counter per one second if $\Phi = 1.1 \cdot 10^{10}$ neutrons/(cm^2·s) and the neutron temperature is 300 K.

14.43. Demonstrate that in a thin target exposed to an isotropic field of neutrons the reaction rate is twice that in the case when a parallel flux of neutrons with the same energy spectrum falls normally on the target's surface. The number of neutrons hitting the target is the same in both cases.

14.44. How long does it take to irradiate a thin layer of ^{10}B nuclide in a field of thermal neutrons with a volume density $n = 4.0 \cdot 10^8$ neutrons/cm^3 to decrease the number of ^{10}B nuclei by 50 percent? It is known that the reaction cross-section $\sigma_{n\alpha} \propto 1/v$.

14.45. A thin sample of metallic sodium of 0.40 g mass was placed in an isotropic field of thermal neutrons with $\Phi = 1.0 \times 10^{10}$ neutrons/(cm^2·s). Assuming the ^{24}Na radionuclide production rate constant, determine: (a) the activity of the saturated sample and the fraction of ^{24}Na nuclei accumulated in such a sample; (b) the irradiation time required to raise the sample's activity up to 75% of its saturation activity.

14.46. The specific activity of a neutron-activated golden foil equals $A = 1.1 \cdot 10^8$ dis/(s·g) = 3.0 mCi/g. For how long has this foil to be additionally exposed to the field of thermal neutrons with $\Phi = 1.0 \cdot 10^{10}$ neutrons/(cm^2·s) to increase its activity by a factor of $\eta = 10$?

14.47. A thin copper plate is exposed to the isotropic field of thermal neutrons with $\Phi = 0.9 \cdot 10^{12}$ neutrons/(cm^2·s). Determine the specific activity of the plate $t = 2.0$ h after the beginning of the exposure.

14.48. A thin ^{115}In foil of mass 0.20 g was exposed to an isotropic thermal neutron flux for $\tau = 2.0$ h. In $t = 0.50$ h after the exposure was discontinued, the foil activity turned out to be $A = 0.07$ mCi. Determine the neutron flux density Φ.

14.49. A ^{51}V sample of mass 0.50 g is activated up to saturation in a thermal neutron field. During $\tau = 5.0$ min immediately after completion of irradiation, $N = 0.8 \cdot 10^9$ pulses were registered, the count efficiency being $\beta = 0.010$. Determine the volume density of neutrons, assuming the activation cross-section to obey the $1/v$ law in this case.

14.50. An ^{115}In foil whose both sides are covered with thin layers of cadmium was exposed to an isotropic neutron field. Taking into account that the cross-section of indium activation obeys the $1/v$ law in the case of thermal neutrons, determine the specific saturation activity of the foil, if the volume density of thermal neutrons $n = 3.1 \cdot 10^4$ cm^{-3} and a cadmium ratio of $R_{Cd} = 20$. Cadmium is supposed to absorb all thermal neutrons and let through above-thermal ones. *Note.* R_{Cd} is the ratio of saturation activities of the naked foil and cadmium-coated one.

MODERATION AND DIFFUSION OF NEUTRONS

14.51. What fraction of its kinetic energy does a neutron lose in: **(a)** an elastic head-on collision with initially stationary nuclei ^2H, ^{12}C, and ^{238}U; **(b)** an elastic scattering through the angle ϑ by an initially stationary deuteron, if the angle ϑ is equal to 30, 90, and 150°?

14.52. Neutrons with the kinetic energy T_0 are elastically scattered by nuclei with the mass number A. Determine: **(a)** the energy of neutrons scattered through the angle $\tilde{\vartheta}$ in the C frame; **(b)** the fraction of neutrons that after single scattering possess a kinetic energy whose value falls within the interval $(T, T + dT)$ provided the scattering in the C frame is isotropic. Plot the distribution of scattered neutrons in terms of energy.

14.53. Neutrons with a kinetic energy of $T_0 = 1.00$ MeV are elastically scattered by initially stationary ^4He nuclei. Determine the mean energy value of singly scattered neutrons, assuming the scattering in the C frame to be isotropic.

14.54. Determine the probability that after a single elastic scattering of a neutron by a deuteron the neutron energy becomes less than half the initial value; the scattering in the C frame is isotropic.

14.55. Neutrons are scattered by initially stationary protons. Assuming this scattering to be isotropic in the C frame, find, using the vector diagram of momenta: **(a)** the probability of a neutron scattering into the angular interval $(\vartheta, \vartheta + d\vartheta)$; **(b)** the fraction of neutrons scattered through angles $\vartheta > 60°$; **(c)** the mean value of neutron scattering angle in the L frame.

14.56. A neutron is scattered by a nucleus with mass number A through the angle defined by the expression

$$\cos\vartheta = \frac{1 + A\cos\tilde{\vartheta}}{\sqrt{1 + A^2 + 2A\cos\tilde{\vartheta}}},$$

where $\tilde{\vartheta}$ is the corresponding scattering angle in the C frame.

(a) Derive this expression.

(b) Determine the fraction of neutrons elastically scattered through angles $\vartheta > \vartheta_1 = 90°$ due to single collisions with ^9Be nuclei; the scattering in the C frame is isotropic.

(c) Demonstrate that the mean value of cosine $\langle\cos\vartheta\rangle = 2/3A$ for the isotropic scattering in the C frame.

14.57. Calculate $\langle\cos\vartheta\rangle$ for neutrons elastically scattered in beryllium oxide provided the scattering in the C frame is isotropic.

14.58. Supposing that the elastic scattering of neutrons by nuclei is isotropic in the C frame, **(a)** derive formula (14.5); simplify this

formula for the case of sufficiently large values of A; **(b)** calculate ξ for a neutron in graphite and heavy water.

14.59. Determine the mean number of elastic collisions that a neutron experiences in the process of its moderation from an energy of 2.00 MeV down to 0.025 eV in uranium, graphite, and heavy water.

14.60. Find the mean time of neutron moderation from energy $T_0 = 2.0$ MeV to $T_t = 0.025$ eV in beryllium, assuming the mean free path of a neutron between two collisions to be independent of energy and equal to $\lambda_s = 1.15$ cm.

14.61. Neutrons with a kinetic energy of 2.0 MeV are thermalized in graphite down to energy 0.025 eV. Calculate the age τ of thermal neutrons and moderation length L.

14.62. Using the expression for moderation density q_E in the case of a point source of fast monochromatic neutrons, demonstrate that the mean squared distance (along the straight line) travelled by a neutron during its thermalization to the energy E is $\langle r^2\rangle = 6\tau$, where τ is the age of the given neutrons.

14.63. To determine the neutron age, a point source of fast neutrons is placed in a large bulk of moderator and thin indium strips cadmium-plated are activated at various distances from that source. The degree of indium activation is primarily effected by its resonance level with an energy of about 1.5 eV. Find the age of resonance indium neutrons in graphite, if the foil activity A (in relative units) at distances r from the source, equal to 50, 100, and 150 mm, is known to be equal to 100, 94, and 85 respectively.

14.64. Demonstrate that for nuclei whose cross-section obeys the $1/v$ law the resonance integral for above-cadmium neutrons (whose energy exceeds 0.40 eV) is equal to $\sigma_0/2$, where σ_0 is the absorption cross-section for a neutron energy of 0.025 eV.

14.65. Thermal neutrons diffuse in a uniform medium whose macroscopic scattering cross-section is Σ_s and absorption cross-section is negligible. Find: **(a)** the probability that a neutron passes in that medium the distance between x and $x + dx$ without collisions; the mean free path λ_s between two successive collisions; **(b)** the mean squared free path $\langle x^2\rangle$ of a neutron in graphite.

14.66. A thermal neutron with an energy of 0.025 eV diffuses in graphite. Determine the mean diffusion time (lifetime) of the given neutron and the mean number of collisions it experiences during that time.

14.67. Calculate for thermal neutrons in graphite: **(a)** the transport length; **(b)** the diffusion length and the mean distance covered by the neutron till its absorption.

14.68. Neutrons diffuse in a medium whose absorption cross-section is negligible. Assuming the neutron scattering to be isotropic in the L frame, determine: **(a)** the number of neutrons crossing 1 cm^2 area from one side per 1 s, if the neutron flux is the same throughout the medium and equal to Φ; **(b)** the resultant density rate of neutrons crossing an element of area oriented normally to $\nabla\Phi$, if $\Phi = \Phi_0 + (d\Phi/dn)_0 x$, where the values subindexed with 0 refer to the points of the considered element of area ($x = 0$).

14.69. A thermal neutron source is located in the infinite uniform medium without multiplication, whose macroscopic absorption cross-section is Σ_a and diffusion coefficient D. Assuming the neutron scattering to be isotropic in the L frame, find the expression describing the steady-state distribution of the neutron flux Φ in the medium, if the neutron source: **(a)** is an infinite plane emitting n neutrons/(cm^2·s); **(b)** is a point with activity n neutrons/s; **(c)** is a sphere of radius R emitting n neutrons/(cm^2·s), with all neutrons getting inside the sphere being absorbed.

14.70. A point source of thermal neutrons is surrounded with a large bulk of heavy water. Calculate the neutron diffusion length in this medium, if the ratio of neutron fluxes at distances $r_1 = 15$ cm and $r_2 = 30$ cm from the source is $\Phi_1/\Phi_2 = \eta = 2.2$.

14.71. Demonstrate that the mean squared displacement of a thermal neutron in a medium from the point at which it became thermal to the point at which it was absorbed is related to the diffusion length as $\langle r^2 \rangle = 6L^2_{\text{dif}}$.

14.72. A neutron diffuses in a medium with albedo β. Determine the probability that the given neutron crosses an imaginary plane in the medium n times exactly, as well as the mean number of times that the neutron crosses the plane.

14.73. An indium foil of thickness 0.13 g/cm^2 is activated in a field of thermal neutrons realized in a water tank. The foil activity turned out to be 6.9 times that of the foil plated by cadmium on one side. Find the thermal neutron albedo in water.

14.74. Making use of the solution of Problem 14.68, find the albedo of a reflector, if a medium producing thermal neutrons: **(a)** is separated by a plane boundary from an infinite graphite reflector having the neutron flux distribution $\Phi = \Phi_0 e^{-x/L}$, where x is the distance from the boundary, L is the diffusion length; **(b)** has the shape of a sphere of radius R surrounded by an infinite reflector; known are the diffusion coefficient D, diffusion length L, and neutron flux distribution in the reflector $\Phi \propto \frac{1}{r} e^{-r/L}$, where r is the distance from the centre of the sphere.

15

NUCLEAR FISSION AND FUSION

- Energy liberated in a single uranium fission is adopted equal to 200 MeV.
- Multiplication constant for neutrons in infinite medium:

$$k_\infty = \varepsilon p f \eta, \tag{15.1}$$

where ε is the fast neutron multiplication, p is the probability of avoiding a resonance capture, f is the thermal neutron utilization coefficient (the probability of absorption of a thermal neutron by a fissionable substance), η is the mean number of fission neutrons per one thermal neutron absorbed by a fissionable substance.

- Energies of the *dd* and *dt* reactions (*d* for deuterium and *t* for tritium):

$$d+d \nearrow t+p+4.0 \text{ MeV}, \qquad d+d \searrow {}^3\text{He}+n+3.3 \text{ MeV}, \qquad d+t \rightarrow \alpha+n+17.6 \text{ MeV}.$$

Both branches of the *dd* reaction are practically equiprobable.

- Effective cross-section of the *dd* reaction:

$$\sigma_{dd} = 1.2\cdot 10^2 \frac{1}{\tilde{T}} \exp\left(-\frac{31.3}{\tilde{T}^{1/2}}\right) \text{ b}, \tag{15.2}$$

where $\tilde{T}$ is the kinetic energy of a relative motion of deuterons, i.e. their total kinetic energy in the C frame, keV.

- In this chapter the plasma particles are supposed to be distributed in accordance with Maxwell's law; the plasma temperature is expressed in energy units $\theta = kT$.
- Mean values of the quantity σv for deuterium and tritium plasma are:

$$\langle\sigma v\rangle_{dd} = 3\cdot 10^{-14} \frac{1}{\theta^{2/3}} \exp\left(-\frac{19}{\theta^{1/3}}\right) \text{ cm}^3/\text{s}, \quad \langle\sigma v\rangle_{dt} = 3\cdot 10^{-12} \frac{1}{\theta^{2/3}} \exp\left(-\frac{20}{\theta^{1/3}}\right) \text{ cm}^3/\text{s}, \tag{15.3}$$

where θ is the plasma temperature, keV. The graphs of these functions are presented in Appendix 13.

- Hydrogen plasma bremsstrahlung intensity

$$w = 4.8\cdot 10^{-31} n_e n_i \sqrt{\theta_e} \text{ W/cm}^3, \tag{15.4}$$

where $n_{e,i}$ is the concentration of electrons and ions, cm^{-3}, θ_e is the electronic temperature, keV.

- Conductivity of totally ionized hydrogen plasma

$$\sigma = 4.0\cdot 10^5 \theta^{3/2} \ \Omega^{-1}\cdot\text{cm}^{-1}, \tag{15.5}$$

where θ is the plasma temperature, keV.

● Basic equation of magnetohydrodynamics for quasisteady state and plasma boundary condition:

$$\nabla p = \frac{1}{c}[\mathbf{jB}]; \qquad p + \frac{B^2}{8\pi} = \text{const}, \tag{15.6}$$

where p is the kinetic pressure of plasma, $\mathbf{j}$ is the electric current density, B is the magnetic induction.

● Relation between the temperature θ and longitudinal current I in an equilibrium cylindrical pinch of totally ionized hydrogen plasma:

$$\theta = I^2/4c^2N, \tag{15.7}$$

where N is the number of electrons per unit length of the pinch.

FISSION OF NUCLEI. CHAIN REACTION

15.1. Determine: **(a)** the energy liberated in fission of 1.0 kg of ^{235}U nuclide; what amount of oil with a calorific value of 42 kJ/g liberates such an energy on combustion? **(b)** the electric power of an atomic power plant if its annual consumption of ^{235}U nuclide comes to 192 kg with an efficiency of 20%; **(c)** the mass of ^{235}U nuclide fissioned in the explosion of an atomic bomb with a trotyl equivalent of 30 kt, if the thermal equivalent of trotyl equals 4.1 kJ/g.

15.2. Find the total neutrino flux and power being lost due to that flux in the case of a reactor with 20 MW thermal power. Each fission is assumed to be accompanied with five β-decays of fission fragments for which the total neutrino energy amounts to about 11 MeV.

15.3. Using the semi-empirical formula for binding energy (10.3), find the ratio Z^2/A at which the fission of an even-even nucleus into two equal fragments becomes feasible in terms of energy.

15.4. A nucleus becomes quite unstable to fission into two equal fragments when the ratio of its electrostatic energy to surface energy equals two. Using formula (10.3), find the value Z^2/A of such nucleus. Compare this value with that of nuclei located at the end of the Periodic Table; explain why these nuclei fission.

15.5. Find the half-life of ^{238}U with respect to spontaneous fission, if it is known that the number of such fissions per one gram of pure ^{238}U equals 25 per hour. How many α-decays occur in this sample during the same interval?

15.6. A ^{235}U nucleus captured a thermal neutron. The unstable nucleus thus formed fissioned to produce three neutrons and two radioactive fragments that transformed into stable ^{89}Y and ^{144}Nd nuclei. Find the energy liberated in this process, if: **(a)** the excess of mass of a neutron and nuclei ^{235}U, ^{89}Y (—0.09415 amu), ^{144}Nd (—0.09010 amu) are known; **(b)** the binding energy per one nucleon in nuclei ^{235}U (7.59 MeV), ^{89}Y (8.71 MeV), ^{144}Nd (8.32 MeV) and binding energy of a neutron in ^{236}U (6.40 MeV) are known.

15.7. The nucleus, appearing after a neutron is captured by a ^{238}U nucleus, fissions provided the neutron energy is not less than 1.4 MeV. Find the activation energy of the fissionable nucleus.

15.8. Determine the most probable and mean kinetic energy of ^{235}U fission neutrons whose energy spectrum is $n(T) \propto \sqrt{T}e^{-0.77T}$, where T is the neutron's kinetic energy, MeV.

15.9. Calculate the fission cross-section of a nucleus in uranium of natural isotopic abundance for thermal neutrons.

15.10. Calculate the fraction of thermal neutrons whose capture by nuclei ^{233}U, ^{235}U, and ^{239}Pu induces their fission.

15.11. Find the mean number of instantaneous fission neutrons per one absorbed thermal neutron in ^{233}U, ^{235}U, and ^{239}Pu.

15.12. Compare the mean number of instantaneous fission neutrons per one absorbed thermal neutron in natural and enriched (1.50% ^{235}U) uranium.

15.13. A ^{235}U plate is exposed to thermal neutrons falling normally on its surface. At what thickness of the plate will each incident thermal neutron produce on the average one fast fission neutron?

15.14. A parallel thermal neutron flux of density $J_0 = 1.2 \times 10^{10}$ neutrons/(cm$^2\cdot$s) falls normally on a plate of natural uranium 8.0 g/cm^2 in thickness. Find the power generated from 1 cm^2 of the plate due to ^{235}U fission.

15.15. Into a constant power reactor a small amount of ^{239}Pu ($m_1 = 0.90$ g) was added. To keep the reactor's power constant, some boron of natural isotopic abundance ($m_2 = 0.060$ g) was also added. Assuming the absorption cross-sections of plutonium and boron to be known, find the mean number of ^{239}Pu fission neutrons per one absorbed thermal neutron.

15.16. A reactor with fissionable isotope ^{235}U operates at a constant power level. Find the fraction of neutrons escaping from the fissile core, if half of fission neutrons are captured by uranium nuclei and impurity nuclei without subsequent fission.

15.17. What is the physical meaning of the multiplication constant k? How many neutrons will there be in the hundredth generation, if the fission process starts from 1000 neutrons and $k = 1.05$?

15.18. Evaluate the time interval during which 1.0 kg of substance fissions in the infinite medium of ^{235}U, if the mean energy of fission neutrons equals 1.6 MeV, the fission cross-section of ^{235}U nucleus for such energy is about 2 b, and the multiplication constant $k = 1.0010$. Suppose that one nucleus fissioned at the initial moment. How will the result change, if $k = 1.010$?

15.19. Each fission of a ^{238}U nucleus releases about 2.6 fission neutrons on the average. The fission cross-section of ^{238}U is equal to about 0.5 b (for fission neutron energies), the inelastic scattering cross-section amounts to a few barns. Taking into account that at least half of fission neutrons possess the energy below ^{238}U fission threshold, demonstrate that a self-sustaining chain reaction is impossible in the medium consisting of these nuclei.

15.20. Explain why a self-sustaining chain reaction is impossible in a medium consisting of natural uranium.

15.21. A homogeneous mixture contains $z = 500$ mol of carbon per each mole of uranium. Calculate: **(a)** the coefficient f, if the uranium of natural isotopic abundance is used; **(b)** the degree of ^{235}U enrichment for which $f = 0.95$.

15.22. In a homogeneous uranium-graphite mixture the probability of avoiding a resonance capture

$$p = \exp\,[-24.7\,(N_a/\Sigma_s)^{0.585}],$$

where N_a is the number of ^{238}U atoms in 1 cm³; Σ_s is the macroscopic scattering cross-section of the medium, b. Calculate the value of p for the mixture containing 400 mol of graphite per each mole of natural uranium.

15.23. Making use of the formula of the foregoing problem and assuming $\varepsilon = 1$, calculate the multiplication constant k_∞ in a homogeneous mixture containing $z = 400$ mol of graphite per each mole of natural uranium.

15.24. The active section of heterogeneous thermal-neutron reactor is a tank filled with moderator into which 200 rods of natural uranium are inserted. The length of each rod is 1.50 m, the diameter 2.0 cm. Neglecting the self-absorption of neutrons in uranium, evaluate the mean flux density Φ and thermal neutron concentration, if the reactor's power is 0.60 MW.

15.25. A thin ^{233}U foil of a mass of 0.10 g was exposed for $\tau = 60$ s to a flux of thermal neutrons falling normally on its surface. The flux is $I = 1.1\cdot10^{10}$ neutrons/(cm²·s). Following $\tau = 10$ s after the end of the exposure, the activity of one of the delayed neutron groups with a half-life of 55 s turned out to be $A = 4.0\cdot10^5$ neutrons/s. Find the yield of delayed neutrons of that group per one fission.

15.26. Evaluate the mean lifetime of a single generation of fission neutrons in a homogeneous medium containing 100 moles of graphite per each mole of natural uranium. Take into account that the neutron thermalization time is much less than the diffusion time.

15.27. The fission of ^{235}U exposed to thermal neutrons produces six groups of delayed neutrons:

T_i, s	55.7	22.7	6.20	2.30	0.61	0.23
w_i, 10^{-3}	0.52	3.46	3.10	6.24	1.82	0.66

Here T_i is the half-life of the fragments emitting the ith group of delayed neutrons, w_i is the yield of these neutrons per one fission. Find the increase in the mean effective lifetime of a single generation of neutrons, in a medium consisting of ^{235}U and moderator, caused by delayed neutrons. The mean lifetime τ of a single generation of fission neutrons is known to be equal to about 1 ms.

15.28. Find the reactor period T (the time interval during which its power increases e-fold), if the multiplication constant $k = 1.010$ and mean lifetime of a single generation of neutrons $\tau = 0.10$ s.

15.29. What amount of energy is liberated in fusion of 1 g of nuclear fuel in the following thermonuclear reactions: **(a)** dt; **(b)** dd; **(c)** $^6Li(d, \alpha)^4He$?

Compare the obtained results with the energy liberated in fission of one gram of uranium.

15.30. How long would the thermonuclear energy liberated in the dd reaction have held out, if 1% of deuterium contained in the Ocean had been used and the world energy consumption rate had been equal to $10Q$ per year (the volume of the Ocean is 10^{18} m³, $1Q = 10^{18}$ kJ)? *Note.* The present energy consumption rate amounts to about $0.1Q$ per year. Still, the above-mentioned rate of $10Q$ per year would not change the Earth climate appreciably, as this value totals 1% of the solar radiation absorbed and emitted by the Earth annually.

15.31. A carbon cycle of thermonuclear reactions, proposed by Bethe as a possible source of stellar energy, comprises the following transformations:

$$p + {}^{12}C \to {}^{13}N + \gamma,\quad p + {}^{14}N \to {}^{15}O + \gamma,$$
$${}^{13}N \to {}^{13}C + e^+ + \nu,\quad {}^{15}O \to {}^{15}N + e^+ + \nu,$$
$$p + {}^{13}C \to {}^{14}N + \gamma,\quad p + {}^{15}N \to {}^{12}C + {}^4He.$$

Calculate the energy liberated in this cycle in the process of production of one mole of helium.

15.32. What fraction of liberated energy is carried from a thermonuclear reaction core by neutrons released as a result of the reaction: **(a)** dt; **(b)** dd?

15.33. The energy of neutrons produced in the thermonuclear reaction dt can be utilized by surrounding the reaction core by an envelope which absorbs the neutrons with positive thermal energy yield, e.g. by the envelope containing ^6Li ($n + {}^6Li \to t + \alpha$). By what factor will the utilized energy increase on the introduction of such an envelope?

15.34. Evaluate the lowest temperature of deuterium plasma in which the deuterons possessing the most probable value of relative velocity are capable of overcoming the Coulomb barrier. The radius of deuteron $R \approx 2.0\cdot10^{-15}$ m.

15.35. When the kinetic energy $\tilde{T}$ of the relative motion of charged particles is considerably less than the Coulomb barrier height, the coefficient of transparency of the barrier

$$D \approx \exp\,(-\alpha/\sqrt{\tilde{T}});\qquad \alpha = \pi q_1 q_2 \sqrt{2\mu}/\hbar,$$

where q_1 and q_2 are the charges of the particles, μ is the reduced mass.

(a) Derive this formula from the general expression (3.5).

(b) Calculate the values of D for deuterons possessing the most probable value of relative velocity at plasma temperatures of 1.0 and 10.0 keV.

15.36. Calculate the cross-section of the *dd* reaction for the most probable value of relative velocity of deuterons in a plasma with temperatures of 1.0 and 10.0 keV.

15.37. In a deuterium plasma with a concentration of nuclei of $n = 1.0 \cdot 10^{15}$ cm^{-3}, the deuterons react with frequency $f = 1.0 \times \times 10^{-3}$ s^{-1}. Find the volume density of the released thermonuclear energy and plasma temperature θ.

15.38. Determine the kinetic energy of the relative motion of deuterons corresponding to the maximum velocity of the *dd* reaction in deuterium plasma with a temperature of $\theta = 2.0$ keV.

15.39. A deuterium plasma with a concentration of nuclei of $n = 1.0 \cdot 10^{15}$ cm^{-3} is heated to the temperature θ. For $\theta = 1.0$ and 10 keV calculate: **(a)** the mean lifetime of a deuteron in terms of the *dd* reaction; **(b)** the number of *dd* reactions per unit time (cm$^{-3}\cdot$s^{-1}) and volume density of the released power.

15.40. In a deuterium-tritium plasma with the temperature θ and concentration of nuclei $n = 1.00 \cdot 10^{15}$ cm^{-3}, the concentration of tritium nuclei equals $n/100$. For $\theta = 1.00$ and 10.0 keV calculate: **(a)** the number of thermonuclear reactions ($dd + dt$) per 1 s per 1 cm^3; **(b)** the volume density of released power.

15.41. A deuterium-tritium plasma with a concentration of nuclei of $n = 1.00 \cdot 10^{15}$ cm^{-3} has the temperature θ. For $\theta = 1.00$ and 10.0 keV find the ratios of nuclear concentrations of tritium and deuterium at which the released thermonuclear power is the highest, as well as the values of this power in W/cm^3. What inference can be made on the relative contributions of the *dd* and *dt* reactions into the power released under the given conditions?

15.42. Find the temperature θ of deuterium-tritium plasma with a concentration of nuclei of $n = 1.0 \cdot 10^{15}$ cm^{-3} ($n_d = n_t$) at which the volume density of released power is $w = 1.0$ W/cm^3. Assume this power to be released primarily due to *dt* reactions.

15.43. What would be the radius of a spherical thermonuclear reactor filled with deuterium plasma with deuteron concentration n at the temperature θ, if the heat was removed from the active section only in the form of thermal radiation in accordance with the Stefan-Boltzmann law? Calculate θ_m at which the reactor's radius is the smallest. What is its value, if $n = 1.0 \cdot 10^{20}$ cm^{-3}? Think over the result obtained.

15.44. Find the deuterium plasma temperature θ at which the released thermonuclear power is equal to the power of bremsstrahlung radiation of electrons.

15.45. A deuterium-tritium plasma is maintained at a temperature of 10.0 keV and constant deuterium concentration $n_d = = 1.00 \cdot 10^{15}$ cm^{-3} is replenished from an outside source. The latter provides deuterons at the rate q nuclei/(cm$^3\cdot$s). Considering only the *dd* and *dt* reactions, find: **(a)** the steady-state concentration of tritium nuclei and the value of q; **(b)** the volume density of released power w. What is the value of n_d for which $w = 100$ W/cm^3?

15.46. A plasma sample is shaped as a plane-parallel layer. Suppose that due to a certain cause all electrons shifted with respect to ions by the distance x in the direction perpendicular to the layer's surface. Using this model, find the frequency of electronic oscillations set in the plasma.

15.47. The dielectric permittivity of plasma in electric field $E = = E_0 \cos \omega t$ is $\varepsilon(\omega) = 1 - (\omega_0/\omega)^2$, ($\omega_0$ is the Langmuir frequency of electrons) under the conditions that collisions between particles and ionic motion can be neglected.

(a) Derive this expression.

(b) Demonstrate that an electromagnetic wave with the frequency $\omega < \omega_0$ goes through total internal reflection in the plasma.

(c) Calculate the electronic concentration in the plasma at which the electromagnetic radiation with wavelengths exceeding $\lambda_0 = = 1.7$ cm is suppressed.

15.48. Calculate the flux of thermonuclear neutrons from a deuterium plasma of volume $V = 1.0$ litre at temperature $\theta = 10.0$ keV, if the suppression of sounding radio waves is known to be observed for wavelengths longer than $\lambda_0 = 5.0$ mm.

15.49. Using Poisson's equation, demonstrate that the mean potential of electrostatic field in the vicinity of an ion in hydrogen plasma is $\varphi \propto \frac{1}{r} e^{-r/d}$, where r is the distance from the ion, d is the Debye length. Find the expression for d provided the concentration of electrons (and ions) is equal to n and plasma temperature to θ ($\theta_e = \theta_i$).

Assume the spatial distribution of particles to obey the Boltzmann law, and, specifically, $|e\varphi| \ll \theta$.

15.50. A hydrogen plasma with concentration of nuclei $n = = 1.0 \cdot 10^{15}$ cm^{-3} is kept at temperature $\theta = 10$ keV. Calculate the Debye length and number of nuclei contained in the sphere whose radius is equal to the Debye length.

15.51. Calculate the cross-section corresponding to scattering of electrons with the kinetic energy $T = 1.00$ keV through the angles $\vartheta > 90°$ due to collisions with ions of hydrogen plasma.

15.52. A hydrogen plasma with concentration of nuclei $n = = 1.0 \cdot 10^{15}$ cm^{-3} is kept at the temperature $\theta = 1.0$ keV. Evaluate the minimum angle ϑ_{min}, through which the electrons with the most probable velocity are scattered, and also the magnitude of the Coulomb logarithm $\ln(2/\vartheta_{min})$. The Coulomb field of nuclei is supposed to reach over the Debye length and then vanish abruptly.

15.53. The effective cross-section for transfer of electronic momentum (when an electron is scattered by plasma ions) is defined by the following expression: $\sigma = \int (1 - \cos \vartheta)\, \sigma(\vartheta)\, d\vartheta$, where $\sigma(\vartheta) = = d\sigma/d\vartheta$ is the differential cross-section given by the Rutherford formula. Using this formula, calculate for the electrons with the most probable velocity (if $\theta = 1.0$ keV and the concentration of nuclei $n = 1.0 \cdot 10^{15}$ cm^{-3}): **(a)** the magnitude of the given effective cross-section; **(b)** the mean "free path", mean time interval between "collisions", and the number of collisions per one second.

Assume the electrons to be scattered due to the Coulomb field of nuclei, which reaches over the Debye length and vanishes abruptly at longer distances.

15.54. An electron in a hydrogen plasma emits a power of $\delta w = 4.5 \cdot 10^{-31} n_i \sqrt{T}$ W, where n_i is the concentration of nuclei, cm^{-3}; T is the kinetic energy of the electron, keV. Find the volume density of bremsstrahlung radiation of electrons in the plasma, if their temperature is equal to θ keV and concentration to n_e, cm^{-3}.

15.55. The power transferred from electrons to ions in unit volume of the deuterium plasma

$$w_{ei} = 1.7 \cdot 10^{-28} n^2 (\theta_e - \theta_i) \theta_e^{-3/2} \text{ W/cm}^3,$$

where n is the concentration of electrons (ions), cm^{-3}; θ_e and θ_i are the electronic and ionic temperatures, keV. Using this expression for deuterium plasma with $n = 1.0 \cdot 10^{15}$ cm^{-3}, find the time interval during which θ_i rises to $2\theta_e/3$, if the initial ionic temperature is negligible and the electronic temperature: **(a)** is maintained at a constant value $\theta_e = 1.0$ keV; **(b)** is $\theta_{e0} = 1.0$ keV at the initial moment and there is no heat exchange with the environment.

15.56. The current I_0 flows along a thin skin layer of a cylindrical plasma filament of pradius r_0. Find the magnetic pressure p, that is, the Lorentz force acting on unit area of the filament's surface. Demonstrate that $p = B_0^2/8\pi$, where B_0 is the magnetic induction at the filament's surface.

15.57. Suppose that the gas-kinetic pressure p of deuterium plasma at a temperature of $\theta = 10.0$ keV is counterbalanced by the magnetic pressure developed by a magnetic field $B = 50$ kG. Calculate the concentration of deuterons, pressure p, and volume density of power released in thermonuclear *dd* reactions.

15.58. A current flows along a thin skin layer of a stable cylindrical filament of hydrogen plasma of radius r_1, and in the opposite direction, along an external coaxial cylinder of radius r_2. Find the ratio of the magnetic energy to gas-kinetic energy of the plasma filament. The plasma is assumed fully ionized.

15.59. A plasma has the shape of a thin cylindrical layer carrying the current I_1. Along the axis of this layer there is a conductor carrying the current I_2 in the opposite direction. Ignoring the magnetic field inside the plasma, determine: **(a)** the current ratio I_1/I_2 at which the plasma layer is at equilibrium; **(b)** the value of I_1 at which the hydrogen plasma temperature $\theta = 1.00$ keV, if the equilibrium radius of the plasma filament $r_0 = 6.0$ cm and concentration of nuclei $n = 1.00 \cdot 10^{15}$ cm^{-3}.

15.60. The current I_0 flows along a hydrogen plasma filament of cylindrical form with equilibrium radius r_0. Ignoring the external gas-kinetic pressure, determine: **(a)** the mean value of gas-kinetic pressure inside the filament; **(b)** the plasma temperature, if $I_0 = 300$ kA and the number of nuclei per unit length of the pinch $N = 0.7 \cdot 10^{17}$ cm^{-1}.

15.61. The current I flows along a cylindrical fully ionized hydrogen plasma filament possessing N electrons per unit length. Assuming the plasma temperature and current density to be constant over the cross-section: **(a)** demonstrate that the concentration of electrons is distributed over the cross-section as $n(r) = \frac{2N}{\pi r_0^2}\left(1 - \frac{r^2}{r_0^2}\right)$, where r_0 is the equilibrium radius of the filament; find also the value $n^2(r)$ averaged over the cross-section; **(b)** find the current whose Joule heat power is equal to the plasma bremsstrahlung radiation power. What is the plasma temperature under this condition, if $N = 5 \cdot 10^{17}$ cm^{-1}?

15.62. A glass tube of radius $r_0 = 2.5$ cm is filled with deuterium with concentration $n_0 = 4.0 \cdot 10^{14}$ molecules/cm^3 and placed inside a one-turn "coil" of length $l = 10$ cm made of flat copper bus. After preliminary ionization of deuterium a capacitor bank is discharged through the coil. The current flowing in the coil generates a magnetic field that pinches the plasma filament to radius $r = 0.50$ cm, with the current in the coil reaching a value $I = 1.0 \cdot 10^6$ A at that moment. Evaluate the plasma temperature neglecting the magnetic field inside the plasma. Explain the mechanism of plasma compression in this case.

15.63. Due to a sharp increase of current to $I_0 = 50$ kA, a cylindrical filament of fully ionized hydrogen plasma is pinched to equilibrium radius $r_0 = 1.0$ cm; while increasing, the current flows along a thin skin layer. Evaluate the time interval during which the magnetic field settles over the filament's cross-section, if the number of nuclei per unit of its length $N = 1.00 \cdot 10^{16}$ cm^{-1}. *Instruction.* The time during which magnetic field diffuses over the length l is equal to about l^2/D, where $D = c^2/4\pi\sigma$ is the diffusion coefficient, σ is the electric conductivity.

15.64. A hydrogen plasma with a concentration of nuclei of $2.0 \cdot 10^{10}$ cm^{-3} filling a glass tube of radius of 2.5 cm is placed in an external longitudinal magnetic field $B_{z0} = 4.0$ kG. On passing through the plasma of a current $I_z = 5.0 \cdot 10^5$ A flowing in a thin skin layer, the plasma gets compressed to a radius of 0.50 cm carrying along the magnetic field B_z confined in the plasma. Evaluate the time interval during which the field gets trapped in the plasma.

15.65. Show that the condition for suppression of sausage-type instabilities in a cylindrical plasma filament carrying the current I in the presence of the longitudinal magnetic field B_z trapped in the plasma takes the following form: $B_I < \sqrt{2}\, B_z$, where B_I is the magnetic field of the current I. The current is supposed to flow in a thin skin layer. Find the value of B_z at which the sausage-type instabilities will be quelled in the plasma filament of radius $r = 1.0$ cm carrying the current $I = 100$ kA.

15.66. Make sure that a doughnut-shaped plasma turn located in the external circular magnetic field B_φ induced by a toroidal solenoid cannot be stable irrespective of whether or not there is a current in the turn.

15.67. A circular turn of hydrogen plasma carrying the current $I = 10$ kA and having a concentration of nuclei $n = 1.0 \cdot 10^{15}$ cm^{-3} is formed in a toroidal quartz chamber with a cross-sectional radius $a = 5.0$ cm and mean great radius $R = 50$ cm. Suppose that at the initial moment the mean great radius of the turn is also equal to R. Evaluate the time interval required to get the turn thrown off to the chamber's walls. Assume that while stretching the turn keeps its cross-sectional radius, equal to $r = 1.0$ cm, and the current I constant, with the gas-kinetic pressure of the plasma turn being counterbalanced by the magnetic pressure of the current I. Also assume that the current I flows along the surface of the turn, so that the inductance of the turn can be adopted to be equal to $L = 4\pi R\left(\ln \frac{8R}{r} - 2\right)$.

15.68. A plasma turn with the current $I = 10$ kA is located in the external uniform magnetic field B_z directed normally to the turn's plane. Assuming that the current flows along the surface of the turn, find the value of B_z at which the turn will be at equilibrium, if its great and cross-sectional radii are equal to $R = 50$ cm and $r = 1.0$ cm. The expression for the inductance of the turn is to be taken from the foregoing problem.

15.69. A circular plasma filament is formed in a toroidal chamber over which a winding is contrived to produce the longitudinal magnetic field B_φ. When the filament carries the current I, the magnetic lines of force take, under these conditions, the helical form, so that the plasma filament can develop instabilities of the helical type. Such instabilities do not occur though, if the filament length is less than the lead of the helical lines of force on the filament's surface. Find the limiting value of the current I, if $B_\varphi = 20$ kG and the great and cross-sectional radii of the filament are $R = 50$ cm and $r = 1.0$ cm.

16

ELEMENTARY PARTICLES

● In all formulas of this chapter, energy, momentum, and mass are expressed in energy units: p and m being an abbreviation of pc and mc^2. Whenever the term "mass" is used, the rest mass is meant.

● Kinetic energy of relative motion is the total kinetic energy of particles in the C frame.

● Lorentz invariant:

$$E^2 - p^2 = \text{inv}, \tag{16.1}$$

where E and p are the total energy and total momentum of a system of particles. On transition from one inertial frame of reference into another this quantity remains constant.

● Velocity of the C frame relative to laboratory one (the L frame):

$$\beta = v/c = p/E, \tag{16.2}$$

where p and E are the total momentum and total energy of a system of particles.

● Lorentz equations transforming the momentum, total energy, and angles on transition from the L to C frame (Fig. 41):

$$\tilde{p}_x = \frac{p_x - E\beta}{\sqrt{1-\beta^2}}; \quad \tilde{E} = \frac{E - p_x\beta}{\sqrt{1-\beta^2}}; \quad \tan\tilde{\vartheta} = \frac{\sqrt{1-\beta^2}\sin\vartheta}{\cos\vartheta - (E/p)\beta}, \tag{16.3}$$

where β is the velocity of the C frame relative to the L frame.

● Threshold kinetic energy of a particle m striking a stationary particle M and activating the reaction $m + M \to \Sigma m_i$

$$T_{m\,\text{th}} = \frac{(\Sigma m_i)^2 - (m+M)^2}{2M}. \tag{16.4}$$

When a particle of mass M decays into two particles, the momenta of the generated particles are equal (in the C frame) to

$$\tilde{p} = \frac{1}{2M}\sqrt{[M^2 - (m_1+m_2)^2]\cdot[M^2 - (m_1-m_2)^2]}, \tag{16.5}$$

where m_1 and m_2 are their masses.

● Vector diagram of momenta for the decay of a relativistic particle of mass M into two particles with masses m_1 and m_2 (Fig. 42). The locus of possible locations of the tip of the momentum vector p_1 of particle m_1 is an ellipse for which

$$b = \tilde{p}; \quad a = \tilde{p}/\sqrt{1-\beta^2}; \quad f = \tilde{p}\beta/\sqrt{1-\beta^2}, \tag{16.6}$$

where b and a are the semi-minor and semi-major axes, f is the focal distance, $\tilde{p}$ is the momentum of generated particles in the C frame, β is the velocity of the decaying particle (in units of c).

The centre of the ellipse divides the section AB into two parts α_1 and α_2 in the ratio $\alpha_1 : \alpha_2 = \tilde{E}_1 : \tilde{E}_2$, where $\tilde{E}_1$ and $\tilde{E}_2$ are the total energies of the generated particles in the C frame.

Maximum angle at which the particle m_1 is ejected is defined by the formula

$$\sin \vartheta_{1\,\max} = \frac{M}{m_1} \cdot \frac{\tilde{p}}{p_M}, \tag{16.7}$$

where p_M is the momentum of the decaying particle.

● In particle interactions the conservation laws for lepton and baryon charges hold. In strong interactions also hold the conservation laws for strangeness S, isotopic spin T, and its projection T_z.

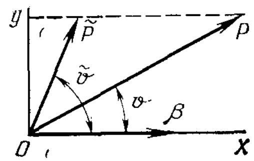


Fig. 41

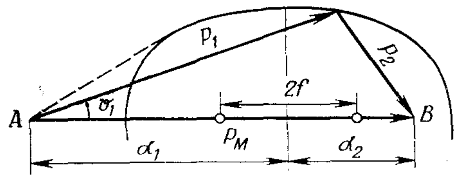


Fig. 42

● It follows from the generalized Pauli exclusion principle that for a system of two particles with identical isotopic spins

$$(-1)^{l+s+T} = \begin{cases} -1 & \text{for half-integer spin particles} \\ 1 & \text{for zero-spin particles,} \end{cases}$$

where l is the orbital moment, s is the spin of the system, T is the isotopic spin.

INTERACTION OF RELATIVISTIC PARTICLES

16.1. Calculate the momenta (GeV/c) of a proton, muon, and electron whose kinetic energies are equal to 1.0 GeV.

16.2. A relativistic particle with mass m and kinetic energy T strikes a stationary particle of the same mass. Find the kinetic energy of their relative motion, momentum of either particle in the C frame, and velocity of this system.

16.3. What amount of kinetic energy should be provided to a proton striking a stationary proton to make the kinetic energy of their relative motion equal to that effected in a collision of two protons moving toward each other with kinetic energies $T = 30$ GeV?

16.4. A relativistic particle with mass m_1 and kinetic energy T strikes a stationary particle with mass m_2. Find: **(a)** the kinetic energy of their relative motion; **(b)** the momentum and total energy of either particle in the C frame.

16.5. Determine the kinetic energies of particles with masses m_1 and m_2 in the C frame, if the kinetic energy of their relative motion is known to be equal to $\tilde{T}$.

16.6. One of the particles of a system moves with momentum p and total energy E at the angle ϑ (in the L frame) relative to the velocity vector β_c of the C frame. Find the corresponding angle $\tilde{\vartheta}$ in the C frame.

16.7. A relativistic proton with the kinetic energy T is scattered through the angle ϑ_1 by a stationary proton. As a result of the collision, the initially stationary proton is ejected at the angle ϑ_2.

(a) Demonstrate that $\cot \vartheta_1 \cot \vartheta_2 = 1 + T/2m$.

(b) Calculate the minimum possible angle of divergence of the two particles.

(c) Determine T and kinetic energies of either particle after collision, if $\vartheta_1 = 30°$ and $\vartheta_2 = 45°$.

16.8. Demonstrate that when a relativistic particle with mass m_1 is elastically scattered by a stationary particle with mass $m_2 < m_1$, the maximum scattering angle of the incoming particle is given by the expression $\sin \vartheta_{\max} = m_2/m_1$.

16.9. A negative muon with the kinetic energy $T = 100$ MeV sustains a head-on collision with a stationary electron. Find the kinetic energy of the recoil electron.

16.10. Relativistic protons with the kinetic energy T are elastically scattered by stationary nuclei of hydrogen atoms. Let $\tilde{\vartheta}$ be the proton scattering angle in the C frame corresponding to the angle ϑ in the L frame. Prove that

(a) $\tan(\tilde{\vartheta}/2) = \sqrt{1 + T/2m}\,\tan \vartheta$, m is the mass of the proton;

(b) the differential cross-sections of this process in the C and L frames are related as

$$\tilde{\sigma}(\tilde{\vartheta}) = \frac{(1+\alpha \sin^2 \vartheta)^2}{4(1+\alpha)\cos \vartheta}\,\sigma(\vartheta)\ \text{cm}^2/\text{sr}; \quad \alpha = T/2m;$$

(c) the scattering in the C frame is anisotropic, if the differential cross-sections σ_1 and σ_2 corresponding to angles $\vartheta_1 = 15°$ and $\vartheta_2 = 30°$ are equal to 26.8 and 12.5 mb/sr respectively at $T = 590$ MeV.

16.11. Relativistic protons with the kinetic energy T_0 are elastically scattered by nuclei of hydrogen atoms.

(a) Demonstrate that the differential cross-section $\sigma(T)$ corresponding to the energy T of the scattered proton in the L frame is defined by the expression

$$\sigma(T) = \tilde{\sigma}(\tilde{\vartheta})\,\frac{4\pi}{T_0}\ \text{cm}^2/\text{MeV},$$

where $\tilde{\sigma}(\tilde{\vartheta})$ is the differential cross-section in the C frame in which the angle $\tilde{\vartheta}$ corresponds to the kinetic energy T.

(b) Find the energy distribution of the scattered protons in the L frame, if their angular distribution in the C frame is isotropic.

16.12. A positron whose kinetic energy is equal to its rest energy and a stationary free electron annihilate. As a result, two γ-quanta emerge, the energy of one being $n = 2$ times that of the other. Calculate the divergence angle between the motion directions of the γ-quanta.

16.13. Demonstrate that when relativistic positrons with momentum p and free electrons annihilate, the differential cross-section of

γ-quanta production with energy E_γ varies inversely with the positrons' momentum, if the angular distribution of γ-quanta in the C frame is isotropic.

16.14. Calculate the threshold energy of a γ-quantum required for $\pi^+\pi^-$ pair production in the field of stationary proton.

16.15. Derive formula (16.4).

16.16. Calculate the threshold kinetic energies of incoming particles in the following reactions (the incoming particles are indicated in the first position):

(1) $p + {}^2\text{H} \to {}^3\text{He} + \pi^0$; (5) $\pi^- + p \to n + K^0 + \tilde{K}^0$;

(2) $p + {}^{10}\text{B} \to {}^{11}\text{B} + \pi^+$; (6) $\tilde{p} + p \to p + \tilde{\Sigma}^0 + \tilde{K}^-$;

(3) $\tilde{p} + p \to \tilde{\Lambda} + \Lambda$; (7) $p + p \to p + p + p + \tilde{p}$;

(4) $\pi^- + p \to \Sigma^- + K^+$; (8) $p + p \to p + p + \Sigma^- + \tilde{\Sigma}^+$.

16.17. Find the kinetic energies of mesons produced in a hydrogen target when a striking particle possesses the threshold energy:

(a) $\gamma + p \to n + \pi^+$; **(b)** $p + p \to p + \Sigma^0 + K^+$.

16.18. Let a relativistic particle a strike a stationary particle A in the direct process, and a particle b strike a stationary particle B in the reverse one ($a + A \rightleftarrows B + b$). Assuming the total energy of interacting particles to be equal for both processes in the C frame, i.e. $\tilde{E}_a + \tilde{E}_A = \tilde{E}_b + \tilde{E}_B$, find: **(a)** how the kinetic energies of the striking particles T_a and T_b in the L frame are related in both direct and reverse processes, if the masses of particles A and B and the threshold kinetic energy of particle a are known; **(b)** the kinetic energy of a pion in the reaction $\gamma + p \rightleftarrows n + \pi^+$ for the reverse process, if the γ-quantum energy in the direct process is $\hbar\omega = 200$ MeV; the masses of a proton and a neutron are assumed to be equal.

16.19. Protons with the kinetic energy $T = 500$ MeV strike a hydrogen target activating the reaction $p + p \to d + \pi^+$. Find the maximum possible angle at which the deuterons are ejected.

16.20. The cross-section of interaction of π^--mesons with the proton target measured as a function of the pion kinetic energy exhibits maximum values at 198, 600, and 900 MeV. These maxima correspond to the unstable particles called *resonances*. Determine their rest masses.

DECAY OF PARTICLES

16.21. A stopped Σ^--particle decayed into a neutron and a pion. Find the kinetic energy and momentum of the neutron.

16.22. Calculate the highest values of the kinetic energy and momentum of an electron produced in the decay of a stopped muon.

16.23. A hypernucleus ${}^5\text{He}_\Lambda$ undergoes the decay ${}^5\text{He}_\Lambda \to {}^4\text{He} + p + \pi^-$. Calculate the binding energy of Λ-hyperon in the given hypernucleus, if its decay energy $Q = 34.9$ MeV.

16.24. A moving pion with a kinetic energy of $T_\pi = 50$ MeV disintegrated into a muon and a neutrino. At what angle was the muon ejected, if the angle of emission of the neutrino is 90°?

16.25. A π^0-meson whose kinetic energy is equal to its rest energy decays during its flight into two γ-quanta. Find: **(a)** the smallest possible divergence angle between the directions of motion of γ-quanta; **(b)** in what limits the energy of either quantum is confined.

16.26. A relativistic K^0-meson with the kinetic energy T decays during its flight into two π^0-mesons. Find: **(a)** at what magnitude of T one of the emerging pions can be produced as stationary; **(b)** the angle between symmetrically diverging pions, if $T = 100$ MeV.

16.27. A Σ^+-hyperon with a momentum of $p_\Sigma = 900$ MeV/c decays during its flight into a positive pion and a neutral particle. The meson is ejected with a momentum of $p_\pi = 200$ MeV/c at an angle $\vartheta = 60°$ to the initial direction of the hyperon's motion. Find the mass of the neutral particle and energy of the given decay.

16.28. A neutral particle decays to produce a proton and a negative pion with the divergence angle between the directions of outcoming particles being equal to $\theta = 60°$. The momenta of the produced particles are equal to 450 and 135 MeV/c. Assuming that there are no other decay products, find the mass of the decaying particle.

16.29. Derive formula (16.5) for the C frame.

16.30. Substantiate the plotting of the vector diagram of momenta for the case of a relativistic particle decaying into two particles (see Fig. 42).

16.31. Calculate the parameters for the ellipse of momenta and draw the appropriate vector diagram for the following cases:

(a) a neutral pion with $T = m_\pi$ decays as $\pi^0 \to 2\gamma$;

(b) a positive pion with $T = m_\pi/2$ decays as $\pi \to \mu + \nu$;

(c) a proton with $T = m_p$ is elastically scattered by a proton;

(d) a deuteron with $T = m_d/2$ is elastically scattered by a proton;

(e) a proton with $T = m_p$ activates the reaction $p + p \to \pi^+ + d$.

Here T is the kinetic energy.

16.32. A moving positive pion with the kinetic energy T disintegrates into a muon and neutrino. Find in the L frame: **(a)** the maximum possible angle of emission of the muon if $T = 50$ MeV; **(b)** the limiting value of T at which the muons are ejected at the limiting angle.

16.33. A relativistic pion moving with the velocity $\beta = v/c$ disintegrates into a muon and a neutrino.

(a) Demonstrate that the angles of emission of the neutrino in the C frame ($\tilde{\vartheta}$) and L frame (ϑ) are related as

$$\cos\tilde{\vartheta} = \frac{\cos\vartheta - \beta}{1 - \beta\cos\vartheta}.$$

(b) Assuming that in the C frame the angular distribution of decay products is isotropic, demonstrate that the differential decay cross-section corresponding to the neutrino outgoing at the angle ϑ in the L frame is

$$\sigma(\vartheta) \propto \frac{1-\beta^2}{(1-\beta\cos\vartheta)^2}\ \text{cm}^2/\text{sr}.$$

(c) Calculate the probability of the neutrino being emitted into the front hemisphere in the L frame if the pion's kinetic energy $T = m_\pi$ and the neutrino's angular distribution is isotropic in the C frame.

16.34. Due to interaction of slow π^--mesons with nuclei of a hydrogen target, the following reactions are observed:

$$\pi^- + p \begin{cases} \nearrow n + \gamma & (1) \\ \searrow n + \pi^0, \quad \pi^0 \to 2\gamma. & (2) \end{cases}$$

The energy spectrum of produced γ-quanta is shown in Fig. 43, where $E_1 = 54$ MeV, $E_2 = 84$ MeV, and $E_3 = 130$ MeV.

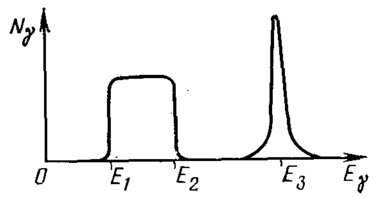


Fig. 43

(a) To what reaction branch does each maximum belong?
(b) Assuming the masses of a proton and a neutron to be known, determine the mass of a π^--meson.
(c) Find the mass of a π^0-meson.

16.35. In studies of the interaction of fast pions with protons, an unstable quasiparticle ρ was observed, whose lifetime is so short that its production and decay occur at practically the same point. How can one establish, having considered many outcomes of that reaction, that the process $\pi^- + p \to \pi^- + \pi^+ + n$ branches partially via the bound state $(\pi^-\pi^+)$, i.e. $\pi^- + p \to \rho + n$, $\rho \to \pi^- + \pi^+$? The total energies E_i and momenta $\mathbf{p}_i$ of emerging pions are assumed to be known in each case in the L frame.

16.36. In studies of the reaction $K^+ + p \to \Lambda + \pi^+ + \pi^-$ activated by K^--mesons with a kinetic energy of $T_K = 790$ MeV, it was observed that the reaction branches partially via the bound state $(\pi^-\Lambda)$ proceeding in two stages as follows: $K^- + p \to (\pi^-\Lambda) + \pi^+$, $(\pi^-\Lambda) \to \pi^- + \Lambda$, with the emerging π^+-mesons possessing a kinetic energy of $\tilde{T}_\pi = 300$ MeV in the C frame. Calculate the rest mass of the $(\pi^-\Lambda)$ resonance and its decay energy.

16.37. Determine the mean proper lifetime of: (a) muons, if, when possessing a kinetic energy $T = 7m_\mu$, their mean lifetime equals $\tau = 17.6$ μs; (b) π^+-mesons, if, possessing momentum $p = 55$ MeV/c, they survive over a distance $l = 3.0$ m on the average.

16.38. Find the probability of a positive pion decaying during its flight from the point at which it was produced to a target (over the distance of 6.00 m), if the pion's kinetic energy is equal to 100 MeV.

16.39. Suppose that a proton remains for some time in the "ideal proton" state with the magnetic moment μ_N and the rest of the time in the "ideal neutron" state ($\mu = 0$) plus a pion ($p \rightleftarrows n + \pi^+$). What is the fraction of time during which the proton remains in the ideal proton state?

16.40. Using the detailed balancing principle (see the introduction to Chapter 13), determine the spin of a positive pion, if the total cross-section σ_{pp} for protons with the kinetic energy $T_p = 500$ MeV (in the L frame) in the reaction $p + p \to d + \pi^+$ for the direct process is known to be one ninth of the total cross-section $\sigma_{\pi d}$ for the reverse process with the corresponding energy. The spins of a proton and a deuteron are known.

16.41. The interaction of γ-quanta with a hydrogen target activates the reaction $\gamma + p \to \pi^0 + p$. The total cross-section of this reaction is $\sigma_{\gamma p} = 0.20$ mb when the energy of γ-quanta is $E_\gamma = 250$ MeV. Using the detailed balancing principle, determine the cross-section of the reverse process (π^0-mesons striking the hydrogen target) with the corresponding kinetic energy of the meson. Find the value of this energy.

16.42. Using the conservation laws for lepton and baryon charges, find out whether the following processes are possible:

(1) $n \to p + e^- + \nu_e$; (4) $K^+ \to \mu^+ + \nu_\mu + \pi^0$;

(2) $\tilde{\nu}_\mu + p \to n + \mu^+$; (5) $\pi^- + n \to K^- + K^0$;

(3) $\mu^+ \to e^+ + \tilde{\nu}_e + \nu_\mu$; (6) $K^- + p \to \Sigma^+ + \pi^-$.

16.43. Which of the reactions written below are forbidden by the strangeness conservation law:

(1) $\pi^- + p \to \Lambda + K^0$; (4) $\tilde{p} + n \to \Lambda + \tilde{\Sigma}^-$;

(2) $\pi^- + p \to K^- + K^+$; (5) $\Sigma^- + p \to \Lambda + n$;

(3) $\tilde{p} + p \to \tilde{\Sigma}^0 + \tilde{K}^0 + n$; (6) $\pi^- + n \to \Xi^- + K^+ + K^-$?

16.44. What branches of the following reactions are forbidden and why?

(a) $\Sigma^- \begin{cases} \nearrow n + \pi^- & (1) \\ \searrow \Lambda + \pi^- & (2) \end{cases}$ (b) $\Xi^- \begin{cases} \nearrow p + 2\pi^- & (1) \\ \searrow \Lambda + \pi^-, \quad \Lambda \to p + \pi^-. & (2) \end{cases}$

16.45. Find possible values of the isotopic spin T and its projection T_z for the systems: nucleon-nucleon; pion-nucleon.

16.46. Using the generalized Pauli principle, find the isotopic spin T of the system:

(a) np in the states ^3P and ^3D;

(b) $\pi^+\pi^0$ in the states ^1P and ^1D;

(c) $\pi^+\pi^-$ in the states ^1P and ^1D.

16.47. Using the Shmushkevich method*, prove that isotopic invariance leads to the following relations between the total cross-sections σ and probabilities w of the processes:

(a) the reactions of the type $\tilde{N} + N \rightarrow \pi + \pi$ (N designates a nucleon):

$$\left.\begin{aligned} \tilde{p}+p &\rightarrow \pi^+ + \pi^- \ (\sigma_1) \\ \tilde{p}+p &\rightarrow \pi^0 + \pi^0 \ (\sigma_2) \\ \tilde{p}+n &\rightarrow \pi^- + \pi^0 \ (\sigma_3) \end{aligned}\right\} \quad 2\sigma_1 = 4\sigma_2 + \sigma_3;$$

(b) the reactions of the type $\pi + N \rightarrow \Lambda + K$:

$$\left.\begin{aligned} \pi^+ + n &\rightarrow \Lambda + K^+ \ (\sigma_1) \\ \pi^0 + p &\rightarrow \Lambda + K^+ \ (\sigma_2) \end{aligned}\right\} \quad \sigma_1 = 2\sigma_2;$$

(c) the reactions of the type $\pi + N \rightarrow \Sigma + K$:

$$\left.\begin{aligned} \pi^+ + p &\rightarrow \Sigma^+ + K^+ \ (\sigma_1) \\ \pi^0 + p &\rightarrow \Sigma^0 + K^+ \ (\sigma_2) \\ \pi^0 + p &\rightarrow \Sigma^+ + K^0 \ (\sigma_3) \\ \pi^- + p &\rightarrow \Sigma^- + K^+ \ (\sigma_4) \\ \pi^- + p &\rightarrow \Sigma^0 + K^0 \ (\sigma_5) \end{aligned}\right\} \quad \sigma_3 = \sigma_5, \ \sigma_1 + \sigma_4 = 2\sigma_2 + \sigma_3;$$

(d) the decay of τ-particles (τ^+, τ^0, τ^-) into three pions:

$$\left.\begin{aligned} \tau^+ &\rightarrow \pi^+ + \pi^0 + \pi^0 \ (w_1) \\ \tau^- &\rightarrow \pi^- + \pi^+ + \pi^- \ (w_2) \\ \tau^0 &\rightarrow \pi^0 + \pi^0 + \pi^0 \ (w_3) \end{aligned}\right\} \quad w_2 = w_1 + w_3;$$

(e) the decay of ω^0-particle into three pions: show that the decay $\omega^0 \rightarrow 3\pi^0$ is impossible (ω^0-isotopic singlet).

16.48. Find the change of isotopic spin T and its projection T_z in the following processes:

(a) $\pi^- + p \rightarrow K^+ + \Sigma^-$; (b) $\pi^- + p \rightarrow K^+ + K^0 + \Xi^-$;

(c) $K^+ \rightarrow \pi^0 + \pi^+$; (d) $K_1^0 \rightarrow 2\pi^0$.

* In this method both a target and a beam are treated as isotopically non-polarized and all possible reactions of the studied process are taken into account; besides, the produced particles of each type are supposed to be isotopically non-polarized as well.

16.49. The exposure of a deuterium target to slow ($l = 0$) π^--mesons activates the reaction $\pi^- + d \rightarrow 2n$. Recalling that the parity of a deuteron is positive, demonstrate using the laws of conservation of momentum and parity that π^--meson has the negative parity.

16.50. It is found experimentally that the isotopic spin of the ρ-particle, representing the bound state of two pions, is equal to 1.

(a) Taking into account that the decay $\rho \rightarrow 2\pi$ belongs to the class of strong interactions, predict the spin and parity of a ρ-particle, using the angular momentum conservation law; the internal parities of pions are identical.

(b) Write the possible decays of ρ^+-, ρ^0-, and ρ^--particles into two pions.

16.51. Below are given the values of quantum numbers of three hypothetical basic particles called quarks:

quarks	z	B	S
q_1	+2/3	+1/3	0
q_2	−1/3	+1/3	0
q_3	−1/3	+1/3	−1

Here z is the electric charge (in units of e), B is the baryon number, S is the strangeness. The quark's spin is equal to 1/2.

(a) From the three quarks construct the following baryons: p, n, Σ^+, Σ^-, Ξ^0, Ξ^-.

(b) Taking into account that the antiquarks $\tilde{q}_1$, $\tilde{q}_2$, and $\tilde{q}_3$ possess the values of z, B, and S that are opposite in sign to those for the corresponding quarks, construct from the two particles (a quark and an antiquark) the following mesons: π^+, π^-, K^+, K^-, and K^0.

(c) Find the ratio of the magnetic moments of a neutron and a proton, assuming the magnetic moment of a quark to be proportional to its electric charge. Take into account that for a particle formed of three quarks the probability of the state in which the spins of two identical quarks are parallel is twice that of the state in which two identical quarks have the antiparallel spins.

17

MOTION OF CHARGED PARTICLES IN EXTERNAL FIELDS

● Equations of motion of a particle with the charge q in axisymmetric electric and magnetic fields (in cylindrical coordinates):

$$\ddot{r} - r\dot{\varphi}^2 = \frac{q}{m}\left(E_r + \frac{v_\varphi}{c} B_z\right);$$

$$\frac{1}{r}\frac{d}{dt}(r^2\dot{\varphi}) = \frac{q}{mc}(v_z B_r - v_r B_z);$$

$$\ddot{z} = \frac{q}{m}\left(E_z - \frac{v_\varphi}{c} B_r\right).$$

LAWS OF MOTION. ANALYSERS

17.1. An electron moves in a uniform electric field whose strength grows at a constant rate $\dot{E} = 20$ MV/(cm·s). What amount of energy will the electron gain after it passes a distance $l = 10.0$ cm, provided that at the initial moment its velocity and the electric field are equal to zero?

17.2. An electron starts moving under the action of uniform electric field $E = 10.0$ kV/cm. Determine the energy that the electron acquires and the distance it covers during a time interval $\tau = 1.00 \cdot 10^{-8}$ s after the beginning of motion.

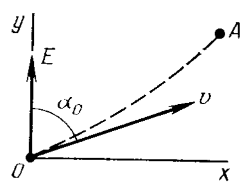


Fig. 44

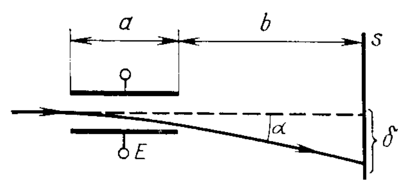


Fig. 45

17.3. A proton outgoing from the point O (Fig. 44) with the kinetic energy $T = 6.0$ keV gets to the point A with coordinates $x = 10.0$ cm and $y = 7.5$ cm due to uniform electric field of strength E. Determine: **(a)** the strength E if the angle α_0 is equal to 60°; **(b)** the values of α_0 and E at which the velocity vector of the proton makes an angle $\alpha = 30°$ with the vector **E** at the point A; **(c)** the time it takes for the proton to reach the point A, if $E = 1.00$ kV/cm

17.4. Having passed through the uniform transverse electric field E, an electron with the kinetic energy T gets onto the screen S (Fig. 45, where $a = 10$ cm, $b = 20$ cm). Determine the deflection angle α of the electron and its displacement δ on the screen, if: **(a)** $E = 20$ V/cm and $T = 1.00$ keV; **(b)** E grows at a constant rate $\dot{E} = 1.00$ MV/(cm·s) and the electron with $T = 40$ eV enters the electric field at the moment when $E = 0$; **(c)** E oscillates harmonically with a frequency $\nu = 10.0$ MHz and amplitude $E_0 = 5.0$ V/cm, and the electron with $T = 100$ eV enters the field at the moment when $E = 0$.

17.5. A beam of electrons accelerated by a potential difference $V = 1.0$ kV passes through two small capacitors separated by a distance $l = 20$ cm. A variable electric field is applied to the capacitors in equiphase from an oscillator. By varying the oscillator frequency, the beam is adjusted to pass this system without deflection. Determine the ratio e/m for an electron, if the two consecutive values of frequency satisfying that condition are equal to 141 and 188 MHz.

17.6. Determine the kinetic energies of a proton and a relativistic electron for which $B\rho = 5.0$ kG·cm.

17.7. A proton with the kinetic energy $T = 50$ keV passes a transverse magnetic field $B = 400$ G and gets onto the screen S (Fig. 46, where $a = 10$ cm and $b = 20$ cm). Determine the deflection angle α of the proton and its displacement δ on the screen.

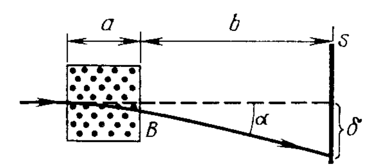


Fig. 46

17.8. From the point A located on the axis of a straight solenoid an electron with a kinetic energy of 500 eV is emitted at an angle $\alpha = 30°$ to its axis. The magnetic induction of the field is equal to $B = 50$ G. Calculate: **(a)** the lead of the helical trajectory of the electron; **(b)** the distance from the axis to the point on a screen to which the electron gets, if the screen is placed at right angles to the axis at a distance $l = 20$ cm from the point A.

17.9. A slightly diverging beam of electrons accelerated by a potential difference $V = 500$ V emerges from a certain point at the axis of a straight solenoid and comes to the focus at a distance $l = 15.0$ cm with two consecutive values of B: 158.0 and 189.6 G. Determine: **(a)** the specific charge of an electron; **(b)** the minimum magnetic induction capable of focussing the beam at that distance.

17.10. A source of monochromatic β-particles is located at the axis of a solenoid. The β-particles emitted at an angle $\alpha = 30°$ to the solenoid's axis are known to be focussed at a point removed from the source by a distance $l = 50$ cm at a minimum value of $B = 200$ G. Find their kinetic energy.

17.11. A source of β-particles is located at the point O on the axis of a straight solenoid, and the inlet of a counter at the point O'. The OO' distance is $l = 50$ cm. At the midpoint between O and O' there is a diaphragm with a narrow ring opening of radius $R = 7.5$ cm. Find: **(a)** the kinetic energy of the β-particles focussed at the point O' at the lowest value $B = 250$ G; **(b)** the first two values of magnetic induction at which β-particles with the kinetic energy equal to their rest energy come to focus at the point O'. Find also the corresponding angles α of emission of such β-particles.

17.12. In a mass spectrometer with semicircle focussing by a uniform transverse magnetic field a source and a focus point are separated by a distance $x = 40.0$ cm. Find the instrument's dispersion: **(a)** with respect to mass $\delta x/\delta A$ for monochromatic uranium ions; **(b)** with respect to energy $\delta x/\delta T$ for β-particles with kinetic energies of about $T = 1.0$ MeV.

17.13. A narrow beam of monochromatic ions passes a sector of a circle in a uniform transverse magnetic field as shown in Fig. 47 (the beam enters and leaves the sector at right angles to the boundary of the field). Find the instrument's angular dispersion with respect to mass $\delta\alpha/\delta A$ (deg/amu) for Ar isotopes, if $\varphi = 60°$.

17.14. A slightly diverging plane beam of ions enters the electric field of a cylindrical capacitor (Fig. 48) at the point A. The potential

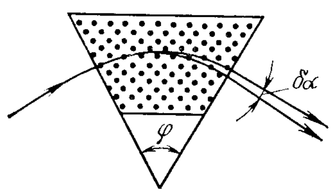


Fig. 47

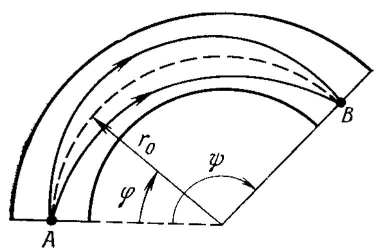


Fig. 48

difference of the capacitor plates is so adjusted that the particles whose velocity vector at the point A is perpendicular to the radius vector of that point continue moving along the circular trajectory of radius r_0. Prove that: **(a)** the sector angle sufficient for focussing the beam is equal to $\Psi = \pi/\sqrt{2}$; **(b)** the instrument's dispersion with respect to velocity is equal to $\Delta r/\Delta v = 2r_0/v$ at the focal point.

17.15. A slightly diverging beam of ions enters the transverse axisymmetric magnetic field diminishing with the distance as r^{-n} (see Fig. 48). The magnetic induction is such that the ions whose velocities are directed at right angles to the radius vector at the point A continue moving along the circular trajectory of radius r_0. Show that: **(a)** the angle at which the beam is focussed in the horizontal plane is $\Psi = \pi/\sqrt{1-n}$; **(b)** when $n = 1/2$, the double focussing takes place (both in the horizontal and vertical directions); **(c)** when $n = 1/2$,

the dispersion with respect to ions' velocity is equal to $\Delta r/\Delta v = 4r_0/v$.

17.16. A beam of deuterons passes through the uniform electric and magnetic fields produced in the same spatial region and crossed at right angles. Find the kinetic energy of the deuterons, if at $E = 1.00$ kV/cm and $B = 500$ G beam's trajectory remains rectilinear.

17.17. Demonstrate that, using the arrangements sketched in Fig. 49, one can simultaneously determine e/m and the velocity of

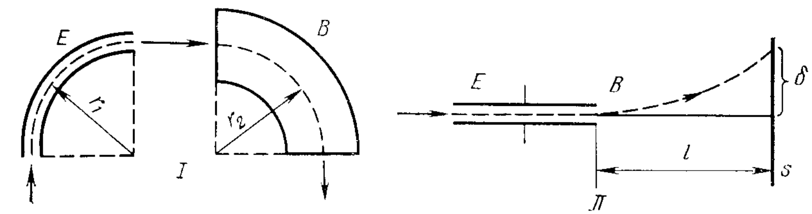


Fig. 49

charged particles. **(a)** In arrangement I a particle passes successively through the electric field E of a cylindrical capacitor and the uniform magnetic field B. The potential difference V across the capacitor plates is known, as well as the plates' radii R_1 and R_2, the magnetic field B, and the curvature radii r_1 and r_2 of the particle's trajectory. **(b)** In arrangement II a particle passes through the electric field of a plane capacitor and falls on the screen S. The whole arrangement (the capacitor and the space between the capacitor and the screen) is placed in the uniform magnetic field B ($\mathbf{B} \perp \mathbf{E}$). The values of E and B, at which the trajectory of the particle in the capacitor is rectilinear, are known, as well as the distance l and displacement δ.

17.18. The cylindrical cathode and anode of a magnetron have radii $r_1 = 1.0$ mm and $r_2 = 20$ cm respectively. The potential difference applied between the anode and cathode is $V = 200$ V. Neglecting the initial velocity of thermions, find the limiting value of the longitudinal uniform magnetic field in the magnetron at which the anode current ceases.

17.19. A cylindrical diode consists of a long straight heating filament and coaxial cathode and anode whose radii are equal to 0.10 and 1.0 cm. A current of 14.5 A flowing through the filament generates a magnetic field in the surrounding space. Neglecting the initial velocity of thermions, find the limiting potential difference between the anode and the cathode at which the anode current ceases.

17.20. A proton with the initial velocity v is ejected in the direction of the x axis from the point O of the region in which the uniform electric and magnetic fields E and B are produced in the direction of the y axis (Fig. 50). Find the equation of its trajectory $x(t)$, $y(t)$, $z(t)$. What is the shape of the trajectory?

17.21. A narrow beam of identical ions with different velocities enters the region with uniform parallel electric and magnetic fields E and B at the point O (see Fig. 50). The beam's direction coincides with the x axis at the entrance point. At the distance l from the point O there is a screen located at right angles to the x axis. Find the equation of the trace that the ions leave on the screen. Prove that this equation is a parabola when $z \ll l$.

17.22. A charged particle moves in the region where the uniform electric and magnetic fields E and B cross at right angles (see Fig. 51).

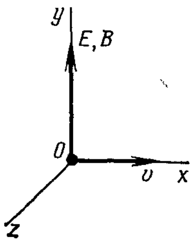


Fig. 50

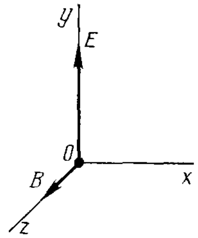


Fig. 51

Assuming that the particle leaves the point O without any initial velocity, find: **(a)** the equation of the particle's trajectory, $x(t)$, $y(t)$; **(b)** the length of the trajectory between the points at which the particle's velocity is equal to zero; **(c)** the mean velocity of the particle in the direction of the x axis.

17.23. A charged particle moves in the region where the uniform electric and magnetic fields E and B cross at right angles. Suppose the particle leaves the point O (see Fig. 51) with an initial velocity whose vector lies in the plane x, y and has the components $\dot{x}_0$ and $\dot{y}_0$. Find the equation of the particle's trajectory $x(t)$, $y(t)$ and draw its approximate plot, if

(a) $\dot{x}_0 = v/2,\ \dot{y}_0 = 0$; **(c)** $\dot{x}_0 = 0,\ \dot{y}_0 = v$;

(b) $\dot{x}_0 = -v,\ \dot{y}_0 = 0$; **(d)** $\dot{x}_0 = \dot{y}_0 = v$.

Here $v = cE/B$.

17.24. Demonstrate that under conditions of the foregoing problem all particles with equal magnitude of e/m will get at the point $x_1 = 2\pi mc^2E/eB^2$ of the x axis irrespective of the value and direction of their initial velocity.

17.25. A proton starts moving in the region where the uniform electric and magnetic fields are produced. The fields cross at right angles, with the magnetic field of induction B being constant and the electric field varying as $E = E_0 \cos \omega t$ with the frequency $\omega = eB/mc$. Find the equation of the proton's trajectory $x(t)$, $y(t)$, if at the initial moment $t = 0$ the proton was at the point O (see Fig. 51).

17.26. A charged particle moves in a longitudinal axisymmetric electric field described by the potential $V(r, z)$. When the particle leaves the source with the zero initial velocity and then moves in the vicinity of the axis of symmetry of the field, the differential equation of its trajectory takes the form:

$$r''V_0 + \frac{r'}{2}V_0' + \frac{r}{4}V_0'' = 0,$$

where $V_0(z)$ is the potential at the axis (relative to the potential of the source, $V_0(0) = 0$; the primes mark the derivatives with respect to z.

(a) Derive this expression, using the equation of motion.

(b) How does the magnitude of the particle's specific charge affect the characteristics of the trajectory? How does the particle's trajectory change when the whole system is increased in size n-fold and the potentials at the plates are kept constant?

17.27. Figure 52 illustrates a trajectory of a charged particle for a thin collecting electrostatic lens whose field is confined practically within a very narrow region between z_1 and z_2 (points *1* and *2*).

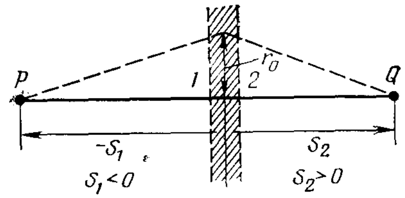


Fig. 52

Making use of the equation of the foregoing problem, demonstrate that:

(a) $$\frac{n_2}{s_2} - \frac{n_1}{s_1} = \frac{1}{4}\int_{-\infty}^{\infty} \frac{V_0''}{\sqrt{V_0}}\,dz;$$

(b) $$\frac{1}{f_2} = \frac{1}{8n_2}\int_{-\infty}^{\infty} \frac{V_0'}{V_0^{2/3}}\,dz.$$

Here $n_{1,2} = \sqrt{V_0(z_{1,2})}$; f_2 is the image-side focal distance.

17.28. A charged particle moves in a longitudinal axisymmetric magnetic field in the vicinity of its axis of symmetry. Using the equations of motion, show that the differential equations of the particle's trajectory take the following form in this case:

$$\varphi' = -\alpha B_0(z); \quad r'' + \alpha^2 r B_0^2(z) = 0,$$

where $\alpha = e/2mvc$, v is the particle's velocity, $B_0(z)$ is the magnetic flux density at the axis; the primes designate the differentiation with respect to z.

17.29. Figure 52 illustrates a trajectory of a charged particle for a thin magnetic lens whose field is confined practically within a very narrow region between z_1 and z_2 (points *1* and *2*). Using the equations of the foregoing problem, demonstrate that the focal length f of such a lens is defined by the following expression:

$$\frac{1}{f} = \alpha^2 \int_{-\infty}^{\infty} B_0^2 (z)\, dz.$$

Find the focal length f of the magnetic lens: **(a)** whose field varies along the axis as $B_0(z) = Ae^{-a^2z^2}$ for particles accelerated by the potential difference V; **(b)** realized as a wire loop of radius $R = 2.0$ cm carrying a current $I = 14$ A for electrons accelerated by potential difference $V = 50$ V. Also find the angle through which the image turns in this case.

ACCELERATORS OF CHARGED PARTICLES

17.30. For an electron and a proton moving along circular orbits in a uniform magnetic field $B = 10.0$ kG determine: **(a)** the orbital periods and radii if the kinetic energy of the particles is $T = 10.0$ MeV; **(b)** the kinetic energies if their orbital radii are $r = 10.0$ cm.

17.31. Suppose that in a betatron the magnetic flux confined by an equilibrium orbit with radius $r = 25.0$ cm grows from the zero value at a constant rate $\dot{\Phi} = 5.0 \cdot 10^9$ Mx/s. Determine: **(a)** the strength of the vortex electric field at the orbit and the energy acquired by the electron during $5.0 \cdot 10^5$ revolutions; **(b)** the distance travelled by an electron for $\tau = 3.00$ ms and the energy acquired during that interval.

17.32. The magnetic induction at the equilibrium orbit of radius $r = 100$ cm in a betatron varies from 0 to $B_m = 5$ kG as $B = B_m \times \sin \omega t$ with a frequency $\nu = 50$ Hz. Find: **(a)** the kinetic energy of the electrons at the end of the acceleration cycle; **(b)** the distance travelled by the electron and the number of revolutions made during the whole acceleration cycle provided the initial velocity of the electrons is equal to zero.

17.33. The condition under which an electron moves along a circular orbit of permanent radius in a betatron requires that at any moment the magnetic field at the orbit should change with the rate equal to half the rate with which the mean magnetic induction within the equilibrium orbit varies, that is, $\dot{B} = \langle \dot{B} \rangle /2$ (the *betatron condition*).

(a) Prove this condition to be true.

(b) How does the orbital radius change in the field $B \propto r^{-n}$, where n is the fall-off index $(0 < n < 1)$, when this condition is not met?

17.34. Using the betatron condition (see the foregoing problem), demonstrate that the vortex electric field in a betatron has the minimum value at the equilibrium orbit. Take into account that the fall-off index of the magnetic field in the vicinity of the equilibrium orbit

$$n = -\frac{r}{B} \cdot \frac{\partial B}{\partial r} < 1.$$

17.35. In a betatron the magnetic field at the plane of symmetry varies in the vicinity of the equilibrium orbit as $B \propto r^{-n}$, where n is the fall-off index. Prove that the motion stability of electrons: **(a)** in the radial direction is effected at $n < 1$; **(b)** in the vertical direction is effected at $n > 0$.

17.36. The magnetic field at the betatron's plane of symmetry varies in the vicinity of the equilibrium orbit as $B \propto r^{-n}$, where n is the fall-off index $(0 < n < 1)$. Let the angular velocity of an electron moving along the equilibrium orbit be equal to ω_0. For the electrons moving in the vicinity of the equilibrium orbit determine the frequencies of: **(a)** radial and **(b)** axial oscillations.

17.37. The unlimited increase in the energy of charged particles in orbit accelerators is inhibited by the effect caused by radiation losses. The amount of energy lost by a particle per one revolution is equal to $\frac{4\pi e^2}{3r}\left(\frac{E}{mc^2}\right)^4$, where r is the orbital radius, E is the total energy of the particle, m is its rest mass. Calculate the energy E that the electrons can be accelerated to in a betatron, if the equilibrium orbital radius is $r = 100$ cm and the magnetic field at the orbit increases at a rate $\dot{B} = 1000$ kG/s.

17.38. For protons, deuterons, and α-particles accelerated in a cyclotron up to a maximum radius of curvature $\rho = 50$ cm, determine: **(a)** the kinetic energy at the end of acceleration, if the magnetic induction is $B = 10.0$ kG; **(b)** the lowest oscillator frequency sufficient to reach the kinetic energy $T = 20$ MeV at the end of acceleration.

17.39. An oscillator drives a cyclotron at a frequency of $\nu = 10$ MHz. Determine the effective accelerating voltage applied to the dees, if the distance between neighbouring trajectories of deuterons is $\Delta\rho = 1.0$ cm for a radius of curvature of $\rho \approx 50$ cm.

17.40. In a cyclotron driven at a frequency of $\nu = 10$ MHz α-particles are accelerated up to a maximum radius of curvature $\rho = 50$ cm. The effective voltage applied to the dees is $V = 50$ kV. Neglecting the gap between the dees, determine: **(a)** the total acceleration time of the particles; **(b)** the total distance covered by the particles during the complete cycle of acceleration.

17.41. At what values of the kinetic energy does the period of revolution of electrons, protons, and α-particles in a uniform magnetic field exceed that at non-relativistic velocities by 1.00%?

17.42. A cyclotron is known to be inapplicable for acceleration of electrons since their orbital period τ increases rapidly with energy and they get out of step with the alternating electric field. This situation can be rectified, however, by making the orbital period increment $\Delta\tau$ of an electron equal to a multiple of the accelerating field period τ_0. An accelerator employing such a principle is called a *microtron*. How many times has the electron to cross the accelerating gap of the microtron to acquire an energy $\Delta E = 4.6$ MeV, if $\Delta\tau = \tau_0$, the magnetic induction is $B = 1.07$ kG, and the frequency of the accelerating field is $f = 3.0 \cdot 10^3$ MHz?

17.43. To counteract dephasing emerging in the process of acceleration of a particle and caused by variation of its orbital period with increase in its energy, the frequency of accelerating field is slowly decreased. Such an accelerator is called a *synchrocyclotron*.

(a) By how many percents should the frequency of the accelerating field of a synchrocyclotron be changed to accelerate protons and α-particles up to an energy $T = 500$ MeV?

(b) What is the time variation function $\omega(t)$ of the frequency of a synchrocyclotron, if the magnetic flux density of the field is B and mean energy acquired by a particle per one revolution is ε?

17.44. In a cyclotron-type accelerator the resonance acceleration of particles can be accomplished, if the magnetic field is changed slowly so that the orbital period of the particle remains constant and equal to the period of the accelerating field. Such an accelerator is called a *synchrotron*. Assuming that the magnetic field of a synchrotron is uniform and changing as $B = B_m \sin \omega t$ and that the frequency of the accelerating field is equal to ω_0, find: **(a)** the particle's orbital radius as a function of time; **(b)** in what limits varies the orbital radius of an electron accelerated from 2.00 to 100.0 MeV, if $\omega_0 = 7.00 \cdot 10^8\ s^{-1}$ and $\omega = 314\ s^{-1}$. What distance does the electron cover during the complete acceleration cycle?

17.45. The accelerator, in which both the frequency of the accelerating electric field $\omega(t)$ and magnetic field $B(t)$ vary simultaneously, is referred to as a *proton synchrotron*. What is the relation between $\omega(t)$ and $B(t)$ allowing the particles to be accelerated along the fixed orbit of radius r? The influence of the vortex electric field is to be ignored.

17.46. In a cyclic proton synchrotron accelerating protons from 0.500 to 1000 MeV, the orbital radius is $r = 4.50$ m. Assuming the magnetic field to grow in the acceleration process with constant rate $\dot{B} = 15.0$ kG/s, determine: **(a)** the limits, within which the frequency of accelerating electric field varies, and the total acceleration time; **(b)** the energy acquired by a proton per one revolution; **(c)** the distance covered and number of revolutions made during the whole acceleration cycle.

The influence of the rotational electric field is to be ignored.

17.47. In the proton synchrotron of the Joint Nuclear Research Institute in Dubna, protons are accelerated from 9.0 to 10,000 MeV. The perimeter of the stable orbit, with rectilinear sections taken into

account, is $\Pi = 208$ m. The orbital radii at the rounded-off sections of the orbit are $r = 28.0$ m. At these sections the magnetic field increases in the acceleration process with constant rate $\dot{B} = 4.00$ kG/s. Consider the same questions as in the foregoing problem.

17.48. The most powerful modern accelerators (e.g. the Serpukhov one) employ the strong focussing principle. What is the essential of this principle? What are its advantages?

17.49. In a linear accelerator, charged particles move through a system of drift tubes connected alternately to the opposite terminals of a high-frequency oscillator G (see Fig. 53). The acceleration of

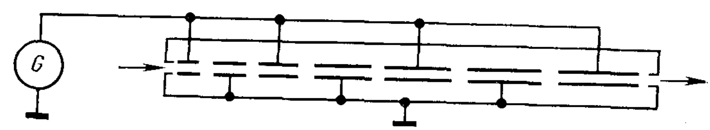


Fig. 53

the particles is effected in the gaps between the tubes. Suppose that protons are injected into an accelerator with an energy $T_0 = 2.0$ MeV to be accelerated up to an energy $T = 20$ MeV. The protons increase their energy by $\Delta E = 0.50$ MeV over each gap; the frequency of the oscillator is $f = 100$ MHz. Ignoring the distance taken up by the gaps between the tubes, determine the length: **(a)** of the nth drift tube, in particular, of the first and the last one; **(b)** of all drift tubes (the length of the accelerator).

17.50. Suppose that all drift tubes in a linear accelerator have the same length $l = 6.00$ cm. Within what limits should the frequency of the oscillator be varied to accelerate the protons and electrons from 5.00 to 50.0 MeV in such an accelerator?

17.51. A travelling wave linac employs a cylindrical diaphragmatic waveguide along which an electromagnetic wave propagates, whose electric axial component is equal to E_x. The application of perforated ring diaphragms increases the phase velocity of the wave travelling along the waveguide, with the accelerated particle being approximately in the same phase all the time. Find: **(a)** the value of E_x sufficient for acceleration of protons from 4.0 to 1000 MeV over the waveguide's length $L = 67.0$ m; **(b)** how the phase velocity of the wave depends on the distance from the entrance opening of the waveguide. By what factor does the phase velocity of the wave change in the case of protons and by how many percents in the case of electrons on their acceleration from 4.0 to 1000 MeV?

17.52. A considerable increase in the energy of colliding particles can be attained by using the colliding beams of these particles. What amount of kinetic energy should be transferred to a proton striking another (stationary) proton to make their total kinetic energy in the C frame equal to the kinetic energy of two protons moving toward each other with kinetic energies of 50 GeV?

ANSWERS AND SOLUTIONS

1.1. The ordinate of the curve $u_2(\omega)$ corresponding to the frequency ω_2 relates to the ordinate of the curve $u_1(\omega)$ corresponding to the frequency ω_1 such that $\omega_1/T_1 = \omega_2/T_2$, as $u_1(\omega_1) = (T_1/T_2)^3 u_2(\omega_2)$.

1.2. **(a)** From the condition $du_\omega/d\omega = 0$, we obtain $3f(x) + xf'_x(x) = 0$, where $x = \omega/T$. The root of this equation is x_0, and, hence, $\omega_{pr} \propto T$. **(b)** According to Eq. (1.1), $M \propto u = \int_0^\infty \omega^3 f(\omega/T)\, d\omega = T^4 \int_0^\infty x^3 f(x)\, dx \propto T^4$, where $x = \omega/T$.

1.3. **(a)** Transform Wien's formula (1.2) from the variable u_ω to u_λ: $u_\lambda = \lambda^{-5} F(\lambda T)$. From the condition $du_\lambda/d\lambda = 0$, we obtain $5F(x) + xF'_x(x) = 0$, where $x = \lambda T$. The root of this equation is x_0, and therefore $\lambda_{pr} \propto 1/T$. **(b)** $(u_\lambda)_{max} = \lambda_{pr}^{-5} F(\lambda_{pr} T) \propto \lambda_{pr}^{-5} \propto T^5$.

1.4. Decreases by $3.0 \cdot 10^2$ K.

1.5. 2.9 μm.

1.6. $4.6 \cdot 10^{20}$ MW ($5 \cdot 10^9$ kg/s); 10^{11} years.

1.7. $T_2 = T_1 \sqrt{r/R} = 3.8 \cdot 10^2$ K.

1.8. **(a)** $\approx 1.6 \cdot 10^4$ GPa; **(b)** $\approx 19 \cdot 10^6$ K.

1.9. $t = c\rho r (\eta^3 - 1)/9\sigma T_0^3 = 1.6$ hours, ρ is the density of copper.

1.10. **(a)** $\omega_{pr} = 3T/a = 7.85 \cdot 10^{14}$ s^{-1}; **(b)** $\langle\omega\rangle = \frac{4T}{a}$.

1.11. **(a)** $\lambda_{pr} = 2\pi ca/5T = 1.44$ μm; **(b)** $\langle\lambda\rangle = 2\pi ca/3T = 2.40$ μm.

1.12. $u_{Cu}/u_{rad} = 3R\rho c/4M\sigma T^3 = 1.8 \cdot 10^{14}$, where R is the universal gas constant, ρ is the density of copper, c is the velocity of light, M is the mass of a mole of copper, σ is the Stefan-Boltzmann constant.

1.13. **(a)** $\varepsilon = kT$, $u_\omega\, d\omega = (kT/\pi^2 c^3)\, \omega^2\, d\omega$ is the Rayleigh-Jeans formula;

(b) $\langle\varepsilon\rangle = \dfrac{\Sigma n\hbar\omega e^{-n\hbar\omega/kT}}{\Sigma e^{-n\hbar\omega/kT}} = \dfrac{\Sigma n\hbar\omega e^{-\alpha n\hbar\omega}}{\Sigma e^{-\alpha n\hbar\omega}}$, where $\alpha = 1/kT$. Here the summation is carried over n from 0 to ∞. The latter expression can be obtained as follows:

$$\langle\varepsilon\rangle = -\frac{\partial}{\partial\alpha} \ln (\Sigma e^{-\alpha n\hbar\omega}) = -\frac{\partial}{\partial\alpha} \ln \frac{1}{1 - e^{-\alpha\hbar\omega}} = \frac{\hbar\omega}{e^{\hbar\omega/kT} - 1};$$

$$u_\omega\, d\omega = \frac{\hbar}{\pi^2 c^3} \cdot \frac{\omega^3\, d\omega}{e^{\hbar\omega/kT} - 1} \text{ is Planck's formula.}$$

1.14. $u_\omega = (kT/\pi^2 c^3)\, \omega^2$ and $u_\omega = (\hbar\omega^3/\pi^2 c^3)\, e^{-\hbar\omega/kT}$.

1.15. **(a)** $u_\nu = \dfrac{16\pi^2\hbar}{c^3} \cdot \dfrac{\nu^3}{e^{2\pi\hbar\nu/kT} - 1}$;

(b) $u_\lambda = 16\pi^2 c\hbar \dfrac{\lambda^{-5}}{e^{2\pi\hbar c/kT\lambda} - 1}$.

1.16. $\Delta u_\omega/u_\omega = e^{-\alpha} \leqslant 0.01$, where $\alpha = 2\pi\hbar c/kT\lambda$, whence $\alpha \geqslant 4.6$; $\lambda \leqslant 7.2$ μm.

1.17. **(a)** by the factor of 4.75; **(b)** 0.60 W/cm^2.

1.18. **(a)** $I = \frac{c}{4} \int u_\omega\, d\omega = \frac{\pi^2}{60} \frac{k^4}{c^2\hbar^3} T^4$; **(b)** from the condition $du_\lambda/d\lambda = 0$, we obtain the equation $5 - x = 5e^{-x}$, where $x = 2\pi\hbar c/kb$. The root of this equation is found by inspection or from the graph: $x_0 = 4.965$. Whence $b \approx 0.29$ cm·K.

1.19. **(a)** $\langle\omega\rangle = 3.84kT/\hbar = 1.0 \cdot 10^{15}$ s^{-1}; **(b)** $T = 2.33\hbar c/k\langle\lambda\rangle = 2.00 \cdot 10^3$ K.

1.20. **(a)** $n_\omega\, d\omega = \dfrac{1}{\pi^2 c^3} \cdot \dfrac{\omega^2\, d\omega}{e^{\hbar\omega/kT} - 1}$; $n_\lambda\, d\lambda = 8\pi \dfrac{\lambda^{-4}\, d\lambda}{e^{2\pi\hbar c/kT\lambda} - 1}$;

(b) $n = 0.243 (kT/\hbar c)^3 = 5.5 \cdot 10^8$ cm^{-3}.

1.21. **(a)** from the condition $dn_\omega/d\omega = 0$, we get $2 - x = 2e^{-x}$, where $x = \hbar\omega/kT$. The root of this equation is found by inspection or from the graph: $x_0 = 1.6$. Whence $\hbar\omega_{pr} = 1.6\, kT = 0.14$ eV; **(b)** $2.7\, kT = 0.23$ eV.

1.22. Of n photons confined within a unit volume, the number of photons that move within the elementary solid angle $d\Omega$ is $dn = n \cdot d\Omega/4\pi$. Then we single out those photons that move within the solid angle $d\Omega$ that makes the angle ϑ with the normal of the area ΔS. These photons move practically parallel to one another, so that the number of photons reaching the area ΔS per unit time Δt can be found from the volume of the oblique cylinder with base ΔS and height $c\,\Delta t \cos\vartheta$ (Fig. 54):

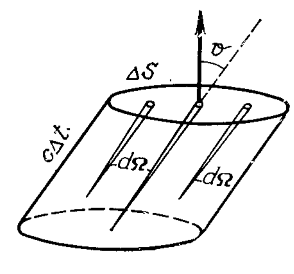


Fig. 54

$$dN = dn\, \Delta S \cos\vartheta \cdot c\, \Delta t.$$

Integrate this expression with respect to ϑ going from 0 to $\pi/2$ and with respect to φ going from 0 to 2π, taking into account that $d\Omega = \sin\vartheta\, d\vartheta\, d\varphi$. Finally we obtain

$$\Delta N = (1/4) nc\, \Delta t\, \Delta S.$$

It follows that the number of photons falling on unit area per unit time is equal to $(1/4)nc$. Multiplying the latter expression by the mean energy of a photon $\langle\hbar\omega\rangle$, we get $(1/4)n\langle\hbar\omega\rangle c = (1/4)uc = M$.

1.23. $2\cdot10^{13}$ $\text{cm}^{-2}\cdot\text{s}^{-1}$ in both cases.

1.24. 2.5 eV/s; 5 keV/s and 0.6 MeV/s.

1.25. $\frac{dp}{dt}=\int\frac{\hbar\omega}{c}\,dN_\omega=\frac{\Phi}{c}$, where $dN_\omega=\frac{\Phi_\omega\,d\omega}{\hbar\omega}$ is the photon flux within the frequency interval $(\omega,\ \omega+d\omega)$.

1.26. $\langle p\rangle=4E\,(1+\rho)/\pi d^2c\tau=5\cdot10^3$ kPa.

1.27. Having drawn the triangle of momenta (at the bottom of Fig. 55, where p_0, p_0', and p are the momenta of the incident beam, reflected beam, and the momentum transferred to the plate), we obtain

$$p=\frac{E}{c}\sqrt{1+\rho^2+2\rho\cos2\vartheta}=3.5\cdot10^{-3}\ \text{g}\cdot\text{cm/s}.$$

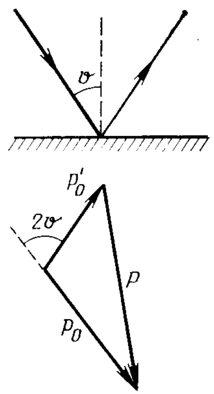


Fig. 55

1.28. (a) $\frac{2}{c}JS\cos^2\vartheta$; (b) $\frac{1}{2c}JS$; (c) $\frac{3}{2c}JS$.

1.29. $F=N/2c\,(1+l^2/r^2)=5\cdot10^{-3}$ dyne.

1.30. The solution is similar to that of Problem 1.22. The normal component of the total momentum transferred to the area dS of the wall during the time interval Δt is

$$\Delta p_n=2\int\delta p\cos\vartheta=\frac{u}{3}\,dS\,\Delta t;$$

$$\delta p=\frac{\hbar\omega}{c}\,n_\omega\,d\omega\,dV\,\frac{d\Omega}{4\pi}.$$

Here the figure 'two' in front of the integral allows for the fact that at thermal equilibrium each incident photon is accompanied with a photon emitted by the wall in the opposite direction.

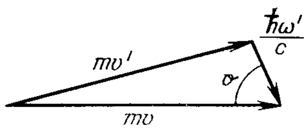


Fig. 56

1.31. From the laws of conservation of energy and momentum (Fig. 56), we obtain

$$\frac{mv^2}{2}+\Delta E=\frac{mv'^2}{2}+\hbar\omega';$$

$$(mv')^2=\left(\frac{\hbar\omega'}{c}\right)^2+(mv)^2-2mv\,\frac{\hbar\omega'}{c}\cos\vartheta.$$

Here $\Delta E=\hbar\omega$ is the decrement of the internal energy of the atom. Eliminating v' from these equations, we get

$$\hbar\,(\omega'-\omega)=\hbar\omega'\left(\frac{v}{c}\cos\vartheta-\frac{\hbar\omega'}{2mc^2}\right).$$

For visible light, when the angle ϑ does not approach $\pi/2$, the last term in parentheses can be neglected. Then

$$\frac{\omega'-\omega}{\omega'}\approx\frac{v}{c}\cos\vartheta.$$

1.32. $d\,(\hbar\omega)=-\gamma\frac{mM}{r^2}\,dr$; $\frac{\Delta\omega}{\omega}=1-e^{-\gamma M/Rc^2}$, where γ is the gravitational constant; $m=\hbar\omega/c^2$ is the mass of the photon; **(a)** $\Delta\lambda/\lambda\approx\gamma M/Rc^2=2.1\cdot10^{-6}$; **(b)** 0.09.

1.33. 12.4 Å·kV.

1.34. 1.0 Å.

1.35. 31 kV.

1.36. $v=c\sqrt{\alpha(\alpha+2)}/(\alpha+1)=0.50c$, $\alpha=2\pi\hbar/mc\lambda_{\min}$.

1.37. **(b)** $\eta=J_{\text{tot}}/P=0.8\cdot10^{-6}ZV_{\text{kV}}\approx0.5\%$.

1.38. $\lambda_{\text{pr}}=1.5\lambda_{\min}=3\pi\hbar c/eV=0.6$ Å.

1.39. **(a)** 0.66 and 0.235 μm; **(b)** $5.5\cdot10^5$ m/s (Zn); $3.4\cdot10^5$ m/s (Ag); electrons are not emitted (Ni).

1.40. 1.7 V.

1.41. $\lambda_0=(2\pi\hbar c/A)\ (\eta-n)/(\eta-1)=0.26$ μm.

1.42. $T_{\max}=\hbar\,(\omega+\omega_0)-A=0.38$ eV.

1.43. The upper levels in both metals are located at the same height (Fig. 57). Therefore, the electrons liberated from the upper

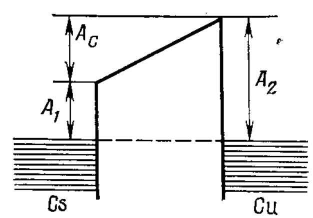


Fig. 57

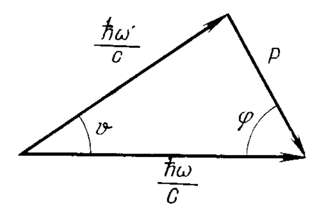


Fig. 58

level in cesium perform the work $A_1+A_c=A_2$, where A_c is the work performed to overcome the external contact potential difference; **(a)** 0.28 μm; **(b)** $6.4\cdot10^5$ m/s.

1.44. From the condition $\hbar\omega=A_{\text{Zn}}+e\,(V_c+V_0)$, we find $V_c=-0.5$ V, i.e. the polarity of the contact potential is opposite to the external difference of potentials.

1.45. 0.196, 0.213, and 0.224 μm.

1.46. $p\approx\sqrt{(\hbar\omega)^2+2m_ec^2\,(\hbar\omega-E)}/c=96$ keV/c.

1.47. From the laws of conservation of energy and momentum $\hbar\omega+mc^2=mc^2/\sqrt{1-\beta^2}$; $\hbar\omega/c=mv/\sqrt{1-\beta^2}$, where $\beta=v/c$, it follows that β is equal either to 0 or to 1. Both results have no physical meaning.

1.48. **(a)** In the general case a recoil electron is relativistic, and the laws of conservation of energy and momentum have therefore to be written in the form

$$\hbar\omega + mc^2 = \hbar\omega' + \sqrt{p^2c^2 + m^2c^4};$$
$$p^2c^2 = (\hbar\omega)^2 + (\hbar\omega')^2 - 2\hbar\omega\hbar\omega' \cos\vartheta,$$

where ω and ω' are the frequencies of the photon before and after scattering; p and m are the momentum and rest mass of the electron. The second relation follows directly from the triangle of momenta (Fig. 58).

Transfer the term $\hbar\omega'$ of the first equation from the right-hand side to the left-hand side and then square both sides. From the expression thus obtained subtract the second equation to get

$$\frac{1}{\omega'} - \frac{1}{\omega} = \frac{2\hbar}{mc^2}\sin^2\frac{\vartheta}{2}; \quad \lambda' - \lambda = 4\pi\frac{\hbar}{mc}\sin^2\frac{\vartheta}{2};$$

(b) $\cot\varphi = (1 + \hbar\omega/mc^2)\tan(\vartheta/2)$.

1.49. **(a)** $\hbar\omega' = \dfrac{\hbar\omega}{1 + 2(\hbar\omega'/mc^2)\sin^2(\vartheta/2)} = 0.20$ MeV;

(b) $T = \dfrac{2\varepsilon^2\sin^2(\vartheta/2)}{1 + 2\varepsilon\sin^2(\vartheta/2)}mc^2 = 0.26$ MeV, where $\varepsilon = \hbar\omega/mc^2$.

1.50. $T = \dfrac{pc}{1 + 2(p/mc)\sin^2(\vartheta/2)} - E_b = 31$ keV.

1.51. $\lambda = \left(\dfrac{2\pi\hbar}{mc}\right)\left(\sqrt{1 + 2mc^2/T_{max}} - 1\right) = 0.020$ Å.

1.52. 0.020 Å; 0.61 and 0.43 MeV.

1.53. 105°.

1.54. $2mc = 1.02$ MeV/c.

1.55. 29°.

1.56. **(a)** 0.012 Å; **(b)** 0.0030 Å.

1.57. $\hbar\omega = (T/2)\left[1 + \sqrt{1 + 2mc^2/T\sin^2(\vartheta/2)}\right] = 0.94$ MeV.

1.58. $T = \hbar\omega\eta/(1 + \eta) = 0.20$ MeV.

1.59. **(a)** $\lambda - \lambda' = (4\pi\hbar/mc)\sin^2(\vartheta/2) = 0.012$ Å; **(b)** 0.17 MeV.

1.60. **(a)** The Compton shift equation is obtained on the assumption that photons are scattered by free electrons. The electrons in a substance behave as free ones when their binding energy is considerably less than the energy transferred to them by the photons. Consequently, the hard radiation should be employed.

(b) Due to scattering by free electrons.

(c) This component is due to scattering of photons by strongly bound electrons and nuclei.

(d) Due to the increase in the number of electrons becoming free (see item (a)).

(e) Due to scattering of photons by moving electrons.

2.1. **(a)** $r = 3e^2/2E = 1.6\cdot10^{-8}$ cm; **(b)** $\omega = \sqrt{e^2/mr^3}$; $3\cdot10^{-8}$ cm.

2.2. **(a)** $5.9\cdot10^{-10}$ cm; **(b)** $r_{min} = (q_1q_2/T)(1 + m_\alpha/m_{Li}) = 3.4\cdot10^{-11}$ cm.

2.3. From the energy conservation law it follows that the magnitude of the momentum of a scattered particle remains the same as before scattering. Hence the scattered particle has the increment of the momentum vector whose modulus is

$$|\Delta\mathbf{p}| = 2p_0\sin(\vartheta/2).$$

On the other hand, from Fig. 59 it follows that

$$|\Delta\mathbf{p}| = \int f_n\,dt = \int\frac{q_1q_2r\cos\chi}{r^3}\,dt = q_1q_2\int\frac{\sin(\varphi - \vartheta/2)\,d\varphi}{r^2\dot\varphi},$$

where f_n is the projection of the interaction force vector on the direction of the vector $\Delta\mathbf{p}$. In accordance with the law of conservation

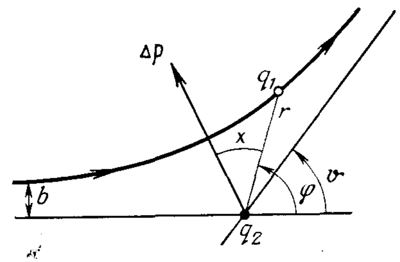


Fig. 59

of angular momentum, the integrand's denominator is $r^2\dot\varphi = -bv_0$, where v_0 is the velocity of the particle far from the nucleus. After integration we obtain

$$|\Delta\mathbf{p}| = (2q_1q_2/bv_0)\cos(\vartheta/2).$$

Comparing the latter expression with the first one, we get formula (2.1).

2.4. $6\cdot10^{-11}$ cm.

2.5. **(a)** $|\Delta\mathbf{p}| = \sqrt{\dfrac{8mT}{1 - (bT/Ze^2)^2}} = 1.3\cdot10^2$ MeV/c;

(b) $T = \dfrac{Ze^2}{b} = 1.3$ MeV; $\pi/2$.

2.6. $r_{min} = \dfrac{Ze^2}{2T}\left(1 + \csc\dfrac{\vartheta}{2}\right) = 1.6\cdot10^{-11}$ cm; $b = 6.6\cdot10^{-12}$ cm.

2.7. $\tilde p = \dfrac{M}{m + M}\sqrt{2mT}$; $\tilde T = \dfrac{M}{m + M}T$.

2.9. $T' = \left(\dfrac{m_2 - m_1}{m_2 + m_1}\right)^2 T$.

2.10. $\vartheta_{max} = \arcsin(m_1/m_2) = 30°$.

2.11. $\tan\vartheta = \dfrac{\sin\tilde\vartheta}{\cos\tilde\vartheta + m_1/m_2}$; $\vartheta = 36°$.

2.12. 20°.

2.13. $v_{rel} = \frac{e}{\sqrt{\mu b}} \sqrt[4]{\frac{1+\eta}{1-\eta}} = 3.8 \cdot 10^5$ m/s, where μ is the reduced mass, $\eta = m_1/m_2$.

2.14. $b = \frac{e^2 \tan \vartheta'}{T}\left(1 + \frac{m_p}{m_\alpha}\right) = 2.4 \cdot 10^{-11}$ cm.

2.15. $b = \frac{e^2}{T \tan(\tilde{\vartheta}/2)}\left(1 + \frac{m_\alpha}{m_d}\right) = 5 \cdot 10^{-11}$ cm, where $\tilde{\vartheta} = \frac{\pi}{2} + \arcsin \frac{m_d}{m_\alpha}$.

2.16. The solution is similar to that of Problem 2.6, only in this case the calculation is performed in the C frame with the substitution $m \to \mu$, $T \to \tilde{T}$, and $\vartheta \to \tilde{\vartheta}$, where μ is the reduced mass; $\tilde{T}$ and $\tilde{\vartheta}$ are the total kinetic energy of the particles and scattering angle in the C frame. $r_{min} = (3e^2/T)(1 + m_\alpha/m_{Li})(1 + \csc \vartheta') = 6 \cdot 10^{-11}$ cm.

2.17. **(a)** $\cos(\vartheta/2) = b/(R + r)$; **(b)** $dw = (1/2) \sin \vartheta \, d\vartheta$; $w = 1/2$.

2.18. $dN/N = n \, d\sigma$, where $d\sigma = (Ze^2/2T)^2 \frac{2\pi \sin \vartheta \, d\vartheta}{\sin^4(\vartheta/2)}$.

2.19. $4 \cdot 10^{-4}$.

2.20. $\Delta\sigma = \pi (Ze^2/2T)^2 \cot^2(\vartheta/2) = 2.1 \cdot 10^{-22}$ cm².

2.21. $d\sigma/d\vartheta = (Ze^2/2T)^2 \frac{2\pi \sin \vartheta}{\sin^4(\vartheta/2)} = 3.0 \cdot 10^{-22}$ cm²/rad.

$d\sigma/d\Omega = (d\sigma/d\vartheta)/2\pi \sin \vartheta = 4.8 \cdot 10^{-23}$ cm²/sr.

2.22. $\Delta\sigma = \pi \sin^2 \vartheta_0 \cdot d\sigma/d\Omega = 5.5 \cdot 10^{-22}$ cm².

2.23. **(a)** $6 \cdot 10^{-5}$; **(b)** $w = \pi n (Ze^2/T)^2 [\csc^2(\vartheta_1/2) - \csc^2(\vartheta_2/2)] = 5 \cdot 10^{-4}$.

2.24. **(a)** $1.3 \cdot 10^4$; **(b)** $\Delta N = N\tau\pi n (Ze^2/T)^2 \cot^2(\vartheta_0/2) = 1.6 \cdot 10^5$; **(c)** $\Delta N = N\tau [1 - \pi n (Ze^2/T)^2 \cot^2(\vartheta_0/2)] = 1.5 \cdot 10^7$.

2.25. $\frac{\Delta N}{N} = \frac{\pi e^4}{4T^2}\left(0.7 \frac{Z_1^2}{A_1} + 0.3 \frac{Z_2^2}{A_2}\right) \rho \, d N_A \cot^2 \frac{\vartheta_0}{2} = 2.7 \cdot 10^{-3}$, where Z_1 and Z_2 are the atomic numbers of copper and zinc; A_1 and A_2 are the masses of their moles; N_A is the Avogadro constant.

2.26. $\frac{d\sigma}{d\Omega}(\vartheta_0) = \frac{\eta}{4\pi n} \frac{\tan^2(\vartheta/2)}{\sin^4(\vartheta_0/2)} = 1.0 \cdot 10^{-23}$ cm²/sr.

2.27. $N' = 4\pi N_0 n (q_1 q_2/4T)^2 [\csc^2(\vartheta_1/2) - \csc^2(\vartheta_2/2)]$.

2.28. $\tau = m^2 c^3 r_0^3/4e^4 \approx 10^{-11}$ s.

2.29. $r_n = \sqrt{(\hbar/m\omega)\, n}$, $E_n = n\hbar\omega$, where $n = 1, 2, \ldots$; $\omega = \sqrt{\varkappa/m}$.

2.30. $2.27 \cdot 10^{39}$.

2.31. $r_n = \frac{\hbar^2}{me^2} \cdot \frac{n^2}{Z}$; $v_n = \frac{e^2}{\hbar} \cdot \frac{Z}{n}$; $T = E_b = \frac{me^4}{2\hbar^2} \cdot \frac{Z^2}{n^2}$.

System	r_1 and r_2, 10^{-8} cm		v_1 and v_2, 10^6 m/s		T and E_b, eV	V_1, V	λ_{12}, Å
H	0.529	2.12	2.19	1.09	13.6	10.2	1215
He⁺	0.264	1.06	4.38	2.19	54.4	40.8	304
Li⁺⁺	0.176	0.70	6.57	3.28	122.5	91.5	135

2.33. 0.116, 0.540, and 1.014 μm.

2.34. Correspondingly: 0.091 ÷ 0.122, 0.365 ÷ 0.657, 0.821 ÷ ÷ 1.875 μm.

2.35. **(a)** 0.657, 0.487, and 0.434 μm; **(b)** from the formula $\lambda/\delta\lambda = kN$, we obtain $N = 2.0 \cdot 10^3$.

2.36. The Brackett series, $\lambda_{6 \to 4} = 2.63$ μm.

2.37. **(a)** 0.122, 0.103, and 0.097 μm (Lyman series); 0.657 and 0.486 μm (Balmer series); 1.875 μm (Paschen series); **(b)** $n(n-1)/2 = 45$.

2.38. 1216, 1026, and 973 Å.

2.39. **(a)** 4; **(b)** 3.

2.40. $Z = 3$, Li^{++}.

2.41. 54.4 eV (He^+).

2.42. $E = E_0 + 4\hbar R^* = 79$ eV.

2.43. $2.3 \cdot 10^6$ m/s.

2.44. $3.1 \cdot 10^6$ m/s.

2.45. $v_{min} = \sqrt{6\pi c\hbar R/m} = 6.25 \cdot 10^4$ m/s, m is the mass of the atom.

2.46. $v \approx 3\hbar R^*/4Mc = 3.25$ m/s; $\Delta\varepsilon/\varepsilon \approx (3/8)\hbar R^*/Mc^2 = 0.55 \times 10^{-6}$%; M is the mass of the atom.

2.47. $7 \cdot 10^5$ m/s.

2.48. From the formula $\omega' = \omega\sqrt{(1+\beta)/(1-\beta)}$, $\beta = v/c$, we get $\beta = 0.29$.

2.49. Let us write the motion equation and Bohr's quantization condition: $\mu\omega^2 a = e^2/a^2$; $\mu a^2 \omega = n\hbar$, where μ is the reduced mass of the system; ω is the angular velocity; a is the distance between the electron and the nucleus. From these equations, we find

$$a_n = \frac{\hbar^2}{\mu e^2} n^2; \quad E_b = \frac{\mu e^4}{2\hbar^2} \frac{1}{n^2}, \quad R^* = \frac{\mu e^4}{2\hbar^3}.$$

When the motion of the nucleus is neglected, E_b and R^* turn out to be greater by $m/M = 0.055$%, where m and M are the masses of the electron and nucleus.

2.50. $m_p/m_e = (n - \eta)/n(\eta - 1) = 1.84 \cdot 10^3$.

2.51. **(a)** $E_D - E_H = 3.7 \cdot 10^{-3}$ eV; **(b)** $V_D - V_H = 2.8$ mV; **(c)** $\lambda_H - \lambda_D = 0.33$ Å.

2.52. **(a)** $2.85 \cdot 10^{-11}$ cm; **(b)** 6.54 Å; **(c)** 2.53 and 2.67 keV.

2.53. **(a)** $1.06 \cdot 10^{-8}$ cm; **(b)** 6.8 and 5.1 V; **(c)** $1.03 \cdot 10^{16}$ s^{-1}; 0.243 μm.

2.54. **(a)** $E_n = (\pi^2\hbar^2/2ml^2)\, n^2$; **(b)** $E_n = \hbar\omega \cdot n$, $\omega = \sqrt{\varkappa/m}$; **(c)** $E_n = (\hbar^2/2mr^2)\, n^2$; **(d)** $E_n = -m\alpha^2/2\hbar^2 n^2$.

3.1. 0.39 and 0.0091 Å; 0.15 keV and 0.082 eV.

3.2. 1.50 Å.

3.3. 1.32 Å.

3.4. 0.12 MeV.

3.5. 0.38 keV.

3.6. $\lambda' = \lambda\sqrt{(n+1)/(n-1)} = 0.022$ Å.

3.7. $\tilde{\lambda} = \lambda_n (1 + m_n/m_{\mathrm{He}}) = 0.7$ Å, where $\lambda_n = 2\pi\hbar/\sqrt{2m_n T}$.

3.8. $\tilde{\lambda} = \lambda (1+\eta)/(1-\eta) = 1.0$ Å, where $\eta = m_{\mathrm{H}}/m_{\mathrm{He}}$.

3.9. **(a)** $\lambda = \dfrac{2\pi\hbar}{\sqrt{2mT}} \cdot \dfrac{1}{\sqrt{1+T/2mc^2}}$; **(b)** $T \leqslant \begin{cases} 20\ \text{keV (electron)}, \\ 37\ \text{MeV (proton)}. \end{cases}$

3.10. $T = mc^2 (\sqrt{1+4\pi^2} - 1) = 2.74$ MeV.

3.11. 0.033 Å.

3.12. $f(\lambda) \propto \lambda^{-4} \exp(-2\lambda_m^2/\lambda^2)$, $\lambda_m = \pi\hbar/\sqrt{mkT} = 0.90$ Å.

3.13. $f(\lambda) \propto \lambda^{-5} \exp(-5\lambda_m^2/2\lambda^2)$, $\lambda_m = 2\pi\hbar/\sqrt{5mkT} = 0.57$ Å.

3.14. $T = (2/m)(\pi\hbar l/d\,\Delta x)^2 = 24$ eV.

3.15. $V_0 = \pi^2\hbar^2/2med^2\, (\sqrt{\eta} - 1)^2 \sin^2\vartheta = 0.15$ kV.

3.16. $d = \pi\hbar k/\sqrt{2mT} \cos(\alpha/2) = 2.1$ Å.

3.17. $d = \pi\hbar k/\sqrt{2mT} \sin\vartheta = 2.3$ Å, with $\tan 2\vartheta = r/l$.

3.18. **(a)** $n = \sqrt{1 + V_i/V} = 1.05$; **(b)** $V/V_i \gg 50$.

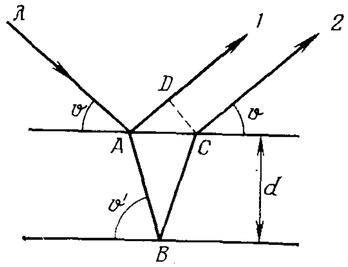


Fig. 60

3.19. **(a)** The maxima are observed when the optical pathlength difference of beams *1* and *2* (Fig. 60)

$$\Delta = (ABC) - (AD) = (2d/\sin\vartheta')\, n - 2d \cot\vartheta' \cos\vartheta = k\lambda,$$

where n is the refractive index. Consequently, $2d\sqrt{n^2 - \cos^2\vartheta} = k\lambda$. **(b)** $V_i = (\pi\hbar k)^2/2med^2 - V \sin^2\vartheta = 15$ V.

3.20. $E_n = (\pi^2\hbar^2/2ml^2) n$; $n = 1, 2, \ldots$.

3.21. $2\pi r = n\lambda$; $n = 1, 2, \ldots$; $\lambda = 2\pi r_1 n$; r_1 is the first Bohr radius.

3.22. **(a)** In accordance with the condition $\Psi(x, t) = \displaystyle\int_{k_0 - \Delta k}^{k_0 + \Delta k} A e^{i(\omega t - kx)}\, dk$. Expand the function $\omega(k)$ into a series of

$k - k_0$, $\omega \approx \omega_0 + (d\omega/dk)_0 (k - k_0)$ and designate $\xi = k - k_0$; then

$$\psi(x, t) = a e^{i(\omega_0 t - k_0 x)} \int_{-\Delta k}^{+\Delta k} e^{i[(d\omega/dk)_0 t - x]\xi}\, d\xi = A(x, t)\, e^{i(\omega_0 t - k_0 x)},$$

where $A(x, t) = 2a \dfrac{\sin[(d\omega/dk)_0 t - x]\,\Delta k}{(d\omega/dk)_0 t - x}$.

(b) The maximum of the function $A(x, t)$ is at the point $x = (d\omega/dk)_0 t$; whence the velocity of displacement of the maximum (group velocity) $v = (d\omega/dk)_0$.

3.23. $v_{\mathrm{gr}} = \dfrac{d\omega}{dk} = \dfrac{dE}{dp} = \dfrac{d}{dp}\sqrt{p^2c^2 + m^2c^4} = v$.

3.25. A scattered photon that passed through an objective lens has $p_x < (\hbar\omega/c) \tan\vartheta$. The right-hand side of this inequality states at the same time the uncertainty Δp_x for an electron: $\Delta p_x \propto (\hbar\omega/c) \times \tan\vartheta \approx (2\pi\hbar/\lambda) \sin\vartheta$. The uncertainty in the electron's coordinate $\Delta x \propto d = \lambda/\sin\vartheta$. Whence, $\Delta x \cdot \Delta p_x \propto 2\pi\hbar$.

3.26. To determine through which slit a particle has passed its y coordinate has to be determined (by means of an indicator I) with uncertainty $\Delta y < d/2$; d is the distance between the slits. In accordance with the uncertainty principle this means that the indicator must introduce the uncertainty in the y projection of the particle's momentum $\Delta p_y' \gtrsim 2\hbar/d$.

On the other hand, the condition for the diffraction pattern not being disturbed is $\Delta p_y' \ll p\vartheta_1$, where $p = 2\pi\hbar/\lambda$; $\vartheta_1 \approx \lambda/d$; λ is the particle's wavelength, i.e. $\Delta p_y' \ll 2\pi\hbar/d$.

Thus, the uncertainty in the momentum Δp_y introduced by the indicator turns out to be much greater than the uncertainty $\Delta p_y'$ at which the diffraction pattern would remain unblurred.

3.27. Assuming $\Delta x = 0.5$ μm, we obtain $2 \cdot 10^4$, $1 \cdot 10$, and 5×10^{-2} cm/s.

3.28. $\Delta v \approx 10^6$ m/s; $v_1 = 2.2 \cdot 10^6$ m/s.

3.29. **(a)** $T_{\min} \propto 2\hbar^2/ml^2 = 15$ eV, here $\Delta x = l/2$ and $p \propto \Delta p$; **(b)** $\Delta v/v \propto 2\hbar/l\sqrt{2mT} = 1.2 \cdot 10^{-4}$; here $\Delta x = l/2$.

3.30. To compress the well by the value δl, the work $\delta A = F\delta l$ should be performed that transforms into the increment dE of the particle's energy. Whence $F = dE/dl \propto 4\hbar^2/ml^3 = 2E_{\min}/l$. It is taken into account here that $\Delta x = l/2$ and $p \propto \Delta p$.

3.31. Assuming that $\Delta x \propto x$ and $v \propto \Delta v$, we obtain the total energy of the particle $E = T + U \approx \hbar^2/2mx^2 + \varkappa x^2/2$. From the condition $dE/dx = 0$, we obtain $E_{\min} \approx \hbar\omega$. The strict solution yields $\hbar\omega/2$.

3.32. Assuming $\Delta r \propto r$ and $v \propto \Delta v$, we obtain

$$E_{\mathrm{b}} = |U| - T \approx \frac{e^2}{r} - \frac{\hbar^2}{2mr^2}.$$

From the condition $dE_{\mathrm{b}}/dr = 0$ we find $r \approx \hbar^2/me^2 = 0.5 \cdot 10^{-8}$ cm and $E_{\mathrm{b}} \approx me^4/2\hbar^2 = 13.6$ eV.

3.33. Assuming for both electrons $\Delta r \propto r$ and $v \propto \Delta v$, write the expression for the total energy

$$E \approx 2\left(\frac{p^2}{2m} - \frac{2e^2}{r}\right) + \frac{e^2}{2r} \approx \frac{\hbar^2}{mr^2} - \frac{7e^2}{2r},$$

where the term $e^2/2r$ accounts for the energy of interaction of the electrons. The minimum of E corresponds to $r \approx 4\hbar^2/7me^2 = 0.3\cdot10^{-8}$ cm; $E_{\min} \approx -\frac{49}{16}\cdot\frac{me^4}{\hbar^2} = -83$ eV. The experiment yields -79 eV.

3.34. $2\cdot10^3$.

3.35. A train of waves spreads owing to the velocity spread $\Delta v \approx \hbar/m\,\Delta x$. After the time dt its width increment is $d(\Delta x) = \Delta v \times dt = (\hbar/m\,\Delta x)\,dt$. Having integrated this equation, we get: $\tau \approx \eta^2 m l^2/2\hbar \approx 10^{-12}$ s.

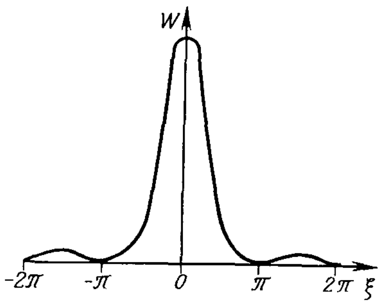


Fig. 61

3.36. The width of the image $s \approx \delta + \delta'$, where δ is the width of the slit, δ' is the additional broadening due to the velocity uncertainty Δv_y caused by the slit; $\delta' \approx (2\hbar/m\delta)l/v$ (it is assumed here that $\Delta x = \delta/2$ and the velocity of spread of the train of waves is directly proportional to Δv_y). The function $s(\delta)$ is minimal at $\delta \approx \sqrt{2\hbar l/mv} = 10^{-3}$ cm.

3.37. **(a)** $\psi(x) = \int_{k_0-\Delta k}^{k_0+\Delta k} a e^{ikx}\,dk = 2a\,\frac{\sin(x\,\Delta k)}{x}\,e^{ik_0x}$;

$$w(x) = |\psi(x)|^2 = 4a^2(\Delta k)^2\frac{\sin^2\xi}{\xi^2};\quad \xi = x\,\Delta k.$$

The function $w(\xi)$ is shown in Fig. 61. It is seen from the figure that the probability of the particle being at a certain location differs from zero practically in the region $\Delta\xi \approx 2\pi$. Whence, $\Delta x \approx 2\pi/\Delta k$, which agrees with the uncertainty principle.

(b) $$\psi(x) = \int_{-\infty}^{+\infty} a_k e^{ikx}\,dk = e^{-\alpha^2k_0^2}\int e^{-\alpha^2k^2 + (2k_0\alpha^2 - ix)k}\,dk.$$

The latter integral is readily transformed to the form

$$\frac{1}{\alpha}\int e^{-\xi^2 + c\xi}\,d\xi,$$

where $\xi = \alpha k$; $c = 2k_0\alpha + ix/\alpha$. To calculate it, supplement the index of the exponent to make a square and put in the substitution $u = \xi - c/2$. Then $\int_{-\infty}^{\infty} e^{-\xi^2+c\xi}\,d\xi = \sqrt{\pi}e^{c^2/4}$ and

$$\psi(x) = (\sqrt{\pi}/\alpha^2)\,e^{-x^2/4\alpha^2}e^{ik_0x},\quad w(x) = (\pi/\alpha^2)\,e^{-x^2/2\alpha^2}.$$

The effective localization region is assumed to be confined between the points at which $w(x)$ is less than in the centre by a factor of "e". Hence, $\Delta x = 2\sqrt{2}\alpha$.

3.38. **(a)** Represent the function $\psi(x)$ by means of the Fourier integral

$$\psi(x) = \int_{-\infty}^{+\infty} a_k e^{ikx}\,dk,\quad a_k = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\psi(x)\,e^{-ikx}\,dx = \frac{l}{\pi}\,\frac{\sin\xi}{\xi},$$

where $\xi = (k_0 - k)l$. Plotting the function $a(\xi) \propto \frac{\sin\xi}{\xi}$ one can easily see that the spectrum of the wave numbers corresponding to the considered train of waves is practically confined within the region $\Delta\xi \approx 2\pi$. Hence, $\Delta k \approx 2\pi/l$ which agrees with the uncertainty principle.

(b) In this case

$$a_k = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\psi(x)\,e^{-ikx}\,dx = \frac{1}{2\pi\alpha}\int_{-\infty}^{+\infty} e^{-\xi^2+c\xi}\,d\xi = \frac{\sqrt{\pi}}{2\pi\alpha}\,e^{-\left(\frac{k-k_0}{2\alpha}\right)^2},$$

where $\xi = \alpha k$, $c = i(k_0 - k)/\alpha$. The latter integral is calculated the way it was done in Problem 3.37, (b).

The effective interval of wave numbers is assumed to be confined between the points at which a_k is less than in the centre by a factor of "e". Hence, $\Delta k \approx 2\alpha$.

3.39. When U depends on time implicitly, the total Schrödinger equation allows the solutions of the form $\Psi(x, t) = \psi(x)f(t)$. Substituting this expression into the total Schrödinger equation, we obtain two equations:

$$\psi'' + \frac{2m}{\hbar^2}(E - U)\psi = 0;\quad \dot{f} + i\frac{E}{\hbar}f = 0.$$

The solution of the first equation gives the eigenfunctions $\psi_n(x)$ corresponding to the energy eigenvalues E_n. The solution of the second equation: $f(t) \sim e^{-i\omega_n t}$, $\omega_n = E_n/\hbar$. Finally, we obtain $\Psi_n(x, t) = \psi(x)e^{-i\omega_n t}$.

3.40. Only the time-dependent coefficient of the total wave function will change. However, the physical meaning can be ascribed only to the square of the modulus of that function, the change in the time-dependent coefficient will not manifest itself in any way.

3.41. Assuming $U = 0$, we look for the solution of the total Schrödinger equation in the form $\Psi(x, t) = \psi(x)f(t)$. As a result,

$$\Psi(x, t) = Ae^{-i(\omega t - kx)};\quad \omega = E/\hbar;\quad k = p/\hbar.$$

3.42. Assuming $U = 0$ in the Schrödinger equation, we obtain the solution $\psi = Ae^{\pm ikx}$; $k = \sqrt{2mE}/\hbar$. This solution is seen to be finite at any values of $E > 0$.

3.43 **(a)** $E_n = \frac{\pi^2\hbar^2 n^2}{2ml^2}$, $\psi_n = \sqrt{\frac{2}{l}} \sin\frac{\pi n x}{l}$;

(b) $w = \frac{1}{3} + \frac{\sqrt{3}}{2\pi} = 0.61$;

(c) $dN_E = (l/\pi\hbar)\sqrt{m/2E}\, dE$.

3.44. **(a)** The solution of the Schrödinger equation in the well can be readily found in the form of a product of sine functions: $\psi(x, y) = A \sin k_1x \cdot \sin k_2y$, since the wave function must turn to zero at $x = 0$ and $y = 0$. The allowed values of k_1 and k_2 are found from the boundary conditions:

$$\psi(a, y) = 0, \quad k_1 = n_1\pi/a, \quad n_1 = 1, 2, 3, \ldots,$$
$$\psi(x, b) = 0, \quad k_2 = n_2\pi/b, \quad n_2 = 1, 2, 3, \ldots .$$

As a result,

$$E_{n_1n_2} = \frac{\pi^2\hbar^2}{2m}\left(\frac{n_1^2}{a^2} + \frac{n_2^2}{b^2}\right); \quad \psi_{n_1n_2} = \sqrt{\frac{4}{ab}} \sin\frac{\pi n_1 x}{a} \cdot \sin\frac{\pi n_2 y}{b}.$$

(b) 0.038.

(c) 2, 5, 8, and 10 units of $\pi^2\hbar^2/2ml^2$. **(d)** Each pair of numbers n_1 and n_2 has a corresponding state. The number of states within the interval dn_1 and dn_2 in the vicinity of the values n_1 and n_2 is $dN = dn_1\, dn_2$.

Let us mark the values $k_1 = n_1\pi/a$ and $k_2 = n_2\pi/b$ on the coordinate axes. Then we draw the circle $k_1^2 + k_2^2 = k^2$ in this k-space. All points of the circle correspond to the same magnitude of energy E. Since the values k_1 and k_2 are positive, we shall consider only one quadrant of the circle. The number of points (states) enclosed within one quarter of the ring, formed by two circles with radii k and $k + dk$, is

$$\delta N = \int dn_1\, dn_2 = \int \frac{ab}{\pi^2}\, dk_1\, dk_2 = \frac{1}{4}\,\frac{S}{\pi^2}\, 2\pi k\, dk; \quad S = ab.$$

Taking into account that $E = (\hbar^2/2m)k^2$, we get $\delta N = (Sm/2\pi\hbar^2)\, dE$.

3.45. **(a)** $E_{n_1n_2n_3} = \frac{\pi^2\hbar^2}{2m}\left(\frac{n_1^2}{a^2} + \frac{n_2^2}{b^2} + \frac{n_3^2}{c^2}\right)$, $\psi_{n_1n_2n_3} = \sqrt{\frac{8}{abc}} \times \sin\frac{\pi n_1 x}{a} \cdot \sin\frac{\pi n_2 y}{b} \cdot \sin\frac{\pi n_3 z}{c}$, where n_1, n_2, n_3 are integers not equal to zero; **(b)** $\Delta E = \pi^2\hbar^2/ml^2$; **(c)** for the sixth level $n_1^2 + n_2^2 + n_3^2 = 14$. It can be easily found by inspection that this number is the sum of squares of the single triad of numbers: 1, 2, and 3. The number of different states corresponding to the given level is equal in our case to the number of permutations of this triad, i.e. six; **(d)** $dN_E = (Vm^{3/2}/\sqrt{2}\,\pi^2\hbar^3)\sqrt{E}\, dE$. The derivation is similar to that presented in item (d) of the foregoing problem.

3.46. Integrate the Schrödinger equation over the narrow interval enclosing the discontinuity of the potential energy:

$$\psi'(+\delta) - \psi'(-\delta) = \int_{-\delta}^{+\delta} \frac{2m}{\hbar^2}(E - U)\psi\, dx.$$

Since the discontinuity of U is finite, the integral approaches zero if $|\delta| \to 0$. Hence $\psi'(+0) = \psi'(-0)$.

3.47. **(a)** Write the Schrödinger equation for two regions:

$$0 \leqslant x \leqslant l, \quad \psi_1'' + k_1^2\psi_1 = 0, \quad k_1 = \sqrt{2mE}/\hbar;$$
$$x \geqslant l, \quad \psi_2'' + k_2^2\psi_2 = 0, \quad k_2 = \sqrt{2m(E - U_0)}/\hbar.$$

The solutions are: $\psi_1 = a \sin k_1x$, $\psi_2 = b \sin(k_2x + \alpha)$. From the condition of continuity of ψ and ψ' at the point $x = l$, we obtain $\tan(k_2l + \alpha) = (k_2/k_1)\tan k_1l$. The latter equation holds for any E because it has an arbitrary constant α.

(b) Write the Schrödinger equation for two regions:

$$0 \leqslant x \leqslant l, \quad \psi_1'' + k^2\psi_1 = 0, \quad k = \sqrt{2mE}/\hbar;$$
$$x \geqslant l, \quad \psi_2'' - \varkappa^2\psi_2 = 0, \quad \varkappa = \sqrt{2m(U_0 - E)}/\hbar.$$

The solutions $\psi_1 = a \sin kx$; $\psi_2 = be^{-\varkappa x}$ satisfy the standard and boundary conditions. From the condition of continuity of ψ and ψ' at the point $x = l$, we obtain

$$\tan kl = -k/\varkappa, \text{ or } \sin kl = \pm\sqrt{\hbar^2/2ml^2U_0}\, kl.$$

The graphical solution of this equation (Fig. 62) gives the roots corresponding to the eigenvalues of E. The roots are found by means of

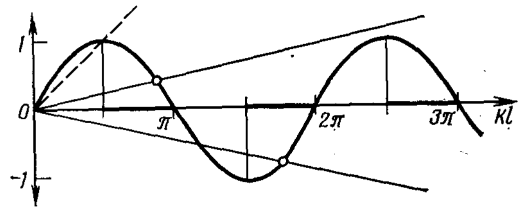


Fig. 62

those intersection points for which $\tan kl < 0$, i.e. they are in the even quarters of a circumference. (These sections of the abscissa axis are shown in Fig. 62 with heavy lines.) It can be seen that in some cases the roots do not exist; the dotted line indicate the ultimate position of the straight line.

(c) The nth level appears at $l^2U_0 = (2n - 1)^2\pi^2\hbar^2/8m$; four levels.

(d) $9\pi^2\hbar^2/16m$; $x_{pr} = 2l/3$; 0.15.

11—0339

(e) The problem reduces to the solution of the equation $\sin kl = \pm(3/5\pi)kl$. Its roots are $k_1 l = 5\pi/6$ and $k_2 l = 5.0$. Respectively, $E_1 = 0.25U_0$ and $E_2 = 0.91U_0$.

3.48. **(a)** Write the solutions of the Schrödinger equation for three regions:

$$x<0,\quad \psi_1 = ae^{\varkappa x},\qquad \varkappa = \sqrt{2m(U_0-E)}/\hbar;$$

$$0\leqslant x\leqslant l,\quad \psi_2 = b\sin(kx+\alpha),\quad k=\sqrt{2mE}/\hbar;$$

$$x>l,\quad \psi_3 = ce^{-\varkappa x}.$$

From the continuity of ψ and ψ' at the points $x = 0$ and $x = l$ we obtain

$$\tan\alpha = k/\varkappa;\quad \tan(kl+\alpha) = -k/\varkappa,$$

whence $\sin\alpha = \hbar k/\sqrt{2mU_0}$; $\sin(kl+\alpha) = -\hbar k/\sqrt{2mU_0}$. Eliminating α from the two latter equations, we get

$$kl = n\pi - 2\arcsin\left(\hbar k/\sqrt{2mU_0}\right),\quad n = 1,\ 2,\ \ldots,$$

where the values of arcsin function are taken for the first quarter (from 0 to $\pi/2$). Since the argument of arcsin function cannot exceed unity, the values of k cannot exceed $k_{max} = \sqrt{2mU_0}/\hbar$.

Let us plot the left-hand and right-hand sides of the latter equation as a function of k (Fig. 63, y_1, y_2, and y_3 being the right-hand side of the equation at $n = 1, 2, 3$). The points at which the straight line crosses the curves y_1, y_2, etc. define the roots of this equation which, as can be seen from the figure, constitute the discrete spectrum of eigenvalues E.

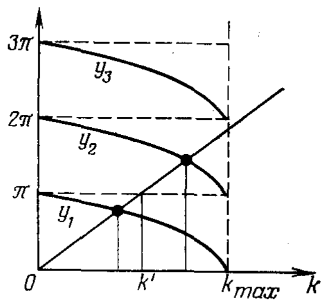


Fig. 63

With U_0 diminishing, k_{max} shifts to the left, and the number of intersection points decreases (for a given l the position of the straight line remains fixed). When k_{max} becomes less than k' (see Fig. 63), the well possesses only one energy level.

Thus, the given well always possesses at least one energy level.

(b) $\pi^2\hbar^2/4m$; 0.28.

(c) $\pi^2\hbar^2/2m$. The energy of the ground state is defined by the equation $2x = \pi - 2\arcsin x$, $x = kl/2$, whence $\cos x = 2x/\pi$. The root of this equation $x \approx 0.93$, $E_1 = 0.35U_0$.

(d) At $l^2U_0 = (\pi^2\hbar^2/2m)n^2$; three levels.

3.49. **(a)** Write the Schrödinger equation for three regions:

$$x<0,\quad \psi_1 = ae^{\varkappa_1 x},\qquad \varkappa_1 = \sqrt{2m(U_1-E)}/\hbar;$$

$$0\leqslant x\leqslant l,\quad \psi_2 = b\sin(kx+\alpha),\qquad k=\sqrt{2mE}/\hbar;$$

$$x>l,\quad \psi_3 = ce^{-\varkappa_2 x},\qquad \varkappa_2 = \sqrt{2m(U_2-E)}/\hbar.$$

From the condition of continuity of ψ and ψ' at the well's boundaries we get $\tan\alpha = k/\varkappa_1$ and $\tan(kl+\alpha) = -k/\varkappa_2$, whence

$$\sin\alpha = \hbar k/\sqrt{2mU_1};\quad \sin(kl+\alpha) = -\hbar k/\sqrt{2mU_2}.$$

Eliminating α from the two latter equations, we obtain

$$kl = n\pi - \arcsin\frac{\hbar k}{\sqrt{2mU_1}} - \arcsin\frac{\hbar k}{\sqrt{2mU_2}},\qquad (1)$$

where $n = 1, 2, 3, \ldots$, and the values of arcsin function are taken for the first quarter (from 0 to $\pi/2$). Figure 64 illustrates the left-hand and right-hand sides of Eq. (1) as a function of k; here y_1, y_2, and y_3 is the right-hand side of the equation at $n = 1, 2, 3$. Since the argument of arcsin function cannot exceed unity, the value of k in Eq. (1) cannot exceed $k_{max} = \sqrt{2mU_1}/\hbar$ if $U_1 < U_2$. The points at which the straight line kl crosses the curves y_1, y_2, etc. define the roots of this equation constituting the discrete spectrum of eigenvalues of E.

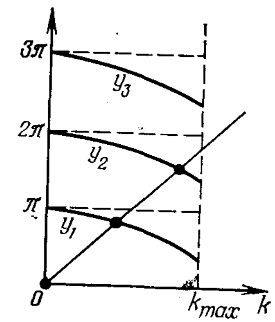


Fig. 64

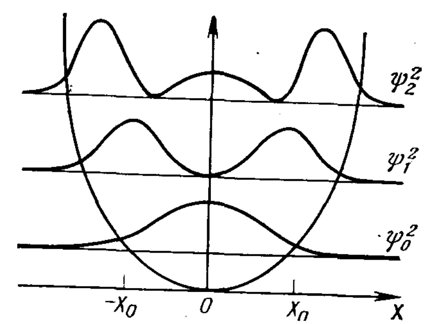


Fig. 65

(b) It can be seen from Fig. 64 that Eq. (1) has at least one root, if at $k = k_{max}$ its left-hand side is not less than the right-hand side: $(l/\hbar)\sqrt{2mU_1} \geqslant (\pi/2) - \arcsin\sqrt{U_1/U_2}$. For $U_2 = 2U_1$ the first level appears at $U_1 = \pi^2\hbar^2/32ml^2$.

3.50. **(a)** $E = \hbar\omega/2$; **(b)** $E = (3/2)\hbar\omega$.

3.51. $E_n = \hbar\omega(n + 1/2)$.

3.52. **(a)** $\psi_0 = (\alpha^2/\pi)^{1/4}e^{-\xi^2/2}$; $\psi_1 = (\alpha^2/4\pi)^{1/4}\,2\xi e^{-\xi^2/2}$; $\psi_2 = (\alpha^2/64\pi)^{1/4}(4\xi^2-2)e^{-\xi^2/2}$.

(b) The values of x_{pr} for the states $n = 0, 1, 2$ are equal to 0, $\pm1/\alpha$, and $\pm2.5/\alpha$. The distribution ψ_n^2 is shown in Fig. 65, $x_0 = 1/\alpha$.

3.53. 0.157.

3.54. The Schrödinger equation for this field in the region $x > 0$ is the same as in the case of a linear oscillator. Consequently, its solutions will be identical with those for the oscillator at odd values of n because $\psi(0) = 0$. The same relates to the energy eigenvalues which can be given in the form $E_{n'} = \hbar\omega(2n' + 3/2)$, $n' = 0, 1, 2, \ldots$. It is obvious that for the same value of ω, the energy of the ground state ($n' = 0$) triples the energy of the oscillator in the ground state.

3.55. **(a)** Let us find the solution in the form $\psi(x, y, z) = X(x) \times Y(y) Z(z)$. After substitution in the Schrödinger equation, we obtain $X''_x + \frac{2m}{\hbar^2}\left(E_x - \frac{k_x x^2}{2}\right)X = 0$ and the similar equations for the functions Y and Z, with $E_x + E_y + E_z = E$. These equations coincide with the equation for a unidimensional oscillator whose eigenfunctions and energy eigenvalues are known. Therefore we can write directly

$$\psi_{n_1 n_2 n_3} = \psi_{n_1}(x)\,\psi_{n_2}(y)\,\psi_{n_3}(z);$$

$$E_n = \hbar\omega(n + 3/2), \quad n = n_1 + n_2 + n_3.$$

(b) The degree of degeneracy of a level with a definite value of n is essentially equal to the number of different combinations of numbers n_1, n_2, and n_3 whose sum is equal to n. To determine the number of combinations, let us first count the number of possible triads n_1, n_2, n_3 for a fixed value of n_1. It is equal to the number of possible values of n_2 (or n_3), i.e. to $n - n_1 + 1$, for n_2 may vary from 0 to $n - n_1$. Then the total number of combinations of n_1, n_2, n_3 (for a given n) is

$$N = \sum_{n_1=0}^{n} (n - n_1 + 1) = \frac{(n+1)(n+2)}{2}.$$

3.56. **(a)** Write the solutions of the Schrödinger equation:

$$x \leqslant 0, \quad \psi_1 = a_1 e^{ik_1 x} + b_1 e^{-ik_1 x}, \quad k_1 = \sqrt{2mE}/\hbar;$$

$$x \geqslant 0, \quad \psi_2 = a_2 e^{ik_2 x} + b_2 e^{-ik_2 x}, \quad k_2 = \sqrt{2m(E - U_0)}/\hbar.$$

Suppose that the incident wave has the real amplitude a_1, and the reflected wave the amplitude b_1. Since in the region $x > 0$ there is only a transmitted wave, $b_2 = 0$. From the condition of continuity of ψ and ψ' at the point $x = 0$, we find b_1/a_1:

$$R = \left(\frac{b_1}{a_1}\right)^2 = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2; \quad D = 1 - R = \frac{4k_1 k_2}{(k_1 + k_2)^2}.$$

(b) In this case the solutions of the Schrödinger equation take the form

$$x < 0, \quad \psi_1 = a_1 e^{ikx} + b_1 e^{-ikx}, \quad k = \sqrt{2mE}/\hbar;$$

$$x > 0, \quad \psi_2 = a_2 e^{\varkappa x} + b_2 e^{-\varkappa x}, \quad \varkappa = \sqrt{2m(U_0 - E)}/\hbar.$$

Suppose the incident wave has the real amplitude a_1. From the finiteness of the wave function, it follows that $a_2 = 0$. From the

condition of continuity of ψ and ψ' at the point $x = 0$, we get

$$R = \left|\frac{b_1}{a_1}\right|^2 = \left|\frac{k - i\varkappa}{k + i\varkappa}\right|^2 = 1.$$

The probability density of finding the particle below the barrier is $w_2(x) \propto e^{-2\varkappa x}$. Hence, $x_{\text{eff}} = 1/2\varkappa$. For an electron $x_{\text{eff}} \approx 1$ Å.

3.57. **(a)** Write the solutions of the Schrödinger equation for three regions:

$$x < 0, \quad \psi_1 = a_1 e^{ikx} + b_1 e^{-ikx}, \quad k = \sqrt{2mE}/\hbar;$$

$$0 < x < l, \quad \psi_2 = a_2 e^{ik_0 x} + b_2 e^{-ik_0 x}, \quad k_0 = \sqrt{2m(E + U_0)}/\hbar;$$

$$x > l, \quad \psi_3 = a_3 e^{ikx}.$$

These expressions are written for the case when the incident wave is characterized by e^{ikx}. Accordingly, the wave function ψ_3 has only one term corresponding to the transmitted wave. From the condition of continuity of ψ and ψ' at the well's boundaries, we find

$$\frac{a_3}{a_1} = \frac{4kk_0 e^{-ikl}}{(k + k_0)^2 e^{ik_0 l} - (k - k_0)^2 e^{ik_0 l}};$$

$$D = \left|\frac{a_3}{a_1}\right|^2 = \left[1 + \frac{(k_0^2 - k^2)^2}{4k^2 k_0^2}\sin^2 k_0 l\right]^{-1} = \left[1 + \frac{U_0^2 \sin^2 k_0 l}{4E(E + U_0)}\right]^{-1};$$

$$R = 1 - D = \left[1 + \frac{4k^2 k_0^2}{(k_0^2 - k^2)^2 \sin^2 k_0 l}\right]^{-1} = \left[1 + \frac{4E(E + U_0)}{U_0^2 \sin^2 k_0 l}\right]^{-1}.$$

(b) From the condition $D = 1$, we obtain $\sin k_0 l = 0$. Hence $k_0 l = n\pi$, or $E = (\pi^2\hbar^2/2ml^2)n^2 - U_0$; n are integers at which $E > 0$.

3.58. **(a)** The solution is similar to that of the foregoing problem (item (a)). Finally, we obtain the same formulas, in which $k_0 = \sqrt{2m(E - U_0)}/\hbar$. When $E \to U_0$, $D \to (1 + ml^2 U_0/2\hbar^2)^{-1}$.

(b) $E_n = (\pi^2\hbar^2/2ml^2)n^2 + U_0 = 11.5$, 16.0, and 23.5 eV. Here $n = 1, 2, 3, \ldots$ ($n \neq 0$, since for $n = 0$ $E = U_0$ and $D < 1$, see item (a) of this problem).

(c) In this case the solution of the Schrödinger equation differs from the case $E > U_0$ only in the region $0 < x < l$:

$$\psi_2 = a_2 e^{\varkappa x} + b_2 e^{-\varkappa x}; \quad \varkappa = \sqrt{2m(U_0 - E)}/\hbar.$$

Therefore, the transparency coefficient can be found by substitution of $i\varkappa$ for k_0 in the expression (see the solution of Problem 3.57, (a)):

$$\frac{a_3}{a_1} = \frac{4i\varkappa k e^{-ikl}}{(k + i\varkappa)^2 e^{\varkappa l} - (k - i\varkappa)^2 e^{-\varkappa l}};$$

$$D = \left|\frac{a_3}{a_1}\right| = \left[1 + \left(\frac{k^2 + \varkappa^2}{2k\varkappa}\right)^2 \sinh^2 \varkappa l\right]^{-1} = \left[1 + \frac{U_0^2 \sinh^2 \varkappa l}{4E(U_0 - E)}\right]^{-1}.$$

$D \ll 1$ when $\varkappa l \gg 1$. In this case $\sinh \varkappa l \approx e^{\varkappa l}/2$ and

$$D \approx \frac{16k^2\varkappa^2}{(k^2 + \varkappa^2)^2} e^{-2\varkappa l} = \frac{16E}{U_0}\left(1 - \frac{E}{U_0}\right) e^{-2l\sqrt{2m(U_0 - E)}/\hbar}.$$

(d) For an electron $D \approx 0.27$, for a proton $D \approx 10^{-17}$.

3.59. (a) From the solution of the Schrödinger equation for the three regions, we find the ratio of the amplitudes of the transmitted and incident waves:

$$\frac{a_3}{a_1} = \frac{4k_1k_2e^{-ik_3l}}{(k_1+k_2)(k_2+k_3)e^{-ik_2l}-(k_1-k_2)(k_3-k_2)e^{ik_2l}},$$

where $k_1 = \sqrt{2mE}/\hbar$; $k_2 = \sqrt{2m(E-U_2)}/\hbar$; $k_3 = \sqrt{2m(E-U_3)}/\hbar$. The transparency coefficient $D = |a_3/a_1|^2 \cdot v_3/v_1$, where v_1 and v_3 are the velocities of the particle before and after tunnelling through the barrier; $v_3/v_1 = k_3/k_1$. Hence,

$$D = \frac{4k_1k_2^2k_3}{k_2^2(k_1+k_3)^2+(k_1^2-k_2^2)(k_3^2-k_2^2)\sin^2 k_2l}.$$

(b) The same expressions as in the foregoing item in which the following substitutions are made: $k_2 \to i\varkappa$ and $\sin k_2l \to \sinh \varkappa l$, where $\varkappa = \sqrt{2m(U_2-E)}/\hbar$:

$$D = \frac{4k_1\varkappa^2k_3}{\varkappa^2(k_1+k_3)^2+(k_1^2-\varkappa^2)(k_3^2-\varkappa^2)\sinh^2 \varkappa l}.$$

3.60. (a) $D = \exp\left[-\frac{8l\sqrt{2m}}{3\hbar U_0}(U_0-E)^{3/2}\right]$;

(b) $D = \exp\left[-\frac{\pi l}{\hbar}\sqrt{\frac{2m}{U_0}}(U_0-E)\right]$.

4.1. *Instruction.* Take into account that $\hat{A}^2\psi = \hat{A}(\hat{A}\psi)$.

4.2. (a) $(2-x^2)\cos x - 4x\sin x$; $(1-x^2)\cos x - 3x\sin x$;
(b) $(2+4x+x^2)\,e^x$, $(1+3x+x^2)\,e^x$.

4.3. (a) $A = 4$; (b) $A = 1$; (c) $A = -\alpha^2$.

4.4. (a) $\psi(x) = Ce^{i\lambda x}$, $\lambda = 2\pi n/a$, $n = 0, \pm1, \pm2, \ldots$;
(b) $\psi(x) = C\sin(\sqrt{\lambda}x)$, $\lambda = (n\pi/l)^2$, $n = \pm1, \pm2, \ldots$.

4.6. (a) $\hat{A}(\Sigma\hat{B}_i)-(\Sigma\hat{B}_i)\hat{A} = \Sigma(\hat{A}\hat{B}_i-\hat{B}_i\hat{A}) = \Sigma[\hat{A}, \hat{B}_i]$;
(b) $\hat{A}(\hat{B}\hat{C})-(\hat{B}\hat{C})\hat{A} = \hat{A}\hat{B}\hat{C}-\hat{B}\hat{C}\hat{A}+\hat{B}\hat{A}\hat{C}-\hat{B}\hat{A}\hat{C} = [\hat{A}, \hat{B}]\hat{C}+\hat{B}[\hat{A}, \hat{C}]$.

4.8. $[\hat{B}, \hat{A}^2] = \Sigma[\hat{B}, \hat{A}_i^2] = 0$, for $[\hat{B}, \hat{A}_i^2] = [\hat{B}, \hat{A}_i]\hat{A}_i+\hat{A}_i[\hat{B}, \hat{A}_i] = 0$.

4.9. (a) Multiplying the equality $\hat{A}\hat{B}-\hat{B}\hat{A} = 1$ by the operator $\hat{B}$ first from the left-hand side and then from the right-hand side, we obtain correspondingly $\hat{B}\hat{A}\hat{B}-\hat{B}^2\hat{A} = \hat{B}$ and $\hat{A}\hat{B}^2-\hat{B}\hat{A}\hat{B} = \hat{B}$. Summing up these equalities, we get: $\hat{A}\hat{B}^2-\hat{B}^2\hat{A} = 2\hat{B}$.

4.12. The operators $\hat{B}$ and $\hat{C}$ are not commutative in the general case. For example, the operator $\hat{p}_y$ commutes with the operators $\hat{x}$ and $\hat{p}_x$ which do not commute with each other.

4.13. (a) If ψ is the common eigenfunction of the operators $\hat{A}$ and $\hat{B}$, then $\hat{A}\hat{B}\psi = \hat{A}B\psi = B\hat{A}\psi = BA\psi$; $\hat{B}\hat{A}\psi = \hat{B}A\psi = A\hat{B}\psi = AB\psi$, i.e. $\hat{A}\hat{B}\psi = \hat{B}\hat{A}\psi$ and $[\hat{A}, \hat{B}] = 0$.

(b) Let ψ be the eigenfunction of the operator $\hat{A}$ and belong to the eigenvalue A. Since $\hat{A}$ and $\hat{B}$ are commutative, $\hat{A}\hat{B}\psi = \hat{B}\hat{A}\psi = \hat{B}A\psi = A\hat{B}\psi$, i.e. $\hat{A}\psi' = A\psi'$, where $\psi' = \hat{B}\psi$. Thus, the eigenvalue A belongs both to the function ψ and to the function ψ' that describe therefore the same state. This can happen only when these functions differ by a constant factor, for example, B: $\psi' = B\psi$. But $\psi' = \hat{B}\psi$, and therefore $\hat{B}\psi = B\psi$, i.e. the function ψ is the common eigenfunction of the operators $\hat{A}$ and $\hat{B}$.

4.14. (a) $f(x, z)\,e^{ik_yy}$; (b) $Ae^{i(k_xx+k_yy+k_zz)}$; (c) $f(y, z)\,e^{\pm ik_xx}$. Here $k_i = p_i/\hbar$ $(i = x, y, z)$; f is an arbitrary function.

4.15. It does only when the function ψ_A is at the same time the eigenfunction of the operator $\hat{B}$. It does not in the general case. For example, in the case of degeneracy (in a unidimensional rectangular potential well two values of the momentum's projection, $+p_x$ and $-p_x$, correspond to each energy level despite the fact that the operators $\hat{H}$ and $\hat{p}_x$ commute).

4.16. Suppose that ψ is an arbitrary eigenfunction of the operator $\hat{A}$, corresponding to the eigenvalue A. Then from hermicity of the operator $\hat{A}$ it follows that $\int \psi^*\hat{A}\psi\,dx = \int \psi\hat{A}^*\psi^*\,dx$ and $A\int \psi^*\psi\,dx = A^*\int \psi\psi^*\,dx$, whence $A = A^*$. The latter is possible only if A is real.

4.17. (a) $\int \psi_1\hat{p}_x\psi_2\,dx = -i\hbar\int \psi_1^*\frac{\partial\psi_2}{\partial x}\,dx = -i\hbar\left[(\psi_1^*\psi_2)_{-\infty}^{+\infty} - \int \psi_2\frac{\partial\psi_1^*}{\partial x}\,dx\right] = \int \psi_2\left(i\hbar\frac{\partial}{\partial x}\right)\psi_1^*\,dx = \int \psi_2\hat{p}_x^*\psi_1^*\,dx$.

4.18. The operator $\hat{A}^+$ conjugated with the operator $\hat{A}$ is defined as follows: $\int \psi_1^*\hat{A}\psi_2\,dx = \int \psi_2(\hat{A}^+\psi_1)^*\,dx$. (a) $\hat{p}_xx$; (b) $-i\hat{p}_x$.

4.19. From the hermicity of the operators $\hat{A}$ and $\hat{B}$ it follows that

$$\int \psi_1^*\hat{A}(\hat{B}\psi_2)\,d\tau = \int \hat{B}\psi_2(\hat{A}^*\psi_1^*)\,d\tau = \int \hat{A}^*\psi_1^*(\hat{B}\psi_2)\,d\tau = \int \psi_2\hat{B}^*(\hat{A}^*\psi^*)\,d\tau.$$

Since $\hat{A}$ and $\hat{B}$ commute, $\hat{B}^*\hat{A}^* = \hat{A}^*\hat{B}^*$ and

$$\int \psi_1^*\hat{A}\hat{B}\psi_2\,d\tau = \int \psi_2\hat{A}^*\hat{B}^*\psi_1^*\,d\tau.$$

4.20. Every operator commutes with itself. Consequently, if the operator $\hat{A}$ is hermitian, the operators $\hat{A}^2 = \hat{A}\hat{A}$ and $\hat{A}^n$ are also hermitian.

4.23. (a) The solution of the equation $\hat{L}_z\psi = L_z\psi$ is $\psi(\varphi) = Ae^{iL_z\varphi/\hbar}$. From the condition of single-valuedness $\psi(\varphi) = \psi(\varphi + 2\pi)$ it follows that $L_z = m\hbar$, where $m = 0, \pm 1, \pm 2, \ldots$. From normalization, we get $A = (2\pi)^{-1/2}$. Finally, $\psi_m(\varphi) = (2\pi)^{-1/2}e^{im\varphi}$.

(b) The eigenvalues are $L_z^2 = m^2\hbar^2$, where $m = 0, \pm 1, \pm 2, \ldots$. The eigenfunctions have the same form as for the operator $\hat{L}_z$, i.e. the function $\psi_m(\varphi) = (2\pi)^{-1/2}e^{im\varphi}$ is the common eigenfunction of the operators $\hat{L}_z$ and $\hat{L}_z^2$. All states with eigenvalues L_z^2 except for $m = 0$ are doubly degenerate (in terms of direction of the rotational moment, $L_z = \pm|m|\hbar$).

4.24. $2\hbar^2$.

4.25. (a) $\int \psi_1^*\hat{L}_z\psi_2\,d\varphi = -i\hbar(\psi_1^*\psi_2)_0^{2\pi} + \int \psi_2\left(i\hbar\frac{\partial\psi_1^*}{\partial\varphi}\right)d\varphi = \int \psi_2\hat{L}_z^*\psi_1\,d\varphi$.

Here $(\psi_1^*\psi_2)_0^{2\pi} = 0$, because the functions ψ_1^* and ψ_2 satisfy the condition of single-valuedness.

(b) $\int \psi_1^*\hat{L}_z\psi_2\,d\tau = \int(\psi_1^*x\hat{p}_y\psi_2 - \psi_1^*y\hat{p}_x\psi_2)\,d\tau$. Since the operators $\hat{p}_x$ and $\hat{p}_y$ are hermitian, the integrand can be transformed as $x\psi_2\hat{p}_y^*\psi_1^* - y\psi_2\hat{p}_x^*\psi_1^* = \psi_2(x\hat{p}_y^* - y\hat{p}_x^*)\psi_1^* = \psi_2\hat{L}_z^*\psi_1^*$.

4.26. $\int \psi_1^*\hat{L}^2\psi_2\,d\tau = \int(\psi_1^*\hat{L}_x^2\psi_2 + \psi_1^*\hat{L}_y^2\psi_2 + \psi_1^*\hat{L}_z^2\psi_2)\,d\tau$. Since the operators $\hat{L}_x$, $\hat{L}_y$, and $\hat{L}_z$ are hermitian, the squares of these operators and consequently the operator $\hat{L}^2$ are also hermitian.

4.29. (a) $[\hat{L}_x, \hat{p}_x^2] = [\hat{L}_x, \hat{p}_x]\hat{p}_x + \hat{p}_x[\hat{L}_x, \hat{p}_x] = 0$, for $[\hat{L}_x, \hat{p}_x] = 0$.

4.30. The operator $\hat{T}$ can be represented in the spherical coordinates as the sum $\hat{T} = \hat{T}_r + \hat{L}^2/2mr^2$, where $\hat{T}_r$ performs the operation only on the variable r. Since the operator $\hat{L}^2 = -\hbar^2\nabla^2_{\vartheta,\varphi}$ operates only on the variables ϑ and φ, $[\hat{L}^2, \hat{T}] = [\hat{L}^2, \hat{T}_r] + [\hat{L}^2, \hat{L}^2/2mr^2] = 0$.

4.31. (a) $[\hat{L}_x, \hat{L}_y] = \hat{L}_x\hat{L}_y - \hat{L}_y\hat{L}_x = (y\hat{p}_z - z\hat{p}_y)(z\hat{p}_x - x\hat{p}_z) - (z\hat{p}_x - x\hat{p}_z)(y\hat{p}_z - z\hat{p}_y) = [z, \hat{p}_z](x\hat{p}_y - y\hat{p}_x) = i\hbar(x\hat{p}_y - y\hat{p}_x) = i\hbar\hat{L}_z$.

4.32. (a) $[\hat{L}^2, \hat{L}_x] = [\hat{L}_x^2, \hat{L}_x] + [\hat{L}_y^2, \hat{L}_x] + [\hat{L}_z^2, \hat{L}_x]$, where $[\hat{L}_x^2, \hat{L}_x] = 0$; $[\hat{L}_y^2, \hat{L}_x] = \hat{L}_y[\hat{L}_y, \hat{L}_x] + [\hat{L}_y, \hat{L}_x]\hat{L}_y = -i\hbar(\hat{L}_y\hat{L}_z + \hat{L}_z\hat{L}_y)$; $[\hat{L}_z^2, \hat{L}_x] = \hat{L}_z[\hat{L}_z, \hat{L}_x] + [\hat{L}_z, \hat{L}_x]\hat{L}_z = i\hbar(\hat{L}_z\hat{L}_y + \hat{L}_y\hat{L}_z)$. Whence it follows that $[\hat{L}^2, \hat{L}_x] = 0$. Similarly for $\hat{L}_y$ and $\hat{L}_z$.

4.33. In the case of $r = r_0 = \text{const}$, the operator

$$\hat{H} = -\frac{\hbar^2}{2\mu r_0^2}\nabla^2_{\vartheta,\varphi} = \frac{1}{2\mu r_0^2}\hat{L}^2.$$

Therefore,

$$\hat{H}\psi = \frac{1}{2\mu r_0^2}\hat{L}^2\psi = E\psi.$$

Since the eigenvalues of the operator $\hat{L}^2$ are equal to $\hbar^2 l(l+1)$, then $E = \hbar^2 l(l+1)/2\mu r_0^2$.

4.34. (a) Since the operator $\hat{A}$ is hermitian, $\int \psi^*\hat{A}\psi\,d\tau = \int \psi\hat{A}^*\psi^*\,d\tau$. Consequently, $\langle A\rangle = \langle A^*\rangle$, which is possible only for a real $\langle A\rangle$.

4.36. $\hat{H}x - x\hat{H} = -\frac{i\hbar}{m}\hat{p}_x$, therefore $\langle p_x\rangle = \int \psi^*\hat{p}_x\psi\,dx = -\frac{m}{i\hbar}\int(\psi^*\hat{H}x\psi - \psi^*x\hat{H}\psi)\,dx$. Due to hermicity of the Hamiltonian the integrand can be written as $x\psi\hat{H}\psi^* - x\psi^*\hat{H}\psi = 0$, for $\hat{H}\psi^* = E\psi^*$ and $\hat{H}\psi = E\psi$.

4.37. (a) From the normalizing conditions $A^2 = 8/3l$.

$$\langle T\rangle = \int \psi\hat{T}\psi\,dx = -\frac{\hbar^2}{2m}\int \psi\frac{d^2\psi}{dx^2}\,dx = \frac{2}{3}\cdot\frac{\pi^2\hbar^2}{ml^2}.$$

(b) $A^2 = 30/l^5$; $\langle T\rangle = 5\hbar^2/ml^2$.

4.38. From the normalizing conditions $A^2 = \sqrt{2/\pi}\,\alpha$; $\langle T\rangle = \langle U\rangle = \hbar\omega/4$.

4.39. (a) Here $\psi_n(x) = \sqrt{2/l}\sin(n\pi x/l)$, $\langle(\Delta x)^2\rangle = \langle x^2\rangle - \langle x\rangle^2 = \frac{l^2}{12}\left(1 - \frac{6}{\pi^2 n^2}\right)$; $\langle(\Delta p_x)^2\rangle = \langle p_x^2\rangle = (\pi\hbar/l)^2 n^2$.

(b) From the normalizing conditions $A^2 = \sqrt{2/\pi}\,\alpha$; $\langle(\Delta x)^2\rangle = 1/4\alpha^2$; $\langle(\Delta p_x)^2\rangle = \alpha^2\hbar^2$.

(c) $A^2 = \sqrt{2/\pi}\,\alpha$; $\langle(\Delta x)^2\rangle = \langle x^2\rangle = 1/4\alpha^2$; $\langle(\Delta p_x)^2\rangle = \langle p_x^2\rangle - \langle p_x\rangle^2 = \alpha^2\hbar^2$.

Instruction. While calculating the mean value of the momentum squared, it is helpful to use the following hermitian property of the operator $\hat{p}_x$: $\langle p_x^2\rangle = \int \psi^*\hat{p}_x^2\psi\,dx = \int |\hat{p}_x\psi|^2\,dx$.

4.40. From the normalizing conditions $A^2 = 4/3\pi$; $\langle L_z^2\rangle = 4\hbar^2/3$.

4.41. From the normalizing conditions $A^2 = 1/\pi$; $\langle(\Delta\varphi)^2\rangle = \langle\varphi^2\rangle - \langle\varphi\rangle^2 = \pi^2/3 - 1/2$; $\langle(\Delta L_z)^2\rangle = \langle L_z^2\rangle = \hbar^2$.

4.42. Recalling that $i\hbar\hat{L}_x = [\hat{L}_y, \hat{L}_z]$, we can write $\langle L_x \rangle = \frac{1}{i\hbar}\int (\psi^*\hat{L}_y\hat{L}_z\psi - \psi^*\hat{L}_z\hat{L}_y\psi)\, d\tau$. Since according to the condition $\hat{L}_z\psi = L_z\psi$ and the operator $\hat{L}_z$ is hermitian, the integrand can be transformed as follows: $\psi^*\hat{L}_y\hat{L}_z\psi - \psi^*\hat{L}_z\hat{L}_y\psi = L_z\psi^*\hat{L}_y\psi - (\hat{L}_y\psi)\hat{L}_z^*\psi^* = (\hat{L}_y\psi)(L_z\psi^* - L_z^*\psi^*)$. The latter expression in parentheses is equal to zero because an eigenvalue of a hermitian operator is real ($L_z = L_z^*$). Similarly for the operator $\hat{L}_y$.

4.43. $\langle L^2 \rangle = \int \psi\hat{L}^2\psi\, d\Omega = 2\hbar^2$, where $d\Omega = \sin\vartheta\, d\vartheta\, d\varphi$.

4.44. Since the axes x, y, z are equivalent, $\langle L^2\rangle = \langle L_x^2\rangle + \langle L_y^2\rangle + \langle L_z^2\rangle = 3\langle L_z^2\rangle$. Allowing for equiprobability of various possible values of L_z, we obtain

$$\langle L_z^2\rangle = \hbar\langle m^2\rangle = \frac{\hbar^2}{2l+1}\sum_{m=-l}^{l} m^2 = \frac{2\hbar^2}{2l+1}\,\frac{l(l+1)(2l+1)}{6}$$

and $\langle L^2\rangle = \hbar^2 l(l+1)$.

4.45. We have $\hat{A}\psi_1 = A_1\psi_1$ and $\hat{A}\psi_2 = A_2\psi_2$. Due to hermicity of the operator $\hat{A}$, its eigenvalues are real and

$$\int \psi_1^*\hat{A}\psi_2\, d\tau = \int \psi_2\hat{A}^*\psi_1^*\, d\tau,$$

or

$$A_2\int \psi_1^*\psi_2\, d\tau = A_1\int \psi_2\psi_1^*\, d\tau.$$

Since $A_1 \neq A_2$, the latter equality is possible only under the condition $\int \psi_1^*\psi_2\, d\tau = 0$, i.e. the functions ψ_1 and ψ_2 are orthogonal.

4.47. **(a)** Multiply both sides of the expansion $\psi(x) = \sum c_k\psi_k(x)$ by $\psi_l^*(x)$ and then integrate with respect to x:

$$\int \psi_l^*\psi\, dx = \sum c_k \int \psi_l^*\psi_k\, dx.$$

The eigenfunctions of the operator $\hat{A}$ are orthonormal, and therefore all integrals in the right-hand side of the equation, with the exception of the one for which $k = l$, turn to zero. Thus, $c_l = \int \psi_l^*(x)\psi(x)\, dx$.

(b) $\langle A\rangle = \int \psi^*\hat{A}\psi\, dx = \int \left(\sum c_k^*\psi_k^*\right)\left(\sum c_l A_l\psi_l\right) dx = \sum_{k,l} c_k^* c_l A_l \times \int \psi_k^*\psi_l\, dx = \sum |c_k|^2 A_k$. Note that $\sum |c_k|^2 = 1$ which follows directly from the normalization of the function $\psi(x)$:

$$\int \psi^*\psi\, dx = \sum c_k^* c_l \int \psi_k^*\psi_l\, dx = 1.$$

From here it follows that the coefficients $|c_k|^2$ are the probabilities of observing definite values of the mechanical quantity A_k.

4.48. First, the normalizing coefficient A should be calculated. The probability of the particle being on the nth level is defined by the squared modulus of the coefficient c_n of expansion of the function $\psi(x)$ in terms of eigenfunctions $\psi_n(x)$ of the operator $\hat{B}$: $c_n = \int \psi(x)\psi_n(x)\, dx$, where $\psi_n(x) = \sqrt{2/l}\,\sin(n\pi x/l)$.

(a) $A^2 = 8/3l$. The probability sought is $w_1 = c_1^2 = 256/27\pi^2 = 0.96$.

(b) $A^2 = 30/l^5$; $w_n = c_n^2 = \frac{240}{(\pi n)^6}[1-(-1)^n]^2$, that is, w_n differs from zero only for odd levels ($n = 1, 3, 5, \ldots$); for them $w_n = 960/(\pi n)^6$; $w_1 \approx 0.999$; $w_3 \approx 0.001$.

4.49. **(a)** First calculate the normalizing coefficient $A = 2/\sqrt{3\pi}$. Then expand the function $\psi(\varphi)$ in terms of eigenfunctions of the operator $\hat{L}_z$ (they have the form $\psi_m(\varphi) = (2\pi)^{-1/2}e^{im\varphi}$):

$$\psi(\varphi) = A\sin^2\varphi = \frac{1}{\sqrt{3\pi}}(1-\cos 2\varphi) = \frac{1}{\sqrt{3\pi}}\left(1 - \frac{1}{2}e^{2i\varphi} - \frac{1}{2}e^{-2i\varphi}\right)$$

$$= \sqrt{\frac{2}{3}}\,\psi_0 - \frac{1}{\sqrt{6}}\,\psi_{+2} - \frac{1}{\sqrt{6}}\,\psi_{-2}.$$

From here it can be seen that $L_z = 0$, $+2\hbar$, and $-2\hbar$. Their probabilities: $w_0 = 2/3$; $w_{+2} = w_{-2} = 1/6$. **(b)** $L_z = 0$, $\pm\hbar$, $\pm 2\hbar$ and $w_0 = 36/70$; $w_{+1} = w_{-1} = 16/70$; $w_{+2} = w_{-2} = 1/70$.

4.50. **(a)** Find the coefficients of expansion of the wave function $\psi_n(x) = \sqrt{2/l}\,\sin(n\pi x/l)$ in terms of eigenfunctions of the operator $\hat{k}$:

$$c_k = \int \psi_n(x)\,\psi_k^*(x)\, dx = n\sqrt{\pi l}\,\frac{1-(-1)^n e^{-ikl}}{\pi^2 n^2 - k^2 l^2},$$

whence

$$w_k = |c_k^2| = \frac{4\pi l n^2}{(\pi^2 n^2 - k^2 l^2)^2}\cdot\begin{cases}\cos^2(kl/2), & \text{if } n \text{ is odd,}\\ \sin^2(kl/2), & \text{if } n \text{ is even.}\end{cases}$$

(b) From the normalizing conditions $A^2 = \sqrt{2/\pi}\,\alpha$; $w_k = |c_k|^2 = \frac{1}{\alpha\sqrt{2\pi}}\,e^{-k^2/2\alpha^2}$.

The corresponding integral can be found by the procedure indicated in the solution of Problem 3.37, (b). It can be readily shown here that the total probability $\int w_k\, dk = 1$.

4.51. It can be a solution of the time-dependent Schrödinger equation.

4.52. Expand the function sought in terms of the eigenfunctions of stationary states

$$\Psi(x, t) = \sum c_n\psi_n(x)\,e^{-i\omega_n t},$$

$\psi_n(x) = \sqrt{2/l}\sin(n\pi x/l)$. The coefficients c_n are found from the initial condition

$$c_n = \int \Psi(x,\ 0)\,\psi_n(x)\,dx = A\frac{2^{3/2}l^{5/2}}{(\pi n)^3}[1-(-1)^n].$$

From here it is seen that $c_n \neq 0$ only for odd n. From the normalizing conditions applied to the function $\Psi(x, 0)$, we find $A^2 = 30/l^5$. As a result,

$$\Psi(x,\ t) = \frac{8}{\pi^3}\sqrt{\frac{30}{l}}\sum\frac{1}{n^3}\sin\frac{n\pi x}{l}e^{-i\omega_n t};$$

$\omega_n = E_n/\hbar = (\pi^2\hbar/2ml^2)n^2$, where $n = 1, 3, 5, \ldots$.

4.53. First, separating the variables φ and t, find the stationary solutions of the Schrödinger equation:

$$i\hbar\frac{\partial\Psi}{\partial t} = \hat{H}\Psi;\quad \Psi_m(\varphi,\ t) = \frac{1}{\sqrt{2}}e^{i(m\varphi-\omega_m t)};$$

$\omega_m = E_m/\hbar = (\hbar/2I)m^2$, where $m = 0, \pm1, \pm2, \ldots$. Then expand the function sought $\Psi(\varphi, t)$ in terms of $\Psi_m(\varphi, t)$: $\Psi(\varphi, t) = \Sigma c_m\Psi_m(\varphi, t)$, where the coefficients c_m are found from the initial condition $\Psi(\varphi, 0) = \Sigma c_m e^{im\varphi}$ (see the solution of Problem 4.49). Finally, we get $\Psi(\varphi, t) = (A/2)(1 + \cos 2\varphi\cdot e^{i2\hbar t/I})$. From this expression it is in particular seen that the rotator comes back to the initial state after the time interval $\Delta t = \pi I/\hbar$.

4.54. **(a)** Recalling that $\langle A\rangle = \int\Psi^*\hat{A}\Psi\,d\tau$, we obtain

$$\frac{d}{dt}\langle A\rangle = \int\frac{\partial\Psi^*}{\partial t}\hat{A}\Psi\,d\tau + \int\Psi^*\frac{\partial\hat{A}}{\partial t}\Psi\,d\tau + \int\Psi^*\hat{A}\frac{\partial\Psi}{\partial t}\,d\tau.$$

But since $\frac{\partial\Psi}{\partial t} = -\frac{i}{\hbar}\hat{H}\Psi$ and $\frac{\partial\Psi^*}{\partial t} = \frac{i}{\hbar}\hat{H}\Psi^*$, then

$$\frac{d}{dt}\langle A\rangle = \frac{i}{\hbar}\int(\hat{H}\Psi^*)\hat{A}\Psi\,d\tau + \int\Psi^*\frac{\partial\hat{A}}{\partial t}\Psi\,d\tau - \frac{i}{\hbar}\int\Psi^*\hat{A}\hat{H}\Psi\,d\tau.$$

Due to hermicity of the operator $\hat{H}$ the first integral of this expression can be rewritten in the form and then

$$\frac{d}{dt}\langle A\rangle = \int\Psi^*\left[\frac{\partial\hat{A}}{\partial t} + \frac{i}{\hbar}(\hat{H}\hat{A} - \hat{A}\hat{H})\right]\Psi\,d\tau.$$

Whence it is seen that $\frac{d\hat{A}}{dt} = \frac{\partial\hat{A}}{\partial t} + \frac{i}{\hbar}(\hat{H}\hat{A} - \hat{A}\hat{H})$.

4.56. Take into account that the operators $\hat{x}$ and $\hat{p}_x$ do not depend on time explicitly.

4.59. The operator $\hat{L}_x$ does not depend on time explicitly, and therefore $\frac{d}{dt}\hat{L}_x = \frac{i}{\hbar}[\hat{H},\ \hat{L}_x] = \frac{i}{\hbar}\left[\frac{\hat{p}^2}{2m},\ \hat{L}_x\right] + \frac{i}{\hbar}[U,\ \hat{L}_x]$. Since

$\hat{p}^2$ and $\hat{L}_x$ are commutative (see Problem 4.29), the first term is equal to zero. Only the second term is to be calculated.

4.60. Differentiating the equation $\hat{A}\Psi = A\Psi$ with respect to time and taking into account that $\frac{\partial\hat{A}}{\partial t} = 0$, we get $\hat{A}\frac{\partial\Psi}{\partial t} = \frac{dA}{dt}\Psi + A\frac{\partial\Psi}{\partial t}$. After the substitution $\frac{\partial\Psi}{\partial t} = -\frac{i}{\hbar}\hat{H}\Psi$, we obtain $\frac{dA}{dt}\Psi = \frac{i}{\hbar}(\hat{A}\hat{H} - A\hat{H})\Psi$. If $\hat{A}$ and $\hat{H}$ commute, then $\hat{A}\hat{H}\Psi = \hat{H}\hat{A}\Psi = A\hat{H}\Psi$ and $dA/dt = 0$.

4.61. The solution is reduced to checking whether the operators of indicated mechanical quantities commute with the Hamiltonian $\hat{H} = \hat{p}^2/2m + U = \hat{T} + U$, where $\hat{T}$ is the kinetic energy operator. The operators $\hat{p}_x$, $\hat{p}_y$, $\hat{p}_z$, $\hat{L}_x$, $\hat{L}_y$, $\hat{L}_z$, and $\hat{L}^2$ all commute with the operator $\hat{T}$ (see Problems 4.29 and 4.30), and, therefore, we have to investigate if all these operators commute with the operator U. **(a)** $\partial\hat{H}/\partial t = 0$ and $U \equiv 0$. All quantities retain their values in the course of time. **(b)** $\partial\hat{H}/\partial t = 0$. The values of E, p_x, p_y, and L_z do not vary with time. **(c)** $\partial\hat{H}/\partial t = 0$. The operators $\hat{L}_x$, $\hat{L}_y$, $\hat{L}_z$, and $\hat{L}^2$ commute with the operator $U(r)$. (This fact becomes evident as soon as the operators are written in spherical coordinates: they operate only on ϑ and φ.) The values of E, L_x, L_y, L_z, L^2 do not vary with time. **(d)** $\partial\hat{H}/\partial t \neq 0$. Only p_x, p_y, and L_z do not vary with time.

4.62. **(a)** $\frac{d}{dt}\langle A\rangle = \frac{i}{\hbar}\int\Psi^*[\hat{H},\ \hat{A}]\Psi\,d\tau = 0$, i.e. $\langle A\rangle = \text{const}$.

(b) Since the operators $\hat{A}$ and $\hat{H}$ commute, they have common eigenfunctions $\psi_n(x)$: $\hat{A}\psi_n = A\psi_n$ and $\hat{H}\psi_n = E_n\psi_n$. Expand the function $\Psi(x, t)$ in terms of the eigenfunctions ψ_n which are the functions of the stationary states ($\hat{H}\psi_n = E_n\psi_n$), so that $\Psi(x, t) = \Sigma c_n\psi_n(x)e^{-i\omega_n t} = \Sigma c_n'(t)\psi_n(x)$; $\omega_n = E_n/\hbar$, where $c_n'(t) = c_n(0)e^{-i\omega_n t}$. The latter summation is an expansion in terms of eigenfunctions of the operator $\hat{A}$, and therefore the squares of moduli of the expansion coefficients define the probabilities of various values of the mechanical quantity A_n at the moment t, i.e. $w(A_n, t)$. Thus $w(A_n,\ t) = |c_n'(t)|^2 = |c_n'(0)|^2 = \text{const}$.

4.63. $\hat{T}_r = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right)$ is the kinetic energy operator in the case of a radial motion.

4.64. **(a)** Write the Hamiltonian in the Schrödinger equation $\hat{H}\psi = E\psi$ in the form $\hat{H} = \hat{T}_r + \hat{L}^2/2\mu r^2 + \hat{U}$, where $\hat{T}_r$ is the kinetic energy operator in the case of a radial motion (see the answer to the foregoing problem). Substituting the function $\psi = RY$ into the Schrödinger equation, we obtain the following expression

$$Y\hat{T}_r R + \frac{R}{2\mu r^2}\hat{L}^2 Y + YUR = YER.$$

Taking into account that $\hat{L}^2 Y = \hbar^2 l(l+1)Y$, we get

$$\left(\hat{T}_r + \frac{\hbar^2 (l+1)}{2\mu r^2} + U\right)R = ER.$$

The latter equation defines the energy eigenvalues E. It can be easily reduced to the sought form.

(b) Substitute the function Y in the form $Y = \Theta(\vartheta)\Phi(\varphi)$ into the equation $\hat{L}^2 Y = \lambda Y$, where $\hat{L}^2 = -\hbar^2\nabla^2_{\vartheta,\varphi}$, and separate the variables ϑ and φ. Denoting the separation constant by m^2, we get the equation for the function $\Phi(\varphi)$:

$$\partial^2\Phi/\partial\varphi^2 = -m^2\Phi; \quad \Phi(\varphi) = Ae^{im\varphi}.$$

From the condition of single-valuedness, it follows that $m = 0, \pm 1, \pm 2, \ldots$. Thus, $\psi = R(r)\,\Theta(\vartheta)\,e^{im\varphi}$.

4.65. The function $|Y_{l,m}|^2$ specifies the probability density, related to a unit of solid angle of the particle with the quantum numbers l and m being in the vicinity of ϑ: $|Y|^2 = d\omega/d\Omega$. (a) $\sqrt{3/4\pi}$; (b) $\sqrt{15/8\pi}$.

4.66. (a) After the substitution into the Schrödinger equation, we obtain $\chi'' + \varkappa^2\chi = 0$; $\varkappa = \sqrt{2mE}/\hbar$. The solution of this equation is to be sought in the form $\chi(r) = A\sin(\varkappa r + \alpha)$. From the finiteness of the function $\psi(r)$ at the point $r = 0$, it follows that $\alpha = 0$. Thus, $\psi(r) = \frac{A\sin\varkappa r}{r}$. From the boundary condition $\psi(r_0) = 0$, we have $\varkappa r_0 = n\pi$, $n = 1, 2, \ldots$, whence

$$E_{ns} = \frac{\pi^2\hbar^2}{2mr_0^2}n^2; \quad \psi_s(r) = \frac{1}{\sqrt{2\pi r_0}}\cdot\frac{\sin\varkappa r}{r}.$$

The coefficient A is found from the normalizing condition $\int_0^{r_0}\psi^2 4\pi r^2\,dr = 1$.

(b) $r_0/2$; 50%. (c) Transform the equation for the function $R_1(r)$ to the form $R_1'' + \frac{2}{r}R_1' + (\varkappa^2 r^2 - 2)R_1 = 0$; $\varkappa = \sqrt{2mE}/\hbar$. Having written the similar equation for $R_0(r)$, differentiate it with respect to r:

$$R_0''' + \frac{2}{r}R_0'' + (\varkappa^2 r^2 - 2)R_0' = 0.$$

From the comparison of these two equations, it can be seen that

$$R_1(r) = R_0'(r) = \frac{A}{r^2}(\varkappa r\cos\varkappa r - \sin\varkappa r),$$

where A is the normalizing coefficient. (d) From the boundary condition, we get $\tan\varkappa r_0 = \varkappa r_0$. The roots of this equation are found either by inspection or by graphical means. The least value is $\varkappa r_0 = 4.5$. Consequently, $E_{1p} \approx 10\hbar^2/mr_0^2 \approx 2E_{1s}$.

4.67. (a) $\langle r\rangle = r_0/2$; $\langle r^2\rangle = \frac{r_0^2}{3}\left(1 - \frac{3}{2\pi^2 n^2}\right)$; $\langle(\Delta r)^2\rangle = \langle r^2\rangle - \langle r\rangle^2 = \frac{r_0^2}{12}\left(1 - \frac{6}{\pi^2 n^2}\right)$;

(b) $\langle T\rangle = \frac{21}{4}\hbar^2/mr_0^2$; (c) expand the function $\psi_{1s}(r)$ in terms of eigenfunctions of the operator $\hat{k}$:

$$\psi_{1s}(r) = (2\pi)^{-3/2}\int c_{\mathbf{k}}e^{i\mathbf{kr}}\,d\mathbf{k},$$

where $c_{\mathbf{k}} = (2\pi)^{-3/2}\int\psi_{1s}(r)\,e^{-i\mathbf{kr}}\,dV$. The latter integral is to be computed in spherical coordinates with the polar axis being directed along the vector $\mathbf{k}$:

$$c_{\mathbf{k}} = \frac{1}{(2\pi)^2\sqrt{r_0}}\int\frac{\sin(\pi r/r_0)}{r}e^{-ikr\cos\vartheta}r^2\,dr\sin\vartheta\,d\vartheta\,d\varphi = \frac{\sqrt{r_0}\sin kr_0}{k(\pi^2 - k^2r_0^2)}.$$

Therefore the probability density of the given wave vector is $|c_{\mathbf{k}}|^2 = \frac{r_0\sin^2 kr_0}{k^2(\pi^2 - k^2r_0^2)^2}$. To find the probability of the modulus of the wave vector being between the values k and $k + dk$, integrate the latter expression with respect to all possible directions of the vector $\mathbf{k}$, i.e. miltiply it by $4\pi k^2\,dk$. Finally we get

$$w(k)\,dk = \frac{4\pi r_0\sin^2 kr_0}{(\pi^2 - k^2r_0^2)^2}\,dk.$$

4.68. (a) The solutions of the Schrödinger equation for the function $\chi(r)$ are

$$r < r_0, \quad \chi_1 = A\sin(kr + \alpha), \quad k = \sqrt{2mE}/\hbar;$$

$$r > r_0, \quad \chi_2 = Be^{\varkappa r} + Ce^{-\varkappa r}, \quad \varkappa = \sqrt{2m(U_0 - E)}/\hbar.$$

From the finiteness of the function $\psi(r)$ throughout the space, it follows that $\alpha = 0$ and $B = 0$. Thus,

$$\psi_1 = A\frac{\sin kr}{r}; \quad \psi_2 = C\frac{e^{-\varkappa r}}{r}.$$

From the condition of continuity of ψ and ψ' at the point $r = r_0$, we obtain $\tan kr_0 = -k/\varkappa$, or $\sin kr_0 = \pm\sqrt{\hbar^2/2mr_0^2U_0}\,kr_0$. This equation, as it is shown in the solution of Problem 3.47, defines the discrete spectrum of energy eigenvalues.

(b) $\pi^2\hbar^2/8m < r_0^2 U_0 < 9\pi^2\hbar^2/8m$. **(c)** In this case there is a single level: $\sin kr_0 = \frac{3\sqrt{3}}{4\pi} kr_0$; $kr_0 = \frac{2}{3}\pi$; $E = \frac{2\pi^2}{9}\cdot\frac{\hbar^2}{mr_0^2}$. From the condition $\partial(r^2\psi^2)/\partial r = 0$, we find $r_{pr} = 3r_0/4$; 34%.

4.69. $\frac{\partial^2 R}{\partial\rho^2} + \frac{2}{\rho}\frac{\partial R}{\partial\rho} + \left(\varepsilon + \frac{2Z}{\rho} - \frac{l(l+1)}{\rho^2}\right) R = 0$, $\rho = r/r_1$, $\varepsilon = E/E_1$.

4.70. **(a)** Neglecting the small values, reduce the Schrödinger equation to the form $\chi'' - \varkappa^2\chi = 0$, $\varkappa = \sqrt{2m\,|E|}/\hbar$. Its solution is $\chi(r) = Ae^{\varkappa r} + Be^{-\varkappa r}$. From the finiteness of $R(r)$, it follows that $A = 0$ and $R(r) \propto e^{-\varkappa r}/r$. **(b)** Transform the Schrödinger equation to the form $\chi'' - \frac{l(l+1)}{r^2}\chi = 0$. Its solution is to be found in the form $\chi = Ar^\alpha$. After the substitution into the equation, we find two values of α: $1 + l$ and $-l$. The function $R(r)$ is finite only if $\alpha = 1 + l$. Hence, $R(r) \propto r^l$.

4.71. **(a)** Substituting this function into the Schrödinger equation, we obtain $B(a, \alpha, E) + rC(a, \alpha, E) + \frac{1}{r}D(a, \alpha) = 0$, where B, C, and D are certain polynomials. This equality holds for any values of r only when $B = C = D = 0$, whence

$$a = \alpha = -1/2r_1 = -me^2/2\hbar^2; \quad E = -me^4/8\hbar^2.$$

(b) $A = \frac{1}{2}(2\pi r_1^3)^{-1/2}$; r_1 is the first Bohr radius.

4.72. **(a)** $r_{pr} = r_1$, where r_1 is the first Bohr radius; 32.3%; **(b)** 23.8%.

4.73. **(a)** $\langle r\rangle = 3r_1/2$; $\langle r^2\rangle = 3r_1^2$; $\langle(\Delta r)^2\rangle = \langle r^2\rangle - \langle r\rangle^2 = 3r_1^2/4$; r_1 is the first Bohr radius; **(b)** $\langle F\rangle = 2e^2/r_1^2$; $\langle U\rangle = -e^2/r_1$; **(c)** $\langle T\rangle = \int \psi\hat{T}\psi\,d\tau = me^4/2\hbar^2$; $\sqrt{\langle v^2\rangle} = e^2/\hbar = 2.2\cdot10^6$ m/s.

4.74. **(a)** $4r_1$ and $9r_1$; **(b)** $5r_1^2$ and $15.75r_1^2$; r_1 is the first Bohr radius.

4.75. $\varphi_0 = \int \frac{\rho(r)}{r} 4\pi r^2\,dr = \frac{e}{r_1}$, where $\rho(r) = e\psi_{1s}^2(r)$ is the volume density of charge; r_1 is the first Bohr radius.

4.76. Write Poisson's equation in spherical coordinates:

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\varphi_e) = 4\pi e\varphi_{1s}^2(r), \quad e > 0.$$

Integrating this equation twice, we get

$$\varphi_e(r) = \left(\frac{e}{r_1} + \frac{e}{r}\right)e^{-2r/r_1} + A + \frac{B}{r},$$

where r_1 is the first Bohr radius, A and B are the integration constants. Choose these constants so that $\varphi_e(\infty) = 0$ and $\varphi_e(0)$ be finite. Hence, $A = 0$, $B = -e$. Adding the potential induced by the nucleus to the expression obtained, we get

$$\varphi(r) = \left(\frac{e}{r_1} + \frac{e}{r}\right)e^{-2r/r_1}.$$

4.77. See the solution of Problem 4.67, (c), $w(k)\,dk = \frac{32r_1^3k^2\,dk}{\pi(1+k^2r_1^2)^4}$, where r_1 is the first Bohr radius.

5.1. 5.14 and 2.1 V.

5.2. 0.41, 0.04, and 0.00.

5.3. Having calculated the quantum defect of S terms, we find $E_b = 5.4$ eV.

5.4. **(a)** 6; **(b)** 12.

5.5. 0.27 and 0.05; 0.178 μm.

5.6. $a = 1.74$; $n = 2$.

5.7. $7.2\cdot10^{-3}$ eV; 1.62 eV.

5.8. 555 cm^{-1}.

5.10. **(a)** $\Delta T = \alpha^2 RZ^4(n-1)/n^4 = 5.85$. 2.31 and 1.10 cm^{-1}; **(b)** 1.73 and 0.58 cm^{-1} (three sublevels).

5.11. $\Delta\lambda = \alpha^2/9R = 5.4\cdot10^{-3}$ Å (equal for H and He^+).

5.12. $Z = 3$, i.e. Li^{++}.

5.13. **(a)** See Fig. 66; $\Delta\tilde{\nu}_{51} = \tilde{\nu}_5 - \tilde{\nu}_1 = 7.58\ \text{cm}^{-1}$, $\Delta\lambda_{51} = 0.204$ Å; **(b)** $\Delta\tilde{\nu} = 2.46\ \text{cm}^{-1}$, $\Delta\lambda = 0.54$ Å.

5.14. $\lambda/\delta\lambda \geqslant \tilde{\nu}/(\tilde{\nu}_3 - \tilde{\nu}_2) = 4.2\cdot10^5$ (see Fig. 66).

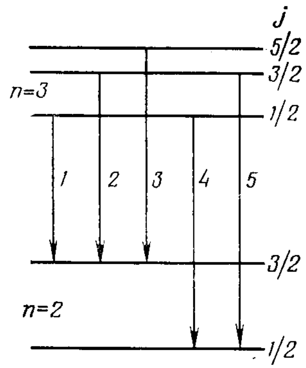


Fig. 66

5.15. In units of $\hbar$: $\sqrt{35}/2$, $\sqrt{15}/2$, and $\sqrt{3}/2$ (^4P); $2\sqrt{5}$, $2\sqrt{3}$, $\sqrt{6}$, $\sqrt{2}$, and 0 (^5D).

5.16. **(a)** $^1P_1$ and $^3P_{0,1,2}$; **(b)** $^1P_1$, $^1D_2$, $^1F_3$, $^3P_{0,1,2}$, $^3D_{1,2,3}$, $^3F_{2,3,4}$; **(c)** $^2P_{1/2,3/2}$, $^2D_{3/2,5/2}$, $^2F_{5/2,7/2}$, $^4P_{1/2,3/2,5/2}$, $^4D_{1/2,3/2,5/2,7/2}$, $^4F_{3/2,5/2,7/2,9/2}$.

5.17. 20 (5 singlet and 15 triplet types).

5.18. $^1S_0$, $^1P_1$, $^1D_2$, $^3S_1$, $^3P_{0,1,2}$, $^3D_{1,2,3}$.

5.19. **(a)** 2, 4, 6, 8; **(b)** respectively, 2; 1, 3; 2, 4; 1, 3, 5.

5.20. $\sqrt{30}\,\hbar$.

5.21. Respectively, $p_S \geqslant \sqrt{2}\,\hbar$ and $p_S = \sqrt{2}\,\hbar$.

5.22. **(a)** 35.2°; **(b)** 34.4°.

5.23. 10 (the number of states with different values of m_J).

5.24. $\sqrt{30}\,\hbar$; $^5H_3$.

5.25. 125°15′.

5.26. **(a)** $\sum_J (2J+1) = (2S+1)\cdot(2L+1)$; **(b)** $2(2l_1+1)\times 2(2l_2+1) = 60$; **(c)** the number of states with identical quantum numbers n and l is $N = 2(2l + 1)$. While distributing k electrons over these states, the Pauli exclusion principle should be taken into account. Consequently, the problem reduces to finding the number of combinations of N elements taken k at a time:

$$C_N^k = \frac{N(N-1)(N-2)\dots(N-k+1)}{k!} = 120.$$

5.27. **(a)** 15; **(b)** 46.

5.28. **(a)** $2(2l+1)$; **(b)** $2n^2$.

5.29. **(a)** C: $1s^2 2s^2 2p^2 (^3P_0)$; N: $1s^2 2s^2 2p^3 (^4S_{3/2})$;
(b) S: $1s^2 2s^2 2p^6 3s^2 3p^4 (^3P_2)$; Cl: $1s^2 2s^2 2p^6 3s^2 3p^5 (^2P_{3/2})$.

5.30. **(a)** $^3F_2$; **(b)** $^4F_{3/2}$.

5.31. $^6S_{5/2}$.

5.32. The basic term $^5D_4$. The degeneracy $2J + 1 = 9$.

5.33. Let us compile the table of possible distributions of electrons over quantum states (numbers) with the Pauli exclusion principle taken into account (Tables 1 and 2). While doing this, we can leave out those distributions that provide the negative values of the sum of projections M_L and M_S; such distributions do not submit anything new, which can be proved directly.

To illustrate, let us denote the spin projection m_s of each electron by the arrow pointing either up (if $m_s = +1/2$) or down (if $m_s = -1/2$).

Table 1

m_l	m_s					
+1	↑	↑	↑	↑	↑↓	—
0	↑	—	↓	—	—	↑↓
−1	—	↑	—	↑	—	—
M_S	1	1	0	0	0	0
M_L	1	0	1	0	2	0

Table 2

m_l	m_s						
+1	↑	↑	↑	↓	↑↓	↑↓	↑
0	↑	↑	↓	↑	↑	—	↑↓
−1	↑	↓	↑	↑	—	↑	—
M_S	3/2	1/2	1/2	1/2	1/2	1/2	1/2
M_L	0	0	0	0	2	1	1

(a) See Table 1. The presence of the state with $M_L = 2$ and $M_S = 0$ indicates that there is a term ^1D; consequently, there must also be two other states: $M_L = 1$ and $M_L = 0$ (for both $M_S = 0$). From other distributions the state with $M_L = 1$ and $M_S = 1$ points to the existence of the ^3P term; therefore, there must be still another state with $M_L = 0$ and $M_S = 1$. The last remaining state with $M_L = 0$ and $M_S = 0$ belongs to the ^1S term. Consequently, three types of terms correspond to the given configuration: ^1S, ^1D, and ^3P.

(b) See Table 2. Via similar reasoning, we get ^2D, ^2P, and ^4S.
(c) ^1S, ^1D, ^1G, ^3P, and ^3F.

5.34. Both configurations have the following identical types of terms: **(a)** ^2P; **(b)** ^1S, ^1D, and ^3P; **(c)** ^2D. This follows from the fact that the absence of an electron in a subshell can be treated as "a hole" whose state is determined by the same quantum numbers as those of the absent electron.

5.35. Let us compile the table of possible distributions of electrons over quantum states, taking into account that the Pauli exclusion principle imposes limitations only on equivalent electrons.

(a) See Table 3 in which the thin arrows indicate spin projections of a p-electron and heavy arrows those of an s-electron.

Table 3

m_l	m_s							
+1	↑	↑	↑	↑	↑	↑	↑↓	—
0	↑↑	↑↓	↑	↓	↓↑	↑	↑	↑↓↑
−1	—	—	↑	↑	—	↓	—	—
M_S	3/2	1/2	3/2	1/2	1/2	1/2	1/2	1/2
M_L	1	1	0	0	1	0	2	0

The possible types of terms: ^2D, ^2P, ^2S, and ^4P.

(b) ^2S, ^2P (three terms), ^2D, ^2F, ^4S, ^4P, and ^4D.

5.36. $N_2/N_1 = (g_2/g_1)\,e^{-\hbar\omega/kT} = 2.4\cdot10^{-3}$, where $g_1 = 2$, $g_2 = 4 + 2$.

5.37. $3\cdot10^{-17}$.

5.38. From the condition $-dN = AN\,dt$, where A is a constant, we find $N = N_0 e^{-At}$. On the other hand, $\tau = \int t\,dN = 1/A$, where the integration is performed with respect to t going from 0 to ∞. The rest of the proof is obvious.

5.39. $\tau = l/v \ln\eta = 1.2\cdot10^{-6}$ s; $\Gamma \approx 5.5\cdot10^{-10}$ eV.

5.40. $N = \tau\lambda I/2\pi\hbar c = 7\cdot10^9$.

5.41. $\tau = \frac{N\hbar\omega}{I}\cdot\frac{g'}{g}\,e^{-\hbar\omega/kT} = 7\cdot10^{-8}$ s, where $g' = 4 + 2$, $g = 2$. It is taken into account here that the concentration of atoms on the ground level practically coincides with the total concentration since $\hbar\omega \gg kT$.

5.42. **(a)** The number of direct and reverse transitions per unit time $Z_{21} = (A_{21} + B_{21}u_\omega)N_2$; $Z_{12} = B_{12}u_\omega N_1$. Taking into account the Boltzmann distribution and the fact that $Z_{21} = Z_{12}$, we obtain

$$u_\omega = \frac{A_{21}}{(g_1/g_2)\,B_{12}e^{\hbar\omega/kT} - B_{21}}.$$

When $T \to \infty$, $u_\omega \to \infty$, and therefore $g_1 B_{12} = g_2 B_{21}$; besides, from the comparison with Planck's formula, it follows that

$$B_{21} = (\pi^2 c^2/\hbar\omega^3) A_{21}.$$

(b) $u_\omega = \frac{A_{21}}{B_{12}} \cdot \frac{g_2}{g_1} e^{-\hbar\omega/kT} = \frac{\hbar\omega^3}{\pi^2 c^3} e^{-\hbar\omega/kT}$ (Wien's formula).

5.43. (a) w_{ind}/w_{sp} is of the order of 10^{-34}; (b) $T = 3\pi\hbar c R/2k \ln 2 = 1.7 \cdot 10^5$ K.

5.44. Let I_ω be the intensity of the transmitted light. On passing through the layer of gas of thickness dx this quantity diminishes as

$$-dI_\omega = \varkappa_\omega I_\omega \, dx = (N_1 B_{12} - N_2 B_{21}) \frac{I_\omega}{c} \hbar\omega \, dx,$$

where N_1 and N_2 are the concentrations of atoms on the lower and upper levels, B_{12} and B_{21} are the Einstein coefficients. Hence,

$$\varkappa_\omega = \frac{\hbar\omega}{c} N_1 B_{12} \left(1 - \frac{g_1 N_2}{g_2 N_1}\right).$$

Then take into account the Boltzmann distribution and the fact that $\hbar\omega \gg kT$ (in this case $N_1 \approx N_0$, the total concentration of atoms).

5.45. It follows from the solution of the foregoing problem that light is amplified if $\varkappa_\omega < 0$, i.e. $g_1 N_2 > g_2 N_1$. This is feasible provided the thermodynamically nonequilibium state is realized. $N_D : N_P = g_D : g_P = 5 : 3$.

5.46. In the stationary case the concentrations of atoms on the upper and lower levels are equal to $N_2 = q/A_{21}$ and $N_1 = q/A_{10}$ respectively. As it follows from the solution of Problem 5.44, the light amplification requires that $g_1 N_2 > g_2 N_1$. The rest of the proof is obvious.

5.47. Solving the system of equations $\dot N_2 = q - A_2 N_2$; $\dot N_1 = A_{21} N_2 - A_{20} N_1$, where $A_2 = A_{20} + A_{21}$, we obtain

$$N_1(t) = \frac{q A_{21}}{A_{10} A_2} \left(1 - \frac{A_2 e^{-A_{10} t} - A_{10} e^{-A_2 t}}{A_2 - A_{10}}\right).$$

5.48. $2 \cdot 10^{-4}$ Å.

5.49. (a) $\delta\omega = \gamma$; (b) $\tau = \lambda^2/2\pi c\delta\lambda = 1.2 \cdot 10^{-9}$ s.

5.50. (b) $I = 2 \int_{\omega_0}^{\infty} I_\omega \, d\omega = \frac{\pi\delta\omega}{2} J_0$.

5.51. (a) Suppose v_x is the projection of the velocity vector of a radiating atom on the direction of observation line. The number of atoms whose velocity projections fall into the interval v_x, $v_x + dv_x$ is

$$n_{v_x} dv_x \propto e^{-mv_x^2/2kT} dv_x.$$

The frequency of a photon emitted by the atom moving with the velocity v_x is $\omega = \omega_0 (1 + v_x/c)$. Using this expression, find the frequency distribution of radiating atoms: $n_\omega \, d\omega = n(v_x) \, dv_x$. Finally, it remains to take into account that the spectral radiation intensity $I_\omega \propto n_\omega$.

5.52. $\delta\lambda_{Dop}/\delta\lambda_{nat} \approx 10^3$.

5.53. $T \approx 1.25 \cdot 10^{-3} \alpha^4 mc^2/k = 39$ K, where α is the fine structure constant, m is the atomic mass.

5.54. About 2′.

5.55. 8.45 and 1.80 Å; 1.47 and 6.9 keV.

5.56. 12.2 Å (Na).

5.57. (a) Fe, Co, Ni, Zn; Cu is omitted (1.54 Å); (b) three elements.

5.58. 0.25, 0.0, and −2.0.

5.59. 15 kV.

5.60. Cu.

5.61. 5.5 and 70 kV.

5.62. In molybdenum, all series; in silver, all series with the exception of K series.

5.63. (b) Ti; 29 Å.

5.64. (a) 5.47 and 0.52 keV; (b) 2.5 Å.

5.65. $E_L = \frac{\hbar\omega}{2\pi c/\omega \, \Delta\lambda - 1} = 0.5$ keV, where $\omega = \frac{3}{4} R^* (Z-1)^2$.

5.66. $a_K = 2.84$; $a_L = 10$.

5.67. 1.54 keV.

5.68. 0.26 keV.

5.69. (a) $T_{photo} = \hbar\omega - E_K = 4.7$ keV; $T_{Auger} = (E_K - E_L) - E_L = 10.4$ keV, where E_K and E_{L_0} are the binding energies of K and L electrons; (b) 0.5 Å.

5.71. (a) ${}^2P_{3/2}$; (b) ${}^2S_{1/2}$ and ${}^2P_{3/2,\,1/2}$; ${}^2S_{1/2}$; ${}^2P_{3/2,\,1/2}$ and ${}^2D_{5/2,\,3/2}$.

5.72. $K \to L, M$, two lines each; $L \to M$, seven lines.

5.73. (a) 0.215 Å (K_{α_1}) and 0.209 Å (K_{α_2}); (b) $4.9 \cdot 10^{-3}$ Å.

5.74. 115.5, 21.9, 21.0, and 17.2 keV.

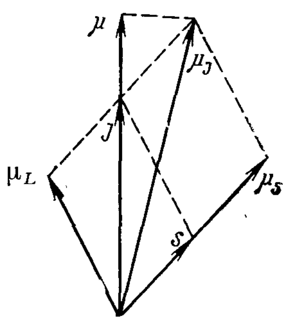


Fig. 67

6.1. From the vector model (Fig. 67 in which μ_S and μ_L are drawn, for the sake of simplicity, coinciding in direction with S and L), it follows that

$$\mu = \mu_L \cos(\mathbf{L}, \mathbf{J}) + \mu_S \cos(\mathbf{S}, \mathbf{J}), \quad (1)$$

where $\mu_L = L^* \mu_B$; $\mu_S = 2S^* \mu_B$; $L = \sqrt{L(L+1)}$; $S^* = \sqrt{S(S+1)}$; $J^* = \sqrt{J(J+1)}$. According to the cosine law

$$\left.\begin{aligned} L^{*2} &= J^{*2} + S^{*2} - 2J^* S^* \cos(\mathbf{S}, \mathbf{J}); \\ S^{*2} &= J^{*2} + L^{*2} - 2J^* L^* \cos(\mathbf{L}, \mathbf{J}). \end{aligned}\right\} \quad (2)$$

Eliminating the cosines from Eqs. (1) and (2), we obtain the sought expression.

6.2. **(a)** $2(S)$, $2/3$ and $4/3(P)$, $4/5$ and $6/5(D)$; **(b)** $0/0(^3P_0)$, $3/2(^3P_1$ and $^3P_2)$; **(c)** $g = 2$ with the exception of the singlet state for which $g = 0/0$; **(d)** $g = 1$.

6.3. **(a)** $^2F_{5/2}$; **(b)** $^3D_3$.

6.4. **(a)** $2\sqrt{3}\mu_B$; **(b)** $2\sqrt{3/5}\mu_B$.

6.5. $S = 3$; the multiplicity $2S + 1 = 7$.

6.6. $4/\sqrt{3}$; $13/\sqrt{15}$, and $4\sqrt{7/5}\,\mu_B$.

6.7. $\sqrt{3}\,\mu_B$.

6.8. For both terms $g = 0$; $\boldsymbol{\mu}_J \perp \mathbf{J}$.

6.9. $\sqrt{2}\,\hbar$ and $\sqrt{6}\,\hbar$.

6.10. **(a)** The ground state $^2P_{3/2}$, $g = 4/3$, $\mu = 2\sqrt{5/3}\,\mu_B$; **(b)** the groud state $^4F_{3/2}$, $g = 2/5$, $\mu = \sqrt{3/5}\,\mu_B$.

6.11. On the one hand, $d\mathbf{J} = [\boldsymbol{\mu}, \mathbf{H}]\,dt$, where $\boldsymbol{\mu}$ is the magnetic moment of the atom. On the other hand (Fig. 68), $|\,d\mathbf{J}\,| =$

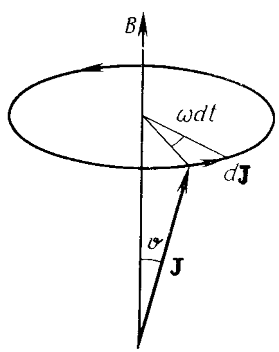


Fig. 68

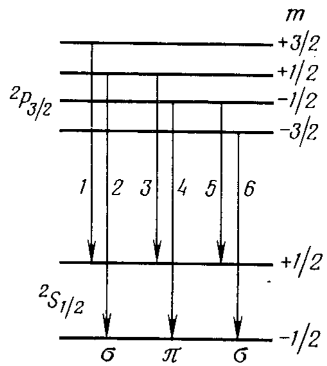


Fig. 69

$= J^* \sin\vartheta \cdot \omega\, dt$, where $J^* = \hbar\sqrt{J(J+1)}$. Comparing the two expressions, we obtain the sought formula.

6.12. **(a)** $0.88\cdot10^{10}$, $1.17\cdot10^{10}$, and $0\ \text{s}^{-1}$; **(b)** $1.32\cdot10^{10}\ \text{s}^{-1}$ $(^3P_2)$.

6.13. $2\sqrt{5}\hbar$ and $5\sqrt{5/4}\,\mu_B$. Here $g = 1.25$; $J = 4$.

6.14. Here $g = -2/3$, that is why $\boldsymbol{\mu} \uparrow\uparrow \mathbf{J}$ (not $\uparrow\downarrow$ as usual).

6.15. $f = \mu_B \dfrac{\partial B}{\partial z} = \dfrac{2\pi i R^2 z \mu_B}{c\,(R^2+z^2)^{5/2}} = 4.1\cdot10^{-27}$ N.

6.16. $\dfrac{\partial B}{\partial z} = \dfrac{mv^2\delta}{a\,(a+2b)\,\mu_B} = 7$ kG/cm.

6.17. **(a)** 0.6, 5, and 6 μ_B; **(b)** five components; no splitting for $\mu = 0$ $(g = 0)$.

6.18. $\delta = \dfrac{a\,(a+2b)\,\mu_B}{2T}\,\dfrac{\partial B}{\partial z} = 5\text{mm}$, $\mu_B = gJ\mu_B$.

6.19. **(a)** $\Delta\tilde{\nu} = L\mu_B B/\pi\hbar c = 0.56\ \text{cm}^{-1}$; **(b)** $^1F_3$.

6.20. Three components in both cases.

6.21. $\Delta\lambda = \lambda^2 eB/2\pi mc^2 = 0.35$ Å.

6.22. $\Delta E = \pi c\hbar\,\Delta\lambda/\lambda^2 = 5\cdot10^{-5}$ eV.

6.23. **(a)** 2 kG; **(b)** 4 kG.

6.24. **(a)** $B = 0.1\,\dfrac{2\pi\hbar c}{g\mu_B}\cdot\dfrac{\Delta\lambda}{\lambda^2}$ $\begin{cases} 28 \text{ kG for } P_{3/2} \text{ term} \\ 55 \text{ kG for } P_{1/2} \text{ term;} \end{cases}$

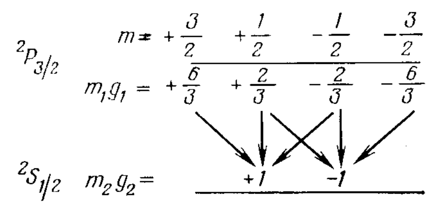
(b) $B = 0.1\,\dfrac{\pi\hbar c\alpha^2 R Z^4}{g\mu_B n^3} = \begin{cases} 0.59 \text{ kG for } P_{3/2} \text{ term} \\ 1.18 \text{ kG for } P_{1/2} \text{ term;} \end{cases}$

(c) 9.4 kG (for $P_{3/2}$) and 19 kG (for $P_{1/2}$).

6.26. **(a)** Normal; anomalous; normal; normal (in the latter case the Landé splitting factor is the same for both terms).

(b) In atoms with an odd number of electrons, the anomalous Zeeman effect; in the remaining atoms, both normal (for singlet lines) and anomalous (for lines of other multiplicity) Zeeman effects.

6.27. See Fig. 69. **(a)** To find the possible shifts, i.e. the values of $m_1g_1 - m_2g_2$, let us draw the following diagram:



The shifts:

$$\Delta\omega = +\frac{5}{3},\ +\frac{3}{3},\ +\frac{1}{3},\ -\frac{1}{3},\ -\frac{3}{3},\ -\frac{5}{3}\left(\text{in } \frac{eB}{2mc} \text{ units}\right).$$

In the diagram the arrows connect only those values of mg whose difference (i.e. the corresponding transition) satisfies the selection rule $\Delta m = 0, \pm1$. The vertical arrows denote π components, the oblique ones, σ components.

(b) $0.78\ \text{cm}^{-1}$.

6.28. $2.7\cdot10^5$.

6.29. **(a)** $\Delta\omega = \dfrac{\pm4,\ \pm8,\ \pm12,\ \pm16,\ \pm24}{15}$;

(b) $\Delta\omega = \dfrac{\pm1,\ \pm3,\ \pm15,\ \pm17,\ \pm19,\ \pm21,}{15}$,

6.30. **(a)** $\Delta\omega = \dfrac{0,\ \pm1,\ \pm2}{6}$; **(b)** $\Delta\omega = \dfrac{\pm1,\ \pm2}{3}$, the central π component is absent, for the transition $\Delta J = 0$, $\Delta m = 0$ is forbidden.

6.31. In the strong magnetic field both vectors, **L** and **S**, behave independently of each other in the first approximation, and the energy of interaction of the atom with the field is

$$\Delta E = -(\mu_L)_B B - (\mu_S)_B B = (m_L + 2m_S)\, \mu_B B.$$

When a transition takes place between two levels, the Zeeman component shift is $\Delta\omega = (\Delta m_L + 2\Delta m_S)\, \mu_B B/\hbar$. The selection rules $\Delta m_L = 0, \pm 1$ and $\Delta m_S = 0$ result in the normal Zeeman effect.

6.32. $B = \frac{\pi\hbar c}{\mu_B} \frac{\Delta\lambda}{\lambda^2} n = 36$ kG.

6.34. In the constant field B the magnetic moments of atoms are oriented in a certain way relative to the vector **B** (spatial quantization). The magnetic moment can change its orientation only due to absorption of a quantum of energy from an ac field. This happens when this quantum of energy is equal to the difference in energies of both states (orientations). Thus, $\hbar\omega = (\mu'_B - \mu''_B)\, B$, where $\mu_B = g m \mu_B$, m is the magnetic quantum number. Taking into account the selection rule $\Delta m = \pm 1$, we obtain $\hbar\omega = g\mu_B B$.

6.35. $B = 2\pi\hbar\nu/g\mu_B = 2.5$ kG.

6.36. $5.6\mu_B$.

6.37. $3.4 \cdot 10^{-6}$ and $7.7 \cdot 10^{-5}\mu_B$.

6.39. $f = \mu \frac{\partial B}{\partial z} = \frac{12\pi^2 I^2 R^4 z}{c^2 (R^2+z^2)^4} \frac{\chi}{N_A} \approx 7 \cdot 10^{-36}$ N.

6.40. The angular frequency of Larmor precession of an electronic shell of atoms is equivalent to the diamagnetic current $I = Ze\omega_L/2\pi$. The magnetic moment of circular current is $\mu = \pi\langle\rho^2\rangle I/c$, where $\langle\rho^2\rangle = \langle x^2\rangle + \langle y^2\rangle$ is the mean squared separation of electrons from the z axis taken in the direction of the field **B**. For the spherically symmetric distribution of the charge in an atom $\langle x^2\rangle = \langle y^2\rangle = \langle z^2\rangle$ and $\langle r^2\rangle = \langle x^2\rangle + \langle y^2\rangle + \langle z^2\rangle = (3/2)\langle\rho^2\rangle$. Whence, $\chi = \mu N/B = -(Ze^2N/6mc^2)\,\langle r^2\rangle$.

6.41. $\langle r^2\rangle = \int_0^\infty r^2\psi^2 4\pi r^2\, dr = 3r_1^2$; $\chi = -2.37 \cdot 10^{-6}$ cm³/mol.

6.42. 0.58, 0.52, and 1.04 Å.

6.43. $B_0 = -\frac{eB}{3mc^2} V_0$, where $V_0 = 4\pi \int_0^\infty \rho(r)\, r\, dr$, $\rho(r)$ is the volume density of the electric charge in the atom at the distance r from the nucleus.

6.44. **(a)** The number of molecules, whose vectors $\boldsymbol{\mu}$ are confined in the elementary solid angle $d\Omega = 2\pi \sin\vartheta\, d\vartheta$, is equal to

$$dN = Ce^{a\cos\vartheta} \sin\vartheta\, d\vartheta, \quad a = \mu B/kT,$$

where C is the constant. This is the number of those molecules whose magnetic moment projections are equal to $\mu_B = \mu\cos\vartheta$. Hence,

$$\langle\mu_B\rangle = \frac{\int \mu_B\, dN}{\int dN} = \mu\left(\coth a - \frac{1}{a}\right).$$

The integration with respect to ϑ is performed between the limits from 0 to π.

(b) $\mu^2 B/3kT$ and μ respectively.

6.45. 0.45 cm³·K/mol, $1.9\mu_B$.

6.46. $1.6 \cdot 10^{-7}$.

6.47. **(a)** $\eta = \tanh a$; $a = mg\mu_B B/kT$; m is the magnetic quantum number; g is the Landé splitting factor. In this case $\eta \approx a = 0.0056$; **(b)** $\eta = \coth b - \operatorname{cosech} b$; $b = g\sqrt{J(J+1)}\,\mu_B B/kT$. In this case $\eta \approx b/2 = 0.0049$.

6.48. $I = N\mu_B \tanh a$; $a = \mu_B B/kT$; $I = N\mu_B^2 B/kT$ at $a \ll 1$.

6.49. **(a)** $\eta = \frac{2\sinh a}{1 + 2\cosh a} \approx \frac{2}{3} a = 0.0037$; $a = Jg\mu_B B/kT$;

(b) $\eta = \frac{\sinh b + \sinh 3b}{\cosh b + \cosh 3b} \approx 2b \approx 0.0060$; $b = g\mu_B B/kT$.

6.50. $\langle\mu_B\rangle = \frac{\sum \mu_B e^{\mu_B B/kT}}{\sum e^{\mu_B B/kT}} = \frac{g\mu_B \sum m e^{\alpha m}}{\sum e^{\alpha m}}$, $\alpha = g\mu_B B/kT$. Here the summation is carried out with respect to m (magnetic quantum number) from $-J$ to $+J$. For a weak magnetic field $\alpha \ll 1$ and therefore $e^{\alpha m} = 1 + \alpha m$. Then $\Sigma\, m e^{\alpha m} = \alpha \Sigma m^2 = \alpha J \times (J+1)(2J+1)/3$; $\Sigma e^{\alpha m} = 2J + 1$. The rest of the proof is obvious.

6.51. **(a)** 0.375 cm³·K/mol; **(b)** 0.18 erg/G.

6.52. $6.6 \cdot 10^{-5}$ cm³/g.

7.1. **(a)** $1.5 \cdot 10^{-2}$ and $4.2 \cdot 10^{-4}$ eV; **(b)** $3.3 \cdot 10^{13}$ and $6.4 \cdot 10^{11}$ s⁻¹.

7.2. 2 and 3.

7.3. $3.46\hbar$.

7.4. 117 and 3.8 K.

7.5. $N_1/N_2 = (g_1/g_2)\, e^{4\hbar B/kT} = 1.9$.

7.6. $5.7 \cdot 10^5$ and $1.9 \cdot 10^6$ dyne/cm.

7.7. $U_0 = D + \hbar\omega/2 = 4.75$ eV; $a = \omega r_0 \sqrt{\mu/2U_0} = 1.43$.

7.8. **(a)** $x_0 = \sqrt{\hbar/\omega\mu} = 0.124$ Å; **(b)** $\sqrt{\langle x^2\rangle} = \sqrt{\hbar/2\omega\mu} = 0.088$ Å.

7.9. $\hbar\omega(1 - 2x) = 0.514$ eV; 33.7 times.

7.10. 534 K.

7.11. $\Delta E = \hbar\omega(1 - 2x) - \hbar BJ(J+1) = 0.37$ eV.

7.12. 13 levels.

7.13. $v_{max} \approx 1/2x$; $E_{max} \approx \hbar\omega/4x$ and $D = \hbar\omega(1 - 2x)/4x$. For a hydrogen molecule $v_{max} = 17$. $E_{max} = 4.8$ eV, $D = 4.5$ eV.

7.14. $x \approx 0.007$.

7.15. $D_D - D_H = (\hbar\omega_H/2)\left(1 - \sqrt{\mu_H/\mu_D}\right) = 0.080$ eV.

7.16. $N_2/N_1 = e^{-\hbar\omega(1-4x)/kT} = 0.02$. At 1545 K.

7.17. $\frac{N_2}{N_1}=\frac{1}{2J+1}e^{-[\hbar\omega(1-2x)-\hbar BJ(J+1)]/kT}=0.01.$

7.18. $\langle E\rangle=\frac{\sum E_v e^{-E_v/kT}}{\sum e^{-E_v/kT}}=\frac{\sum E_v e^{-\alpha E_v}}{\sum e^{-\alpha E_v}},$

where $E_v = \hbar\omega\,(v + 1/2)$, $\alpha = 1/kT$, with the summation being carried out with respect to v in the interval from 0 to ∞. The calculation is performed as follows

$$\langle E\rangle=-\frac{d}{d\alpha}\ln\left(\Sigma e^{-\alpha E_v}\right)=-\frac{d}{d\alpha}\ln\frac{e^{-\alpha\hbar\omega/2}}{1-e^{-\alpha\hbar\omega}}=\frac{\hbar\omega}{2}+\frac{\hbar\omega}{e^{\hbar\omega/kT}-1}.$$

7.19. (a) $T\approx\frac{\hbar\omega}{k\ln 3}=740$ K;

(b) $T=\frac{\hbar\omega}{k\ln[1+\omega/BJ(J+1)]}=630$ K.

7.20. $C_{\text{vibr}}=\frac{R(\hbar\omega/kT)^2e^{\hbar\omega/kT}}{(e^{\hbar\omega/kT}-1)^2}\approx\begin{cases}R(\hbar\omega/kT)^2e^{-\hbar\omega/kT}\\ R.\end{cases}$

Here R is the universal gas constant.

7.21. 0.134, 0.56, and $0.77R$; R is the universal gas constant.

7.22. $1.93\cdot10^{-40}$ g·cm²; 1.12 Å.

7.23. (a) $B' = (\lambda_2 - \lambda_1)/2\lambda_1\lambda_2 = 11$ cm^{-1}; $2.6\cdot10^{-40}$ g·cm²; (b) $4\to3$ and $3\to2$ respectively.

7.24. Decreases by $1.0\hbar$ ($J = 2 \to J = 1$).

7.25. 13 lines.

7.26. $\omega = 2\pi c\,(3\tilde{\nu}_{01}-\tilde{\nu}_{02})=5.0\cdot10^{14}$ s^{-1}; $x=\frac{1}{2}\frac{2\tilde{\nu}_{01}-\tilde{\nu}_{02}}{3\tilde{\nu}_{01}-\tilde{\nu}_{02}}=0.017.$

7.27. From the condition $\hbar\omega = \hbar\omega_0 + \Delta E_{J'J}$, we get

$$\omega = \omega_0 + B\,[J'\,(J'+1) - J\,(J+1)].$$

Taking into account the selection rules $\Delta J = \pm1$, we find

$$J' = J+1,\quad \omega=\omega_0+2B\,(J+1),\quad J=0,1,2,\ldots,$$
$$J' = J-1,\quad \omega=\omega_0-2BJ,\quad J=1,2,3,\ldots.$$

It can be readily noticed that both formulas can be combined into the one given in the problem.

7.28. $B' = 21$ cm^{-1}, $I = \hbar/4\pi cB' = 1.33\cdot10^{-40}$ g·cm². The wave number of the "zero" line which is absent due to exclusion $\Delta J \neq 0$ is $\tilde{\nu}_{10} = 3958$ cm^{-1}. From the ratio $\tilde{\nu}_{10}=\tilde{\nu}\,(1-2x)$, we obtain $x = 0.022$.

7.29. $\Delta\lambda/\lambda = \Delta\mu/\mu = 1.5\cdot10^{-3}$; μ is the reduced mass of the molecule.

7.30. $|\Delta\tilde{\nu}_{\text{vib}}|=\frac{\Delta\mu}{2\mu}\tilde{\nu}_{\text{vib}}=28$ cm^{-1}; $|\Delta\tilde{\nu}_{\text{rot}}|=\frac{\Delta\mu}{\mu}\tilde{\nu}_{\text{rot}}=0.10$ cm^{-1}; $\Delta\tilde{\nu}_{\text{vib}}/\Delta\tilde{\nu}_{\text{rot}}=280$. Here μ is the reduced mass of the molecule.

7.31. $\omega=\pi c\left(\frac{1}{\lambda_v}-\frac{1}{\lambda_r}\right)=1.37\cdot10^{14}$ s^{-1}; $5.0\cdot10^5$ dyne/cm.

7.32. $\omega=2\pi c\frac{\sqrt{1+(\Delta\lambda/\lambda)^2}-1}{(1-2x)\,\Delta\lambda}=7.8\cdot10^{14}$ s^{-1}.

7.33. $I_v/I_r\approx e^{-\hbar\omega(1-2x)/kT}\approx0.07$. Will increase 3.8 times.

7.34. In the transition $E_0\to E_x$ (the first stage of the process) $J_x = J_0 \pm 1$. In the transition to the final state $E_x\to E$ (the second stage) $J = J_x \pm 1 = (J_0\pm1)\pm1$, i.e. $\Delta J = 0, \pm2$.

7.35. (a) From the condition $\hbar\omega=\hbar\omega_0-\Delta E_{J'J}$, we get

$$\omega=\omega_0-B\,[J'\,(J'+1)-J\,(J+1)].$$

Thus, taking into account the selection rule $\Delta J = \pm2$ (for shifted components), we obtain

$$J'=J+2,\quad \omega=\omega_0-2B\,(2J+3),\quad J=0,1,2,\ldots,$$
$$J'=J-2,\quad \omega=\omega_0+2B\,(2J-1),\quad J=2,3,4,\ldots.$$

Both formulas, as one can easily see, can be combined into one given in the text of the Problem; (b) $1.9\cdot10^{-39}$ g·cm², 1.2 Å.

7.36. $B'=\Delta\lambda/12\lambda_0^2=2.0$ cm^{-1}, $1.4\cdot10^{-39}$ g·cm².

8.1. 4.29 and 3.62 Å.

8.2. 2.17 and 1.65 g/cm³.

8.3. The plane (hkl) lying closest to the origin placed at one of the sites of the lattice cuts off the sections a/h, a/k, and a/l on the coordinate axes. The distance between that plane and the origin is equal to the interplanar distance d. Denoting the angles between the plane's normal and the coordinate axes x, y, z by α, β, γ respectively, we obtain: $\cos\alpha = hd/a$; $\cos\beta = kd/a$; $\cos\gamma = ld/a$. Now take into account that the sum of the squares of these cosines is equal to unity.

8.4. (a) a, $\frac{a}{\sqrt2}$, $\frac{a}{\sqrt3}$; (b) $\frac{a}{2}$, $\frac{a}{\sqrt3}$, $\frac{a}{2\sqrt3}$; (c) $\frac{a}{2}$, $\frac{a}{2\sqrt2}$, $\frac{a}{\sqrt3}$.

8.5. 10 and 8 Å.

8.6. $I_{100}:I_{110}:I_{111}=\begin{cases}1:\sqrt2:\sqrt3 \text{ (simple)},\\ 1:\sqrt2:\sqrt3/2 \text{ (space-centered)},\\ 1:\sqrt2/2:\sqrt3 \text{ (face-centered)}.\end{cases}$

8.7. Suppose the edge of the lattice cell is $a = nd_1$, where n is an integer. It can be easily found that when $n = 1$, the cell contains 1/4 of an atom which is impossible. When $n = 2$, the cell has two atoms. In our case the crystal belongs to the cubic system with 4-fold symmetry axes, and therefore the second atom should be located at the cell's centre. If it is the case, then d_2 must be equal to $d_1\sqrt2$, which is indeed so according to the condition of the problem. Consequently, the lattice is space-centered cubic.

8.8. The diffraction maxima are located at the intersection points of two sets of hyperbolas: $a(\cos\alpha - \cos\alpha_0) = k_1\lambda$, $b(\cos\beta - \cos\beta_0) = k_2\lambda$, where α_0, β_0 are the angles between the direction of the incident and the lattice directions along the periods a and b respectively; α, β are the angles between the diffracted beam and the same lattice directions.

8.9. $a(\cos\alpha - 1) = k_1\lambda$; $b\cos\beta = k_2\lambda$; $c\cos\gamma = k_3\lambda$. Taking into account that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, we obtain:

$$\lambda = -\frac{2(k_1/a)}{(k_1/a)^2+(k_2/b)^2+(k_3/c)^2}.$$

8.10. Taking into account Laue's equations $a(\cos\alpha - \cos\alpha_0) = k_1\lambda$; $a(\cos\beta - \cos\beta_0) = k_2\lambda$; $a(\cos\gamma - \cos\gamma_0) = k_3\lambda$ and the relations $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, $\cos^2\alpha_0 + \cos^2\beta_0 + \cos^2\gamma_0 = 1$, we get

$$\lambda = -2a\frac{k_1\cos\alpha_0 + k_2\cos\beta_0 + k_3\cos\gamma_0}{k_1^2+k_2^2+k_3^2}.$$

8.11. Find the sum of squares of left-hand and right-hand sides of the Laue equations:

$$2a^2[1 - (\cos\alpha_0\cos\alpha + \cos\beta_0\cos\beta + \cos\gamma_0\cos\gamma)] = (k_1^2 + k_2^2 + k_3^2)\lambda^2.$$

It can be easily seen that the sum of cosine products equals $\mathbf{n}_0\mathbf{n} = \cos 2\vartheta$, where $\mathbf{n}_0$ and $\mathbf{n}$ are the unit vectors oriented along the directions of the incident and diffracted beams forming the angle 2ϑ equal to the doubled Bragg's angle. Then the former expression takes the following form:

$$2a\sin\vartheta/\sqrt{k_1^2+k_2^2+k_3^2} = \lambda.$$

Since $a/\sqrt{k_1^2 + k_2^2 + k_3^2} = d/n$, where n is the greatest common divisor of the numbers k_1, k_2, k_3 ($k_1 = nh$, $k_2 = nk$, $k_3 = nl$; n, k, l are the Miller indices), we obtain $2d\sin\vartheta = n\lambda$.

8.12. 5.8 Å.

8.13. 1.19 Å; 58°.

8.14. **(a)** 37 and 40 mm respectively.

(b) $$\lambda = \frac{a\sin\vartheta}{n\sqrt{h^2+k^2+l^2}} = \begin{cases} 0.563/n \text{ Å for } (031); \\ 0.626/n \text{ Å for } (221),\ n = 1, 2, \ldots \end{cases}$$

8.15. $$\lambda = \frac{a\sin(\alpha/2)}{\sqrt{k_1^2+k_2^2-2k_1k_2\cos(\alpha/2)}} = 1.7 \text{ Å},$$

k_1 and k_2 are the reflection orders.

8.16. First find the periods of identity I along the [110] and [111] directions. According to Laue $I\cos\vartheta_n = n\lambda$, where ϑ_n is the angle between the rotation axis and the direction to nth layer line; $I_{110} = 2.9$ Å, $I_{111} = 7.1$ Å. Their ratio corresponds to a face-centered lattice (see the solution of Problem 8.6); $a = \sqrt{2}/I_{110} = 4.1$ Å.

8.17.

Lattice type	(100)	(110)	(111)
Space-centered	odd	—	odd
Face-centered	odd	odd	—

8.18. Face-centered: (111), (100), (110), (311), (111).
Space-centered: (110), (100), (211), (110), (310).

8.19. **(a)** 38; 45; 63; 78, and 82°; **(b)** 42; 61; 77; 92, and 107°.

8.20. From the formula $\sin\vartheta = \frac{\lambda}{2a}\sqrt{h^{*2} + k^{*2} + l^{*2}}$, determine the values of the sum of the squares of indices h^*, k^*, l^* and then find (by inspection) the indices themselves: (111), (311), (511), (333). Respectively, 2.33, 1.22, 0.78, and 2.33 Å.

8.21. The first diffraction ring corresponds to the reflection of the first order from the set of planes (111): $a = \frac{2\lambda L}{D}\sqrt{h^2+k^2+l^2} = 4.1$ Å.

8.22. Space-centered.

8.23. The energy of interaction of an ion with all other ions of the chain is

$$U = 2e^2\left(\frac{1}{a} - \frac{1}{2a} + \frac{1}{3a} - \frac{1}{4a}\ldots\right) = \frac{\alpha e^2}{a},$$

where $\alpha = 2\ln 2 = 1.385$, a is the ionic separation.

8.24. **(a)** $|U| = N\alpha\frac{q^2}{r_0}\left(1 - \frac{1}{n}\right)$, where N is the number of ionic pairs in the crystal, r_0 is the equilibrium distance between neighbouring ions of opposite sign; **(b)** 8.85 and 11.4.

8.25. **(a)** $p = (\Delta V/V)/K = 0.3$ GPa; **(b)** expand the function $U(V)$, the binding energy of the crystal, into series in the vicinity of the equilibrium value U_0:

$$U = U_0 + (\partial U/\partial V)_0\,\Delta V + (\partial^2 U/\partial V^2)_0^2\,(\Delta V)^2/2 + \ldots.$$

Taking into account that at equilibrium $(\partial U/\partial V)_0 = 0$ and $1/K = V_0(\partial^2 U/\partial V^2)_0$, we get the expression for the energy increment $U - U_0$, whence for the volume density of energy we have

$$u - u_0 = (\Delta V/V)^2/2K = 1.4 \text{ J/cm}^3.$$

8.26. **(a)** Taking into account that $p = -\partial U/\partial V$, we obtain

$$\frac{1}{K} = V\frac{\partial^2 U}{\partial V^2} = \frac{\alpha e^2(n-1)}{18r_0^4}, \quad n = 1 + \frac{9a^4}{8\alpha e^2 K} = 9.1,$$

where r_0 is the equilibrium separation of neighbouring ions, a is the lattice constant; **(b)** $0.77\cdot 10^3$ kJ/mol.

8.27. $n=1+\dfrac{27a^4}{2\sqrt{3}\,\alpha e^2K}=11.9$; $U=0.63\cdot 10^3$ kJ/mol.

8.28. From the condition of the maximum value of $p=-\partial U/\partial V$ we obtain

$$\left(\frac{r_m}{r_0}\right)^{n-1}=\frac{n+3}{4},\quad n=1+\frac{9a^4}{8\alpha e^2K}=9.1,$$

where r_0 is the equilibrium separation of neighbouring ions, a is the lattice constant. As a result, $r_m/r_0=1.147$. The corresponding pressure is

$$|p_{\max}|=\frac{\alpha e^2}{6r_m^4}\left[1-\left(\frac{r_0}{r_m}\right)^{n-1}\right]=4.2\text{ GPa}.$$

8.29. (a) $U=N\dfrac{\alpha e^2}{r_0}\left(1-\dfrac{\rho}{r_0}\right)$, $\rho=0.112r_0=0.315$ Å;
(b) $K=9\rho a^4/4\alpha e^2(a-4\rho)$. Here r_0 is the equilibrium separation of neighbouring ions, a is the lattice constant.

8.30. $E=3N\left(\dfrac{\hbar\omega}{2}+\dfrac{\hbar\omega}{e^{\hbar\omega/kT}-1}\right)$;

$$C=3Nk\left(\frac{\hbar\omega}{kT}\right)^2\frac{e^{\hbar\omega/kT}}{(e^{\hbar\omega/kT}-1)^2}=\begin{cases}3Nk,\\ 3Nk\,(\hbar\omega/kT)^2\,e^{-\hbar\omega/kT}.\end{cases}$$

8.31. (a) Write the equation of motion of the nth atom

$$m\ddot{u}_n=\varkappa(u_{n+1}-u_n)+\varkappa(u_{n-1}-u_n)=\varkappa(u_{n+1}-2u_n+u_{n-1}).$$

The solution of that equation is to be found in the form of a standing wave: $u_n=A\sin kx\sin\omega t$, where k is the wave number equal to $2\pi/\lambda$, $x=na$ is the coordinate of the nth atom ($n=0, 1, 2, \ldots, N-1$). Such a solution satisfies immediately the boundary condition $u_0=0$. The boundary condition for the other end of the chain $u_{N-1}=0$ is satisfied provided $\sin ka(N-1)=0$. Thus we obtain the spectrum of eigenvalues of the wave number:

$$k_i=\frac{\pi i}{a(N-1)},\quad i=1, 2, \ldots, N-2$$

(when $i=0$, then $N-1$ $\sin kx\equiv 0$, i.e. the solution allows no motion at all). Thus, the displacement of the nth atom can be represented as a superposition of standing waves of the form

$$u_{ni}=A_i\sin k_i na\cdot\sin\omega_i t.$$

(b) Substituting the expression for u_{ni} into the equation of motion, we find

$$\omega_i=2\sqrt{\varkappa/m}\,\sin(k_i a/2).$$

It is seen from this equation that the number of different oscillations is equal to the number of possible values of the wave number k_i, i.e. $N-2$, or, in other words, to the number of oscillatory degrees of freedom of the given chain. $\omega_{\max}=2\sqrt{\varkappa/m}$; $\lambda_{\min}=2a$.

(c) $v_i=\dfrac{\omega_i}{k_i}=2\sqrt{\dfrac{\varkappa}{m}}\dfrac{\sin(k_ia/2)}{k_i}$; $v_1=a\sqrt{\varkappa/m}=$ const; $\dfrac{v_1}{v_{sh}}=\dfrac{\pi}{2}$.

(d) $dZ_\omega=\dfrac{2N}{\pi\sqrt{w_{\max}^2-\omega^2}}\,d\omega.$

8.32. (a) $dZ_\omega=\dfrac{L}{\pi v}\,d\omega$; (b) $\theta=\dfrac{\pi\hbar Nv}{kL}$; (c) taking into account that $E=\int_0^{\omega_m}\langle\varepsilon_\omega\rangle\,dZ_\omega$, where $\langle\varepsilon_\omega\rangle$ is the mean energy of the quantum harmonic oscillator with frequency ω, we obtain

$$E=R\theta\left(\frac{1}{4}+\frac{T^2}{\theta^2}\int_0^{\theta/T}\frac{x\,dx}{e^x-1}\right).$$

To determine $C=\partial E/\partial T$, the integral should be differentiated with respect to T (see Appendix 14). Finally we get

$$C=R\left(2\frac{T}{\theta}\int_0^{\theta/T}\frac{x\,dx}{e^x-1}-\frac{\theta/T}{e^{\theta/T}-1}\right)\approx\begin{cases}R,\\ (\pi^2/3)RT/\theta.\end{cases}$$

The value of the integral in the case of $\theta/T\to\infty$ can be taken in the same Appendix.

8.33. (a) $dZ_\omega=(S/\pi v^2)\,\omega\,d\omega$; (b) $\theta=(\hbar/k)\sqrt{4\pi v^2N/S}$;

(c) $E=4R\theta\left(\dfrac{1}{6}+\dfrac{T^3}{\theta^3}\displaystyle\int_0^{\theta/T}\dfrac{x^2\,dx}{e^x-1}\right)$; $C=4R\left(3\dfrac{T^2}{\theta^2}\displaystyle\int_0^{\theta/T}\dfrac{x^2\,dx}{e^x-1}-\dfrac{\theta/T}{e^{\theta/T}-1}\right)\approx$
$\approx\begin{cases}2R,\\ 28.9RT^2/\theta^2.\end{cases}$ See the solution of the foregoing problem.

8.34. (a) $dZ_\omega=(3V/2\pi^2v^3)\,\omega^2\,d\omega$; (b) $\theta=(\hbar/k)\sqrt{6\pi^2v^3N/V}$;

(c) $E=9R\theta\left(\dfrac{1}{8}+\dfrac{T^4}{\theta^4}\displaystyle\int_0^{\theta/T}\dfrac{x^3\,dx}{e^x-1}\right)$; $C=9R\left(4\dfrac{T^3}{\theta^3}\displaystyle\int_0^{\theta/T}\dfrac{x^3\,dx}{e^x-1}-\dfrac{\theta/T}{e^{\theta/T}-1}\right)\approx$
$\approx\begin{cases}3R,\\ (12/5)\,\pi^2RT^3/\theta^3.\end{cases}$ See the solution of Problem 8.32.

8.35. (a) $dZ_\omega=\dfrac{S}{2\pi}\left(\dfrac{1}{v_l^2}+\dfrac{1}{v_t^2}\right)\omega\,d\omega$; $\theta=\dfrac{\hbar}{k}\sqrt{\dfrac{8\pi N}{S(v_l^{-2}+v_t^{-2})}}$;

(b) $dZ_\omega=\dfrac{V}{2\pi^2}\left(\dfrac{1}{v_l^3}+\dfrac{2}{v_t^3}\right)\omega^2\,d\omega$; $\theta=\dfrac{\hbar}{k}\sqrt[3]{\dfrac{18\pi^2N}{V(v_l^{-3}+2v_t^{-3})}}$.

8.36. 470 K (see the formula for θ from the solution of the foregoing problem).

8.37. (a) 1.8; (b) 4.23 kJ/mol.

8.38. 20.7 and 23.8 J/(mol·K); 5% less.

8.39. It can be easily checked that in this temperature range the heat capacity $C \propto T^3$, and therefore one can use the low temperature formula for heat capacity. $\theta \approx 210$ K; $E_0 = 1.9$ kJ/mol.

8.40. **(a)** $\theta = 2.2 \cdot 10^2$ K; **(b)** $C = 12.5$ J/(mol·K); **(c)** $\omega_{max} = 4.1 \cdot 10^{13}\ s^{-1}$.

8.41. $\hbar\omega_{max} = 5.2 \cdot 10^{-14}$ erg; $p_{max} = \hbar k_{max} \approx \pi\hbar/r_0 \approx 10^{-19}$ g·cm/s.

8.42. **(b)** From the condition $dn/d\omega = 0$ we get the equation $e^x(2 - x) = 2$, where $x = \hbar\omega/kT$. Its root is found either from its graph or by inspection: $x_0 \approx 1.6$. Thus, $\hbar\omega_{pr} = 0.8k\theta$; **(c)** $T = 0.625\theta$; **(d)** $n \propto T^3$ and $n \propto T$ respectively.

8.43. Due to the photon-phonon interaction the energy of the photon changes by the value of the phonon energy: $\hbar\omega' = \hbar\omega \pm \hbar\omega_s$. On the other hand, from the triangle of momenta it follows that

$$(\hbar\omega_s/v)^2 = (\hbar\omega'/c')^2 + (\hbar\omega/c')^2 - 2(\hbar\omega'/c')(\hbar\omega/c')\cos\vartheta.$$

Eliminating ω' from these two equations, we obtain

$$\left(\frac{1}{v^2} - \frac{1}{c'^2}\right)\omega_s^2 = 2\left(\frac{\omega}{c'}\right)^2\left(1 \pm \frac{\omega_s}{\omega}\right)(1 - \cos\vartheta).$$

Taking into account that $v \ll c'$ and $\omega_s \ll \omega$, we can omit the corresponding infinitesimals in the latter expression and thus get the sought formula.

8.44. **(a)** At thermal equilibrium the ratio of the number of atoms N_2 on the upper level to N_1 on the lower one is equal (in accordance with the Boltzmann distribution)

$$N_2/N_1 = e^{-\Delta E/kT}; \quad N_2 = N/(1 + e^{\Delta E/kT}),$$

where $N = N_1 + N_2$ is the total number of atoms. The internal energy of the system is $E = N_2\,\Delta E$, whence

$$C_i = \frac{\partial E}{\partial T} = Nk\left(\frac{\Delta E}{kT}\right)^2 \frac{e^{\Delta E/kT}}{(1 + e^{\Delta E/kT})^2}.$$

(b) Designate $kT/\Delta E = x$. From the condition $\partial C_i/\partial x = 0$, we obtain the equation $e^{1/x}(1 - 2x) = 1 + 2x$. Its root is found either from its graph or by inspection: $x_0 \approx 0.42$.

(c) $\frac{C_{i\,max}}{C_{lat}} = \frac{0.44}{2.34 \cdot 10^{-4}} \approx 2 \cdot 10^3$.

9.1. The number of states within the interval of momenta $(p, p + dp)$ is

$$dZ_p = \frac{4\pi p^2\,dp}{\Delta p_x\,\Delta p_y\,\Delta p_z} = \frac{V}{2\pi^2\hbar^3}p^2 dp.$$

Since each phase element of volume $\Delta p_x\,\Delta p_y\,\Delta p_z$ can contain two electrons with antiparallel spins, the number of electrons in the given interval of momenta is $n(p)\,dp = 2dZ_p$. Transforming to kinetic energies, we obtain

$$n(T)\,dT = \frac{V\sqrt{2m^3}}{\pi^2\hbar^3}\sqrt{T}\,dT.$$

9.2. $T_{max} = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3} = 5.5$ eV.

9.3. **(a)** $(3/5)T_{max}$; **(b)** 31.2 kJ/cm^3.

9.4. 0.65.

9.5. $3.24 \cdot 10^4$ K.

9.6. $\Delta E = 2\pi^2\hbar^2/mV(3\pi^2 n)^{1/3} = 1.8 \cdot 10^{-22}$ eV.

9.7. By 0.1%.

9.8. $n(v)\,dv = \pi(m/\pi\hbar)^3 v^2\,dv$; **(a)** $(3/4)v_m$; **(b)** $3/2v_m$.

9.9. $1.6 \cdot 10^6$ and $1.2 \cdot 10^6$ m/s.

9.11. $n(\lambda)\,d\lambda = 8\pi\lambda^{-4}\,d\lambda$.

9.12. **(a)** $C_{el} = \frac{\pi^2}{2}R\frac{kT}{E_{f0}}$; $\frac{C_{el}}{C_{lat}} = \frac{\pi^2}{6}\cdot\frac{kT}{E_{f0}} = 7.6 \cdot 10^{-3}$. Here we took into account that the given temperature exceeds the Debye temperature, so that $C_{lat} = 3R$ (Dulong and Petit's law).

(b) From the nature of the temperature dependence of lattice heat capacity, it follows that the indicated heat capacities become equal at low temperatures. Making use of Eq. (8.6), we obtain $T = (5k\theta^3/24\pi^2 E_{f0})^{1/2} = 1.7$ K.

9.13. The number of free electrons with velocities $(v, v + dv)$ falling per 1 s per 1 cm^2 of metallic surface at the angles $(\vartheta, \vartheta + d\vartheta)$ to the surface's normal is

$$d\nu = n(v)\,dv\,\frac{2\pi\sin\vartheta\,d\vartheta}{4\pi}\,v\cos\vartheta.$$

Multiplying this expression by the momentum transferred to the wall of metal on reflection of each electron $(2mv\cos\vartheta)$ and integrating, we get

$$p = \int 2mv\cos\vartheta\,d\nu = \frac{\hbar^2}{15\pi^2 m}(3\pi^2 n)^{5/3} \approx 5 \text{ GPa}$$

The integration is performed with respect to ϑ in the interval from 0 to $\pi/2$ and with respect to v from 0 to v_{max}.

9.14. (a) $n(\mathrm{v})\,d\mathrm{v} = \frac{n(v)\,dv}{4\pi v^2\,dv}d\mathrm{v} = \frac{n(E)\,dE}{4\pi v^2\,dv}d\mathrm{v}$. The following derivation is obvious.

(b) $n(v_x)\,dv_x = 2(m/2\pi\hbar)^3\,dv_x\int dv_y\,dv_z = 2\pi(m/2\pi\hbar)^3(v_m^2 - v_x^2) \times dv_x$. The integration is convenient to perform in polar coordinates $dv_y\,dv_z = \rho\,d\rho\,d\varphi$, where $\rho = \sqrt{v_y^2 + v_z^2}$ (with ρ going from 0 to $\rho_m = \sqrt{v_m^2 - v_x^2}$).

9.15. Orient the x axis along the normal of the contact surface and write the conditions which the electrons passing from one metal into the other must satisfy:

$$\frac{mv_{x1}^2}{2} + \varphi_1 = \frac{mv_{x2}^2}{2} + \varphi_2; \quad v_{y1} = v_{y2}; \quad v_{z1} = v_{z2}, \qquad (1)$$

where φ is the potential energy of free electrons. The number of electrons falling per 1 s per 1 cm² of contact surface is

$$d\nu_1 = v_{x1} n(\mathbf{v}_1)\, d\mathbf{v}_1; \quad d\nu_2 = v_{x2} n(\mathbf{v}_2)\, d\mathbf{v}_2.$$

At dynamic equilibrium $d\nu_1 = d\nu_2$, and since according to Eq. (1) $v_{x1}\, d\mathbf{v}_1 = v_{x2}\, d\mathbf{v}_2$, then $n(\mathbf{v}_1) = n(\mathbf{v}_2)$. Consequently, $E_1 - E_{f1} = E_2 - E_{f2}$. Since $E_1 + \varphi_1 = E_2 + \varphi_2$, we get $E_{f1} + \varphi_1 = E_{f2} + \varphi_2$.

9.16. Orient the x axis along the normal of metallic surface and write the conditions which the electrons leaving the metal must satisfy:

$$\frac{mv_x'^2}{2} = \frac{mv_x^2}{2} + U; \quad v_y' = v_y; \quad v_z' = v_z, \tag{1}$$

where the primes mark the electronic velocity components *inside* the metal; U is the potential barrier at the metal's boundary $(E_f + A)$. The number of electrons leaving 1 cm² of the metallic surface per 1 s with velocities $(\mathbf{v}, \mathbf{v} + d\mathbf{v})$ is

$$d\nu = v_x' n(\mathbf{v}')\, d\mathbf{v}' = 2\left(\frac{m}{2\pi\hbar}\right)^3 \frac{v_x'\, d\mathbf{v}'}{1 + e^{(E' - E_f)/kT}}$$

$$= 2\left(\frac{m}{2\pi\hbar}\right)^3 e^{-(A+E)/kT} v_x\, d\mathbf{v}. \tag{2}$$

Here we took into account that according to Eq. (1) $v_x'\, d\mathbf{v}' = v_x\, d\mathbf{v}$ and that $E' - E_f = E + A$ and $kT \ll A$. Write Eq. (2) in spherical coordinates ($v_x = v\cos\vartheta$, $d\mathbf{v} = v^2 \sin\vartheta\, d\vartheta\, dv\, d\varphi$) and integrate with respect to φ from 0 to 2π and with respect to ϑ from 0 to $\pi/2$.

9.17. (a) $2kT$; (b) $j = \frac{emk^2}{2\pi^2\hbar^3} T^2 e^{-A/kT}$; (c) 4.1 eV.

9.18. Let us count the energy values off the top of the valence band. Ignoring unity in the denominator of the Fermi-Dirac function, we obtain the following free electron concentration

$$n_e = \int_{E_g}^{\infty} n(E)\, dE = 2\left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2} e^{(E_f - E_g)/kT},$$

where E_g is the level corresponding to the bottom of the conduction band. On the other hand, the hole concentration is

$$n_h = \int_{-\infty}^{0} f_h g_h\, dE = 2\left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2} e^{-E_f/kT},$$

where $f_h = 1 - f_e = e^{(E - E_f)/kT}$ and $g_h\, dE = g_e\, dE = \frac{\sqrt{2m^3}}{\pi^2\hbar^3}\sqrt{-E}\, dE$. Since $n_e = n_h$, $E_f - E_g = -E_f$ and $E_f = E_g/2$, that is the Fermi level lies in the middle of the forbidden band. Hence,

$$n_e = n_h = 2\,(mkT/2\pi\hbar^2)^{3/2}\, e^{-\Delta E_0/2kT},$$

where ΔE_0 is the width of the forbidden band.

9.19. (a) Counting the energy values from the level of donor atoms, we find the concentration of conduction electrons:

$$n_e = \int_{E_g}^{\infty} n(E)\, dE = 2\left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2} e^{(E_f - E_g)/kT}, \tag{1}$$

where E_g is the level corresponding to the bottom of the conduction band. On the other hand,

$$n_e = n_0\,[1 - f(0)] \approx e^{-E_f/kT}. \tag{2}$$

Multiplying Eqs. (1) and (2), we obtain

$$n_e^2 = 2n_0\,(mkT/2\pi\hbar^2)^{3/2}\, e^{-E_g/kT},$$

whence follows the formula given in the problem.

(b) From the comparison of Eqs. (1) and (2), we get

$$E_f = \frac{1}{2} E_g - \frac{kT}{2} \ln\left[\frac{2}{n_0}\left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2}\right].$$

It is seen from here that at $T \to 0$ $E_f = E_g/2$, i.e. the Fermi level lies in the middle between the level of donor atoms and the bottom of the conduction band.

9.20. $2.6\cdot 10^{-14}$ s, $3.1\cdot 10^{-6}$ cm, 46 cm²/(V·s).

9.21. $n = \sqrt{1 + U/T} = 1.02$, where $U = T_{max} + A$, A is the work function.

9.22. Since $m\ddot{x} = -eE$, where $E = -4\pi P = 4\pi nex$, then

$$\omega_0 = \sqrt{4\pi ne^2/m} = 1.6\cdot 10^{16}\ \text{s}^{-1}; \quad \varepsilon = \hbar\omega \approx 11\ \text{eV}.$$

9.23. Since $m\ddot{x} = eE_0 \cos\omega t$, then $x = -(eE_0/m\omega^2)\cos\omega t$. Taking into account that the electric polarization $P = nex$, we obtain:

$$\varepsilon = 1 + 4\pi\frac{P}{E} = 1 - \frac{4\pi ne^2/m}{\omega^2} = 1 - \left(\frac{\omega_0}{\omega}\right)^2,$$

where ω_0 is the electronic plasma oscillation frequency.

A metal is transparent to light if its refractive index $n = \sqrt{\varepsilon}$ is real (otherwise the reflection of light is observed). Hence,

$$\lambda_0 \leqslant 2\pi c\sqrt{m/4\pi ne^2} = 0.21.$$

9.24. When ν electrons (ν is considerably less than the total number of free electrons) are promoted to non-occupied levels, their kinetic energy increases by $\nu^2\,\Delta E$, where ΔE is the interval between the neighbouring levels (see the solution of Problem 9.6). On transition of the next electron, the kinetic energy increases by $2\nu\,\Delta E$, and the magnetic energy decreases by $2\mu B$, where μ is the magnetic moment of the electron. From the equation $2\nu\,\Delta E = 2\mu B$, we find ν, then the total magnetic moment of unpaired electrons $I = 2\nu\mu$ and paramagnetic susceptibility χ:

$$\chi = \frac{I}{B} = \frac{m\mu^2}{\pi^2\hbar^2}(3\pi^2 n)^{1/3} = 6\cdot 10^{-7}.$$

9.25. $\alpha = \frac{1}{\rho}\cdot\frac{d\rho}{dT} = -\frac{\Delta E_0}{2kT^2} = -\frac{\pi\hbar c}{\lambda kT^2} \approx -0.047\ \text{K}^{-1}$,

where $\rho \sim e^{\Delta E_0/2kT}$; ΔE_0 is the width of the forbidden band.

9.26. $E = \frac{2kT_1T_2}{T_2 - T_1}\ln\eta = 0.34$ eV.

9.27. 1.2 and 0.06 eV.

9.28. $\Delta\sigma/\sigma = 1 - en(b_e + b_h)\rho = 0.15$; $n = 2(mkT/2\pi\hbar)^{3/2} e^{-\Delta E/kT}$.

9.29. $\tau = t/\ln\left[\frac{(\rho-\rho_1)\rho_2}{(\rho-\rho_2)\rho_1}\right] = 0.010$ s.

9.30. 0.10 eV.

9.31. **(a)** $1.0\cdot 10^{15}$ cm^{-3}; $3.7\cdot 10^3$ cm^2/(V·s); **(b)** from the conductivity formula $\sigma = ne^2\tau/m$, where $\tau = \langle\lambda\rangle/\langle v\rangle$, we obtain

$$\langle\lambda\rangle = \frac{cR}{e\rho}\sqrt{8mkT/\pi} = 2.3\cdot 10^{-5}\ \text{cm}.$$

9.32. $R = l\rho V/dBU = 1.4\cdot 10^{-17}$ CGSE unit; $5\cdot 10^{15}$ cm^{-3}; 5×10^2 cm^2/(V·s).

9.33. In the presence of current, both electrons and holes are deflected by a magnetic field in the same direction. At dynamic equilibrium their flux densities in the transverse direction are equal:

$$n_e u_e = n_h u_h, \tag{1}$$

where u are the transverse velocities of directional motion of charge carriers. As $u = bE_\perp^* = \frac{b}{c}(f_L \mp eE_\perp)$, where b is the mobility, f_L is the Lorentz force, and $E_\perp$ is the transverse electric field strength, Eq. (1) can be rewritten as

$$n_e b_e\left(\frac{e}{c}v_e B - eE_\perp\right) = n_h b_h\left(\frac{e}{c}v_h B + eE_\perp\right),$$

where $v = bE$, E is the longitudinal electric field strength. Finding the ratio $E_\perp/EB$, we obtain

$$R = \frac{E_\perp}{jB} = \frac{E_\perp}{\sigma EB} = \frac{1}{ec}\,\frac{n_e b_e^2 - n_h b_h^2}{(n_e b_e + n_h b_h)^2}.$$

9.34. $b_e - b_h = cE_\perp/EB = 2.0\cdot 10^3$ cm^2/(V·s).

9.35. **(a)** 1 : 4.4; **(b)** 0.32.

10.1. $1.5\cdot 10^{14}$ g/cm^3; $8.7\cdot 10^{37}$ N/cm^3; $7\cdot 10^{18}$.

10.2. $1.2\cdot 10^{-12}$ cm.

10.3. $4.5\cdot 10^{-13}$ cm.

10.4. 1 a.m.u. = 1.00032 of old unit; decreased by a factor of 1.00032.

10.5. The atomic percentage is 1.11%; the mass percentage is 1.2%.

10.6. 1.007825, 2.014102, and 15.994915 a.m.u.

10.7. **(a)** ^{8}Be, $E_b = 56.5$ MeV; **(b)** 5.33, 8.60, 8.55, and 7.87 MeV.

10.8. **(a)** 6.76 and 7.34 MeV; **(b)** 14.4 MeV.

10.9. 6.73 MeV.

10.10. 10.56 MeV.

10.11. 7.16 MeV.

10.12. 22.44 MeV.

10.13. $\Delta E_b = 6.36$ MeV; $\Delta U_C = 6.34$ MeV. The coincidence is due to the approximate equality of nuclear forces between nucleons.

10.15. $4.1\cdot 10^{-15}$ m ($\Delta E_b = 4.84$ MeV).

10.16. **(a)** 341.8 and 904.5 MeV (table values: 342.05 and 915.36); **(b)** 8.65 and 7.81 MeV (table values: 8.70 and 7.91); **(c)** 44.955 and 69.932 a.m.u. (table values: 44.956 and 69.925).

10.17. From the condition $dM_N/dZ = 0$, we obtain $Z_m = \frac{A}{1.97 + 0.0149\,A^{2/3}}$. Calculating Z_m from this formula, we obtain 44.9 (47), 54.1 (50), and 59.5 (55) respectively, where the values of Z of the given nuclei are indicated in parentheses. Consequently, the former nucleus possesses the positron activity, and the remaining nuclei, the electron activity.

10.18. 2; 2; 1; 2 and 4.

10.19. 7/2.

10.20. Four components.

10.21. N is equal to the number of different values of the quantum number F, i.e. $2I + 1$ or $2J + 1$ respectively for $I < J$ and $I > J$. If at different values of J of either term **(a)** $N_1 = N_2$, then $N = 2I + 1$; **(b)** $N_1 \neq N_2$, then $N_i = 2J_i + 1$.

10.22. Here the ratio of component intensities is equal to the ratio of statistical weights of sublevels of the splitted term:

$$10/6 = (2F_1 + 1)/(2F_2 + 1) = (I + 1)/I;\quad I = 3/2.$$

10.23. The energy of magnetic interaction is $E = \mu_I B_0 \cos(\mathbf{I}, \mathbf{J})$, where

$$\cos(\mathbf{I}, \mathbf{J}) = \frac{F(F+1) - I(I+1) - J(J+1)}{2\sqrt{I(I+1)J(J+1)}}.$$

Since the values of I and J are the same for all sublevels, we find that $E \propto [F(F+1) - I(I+1) - J(J+1)]$. Thus, the interval between neighbouring sublevels $\delta E_{F,\,F+1} \propto F + 1$.

10.24. It is easy to notice that the number of the components of the given term is determined by the expression $2J + 1$. It can be thus concluded that $I \geqslant 3/2$. From the rule of intervals, we have

$4:5:6 = (F+1):(F+2):(F+3)$, where $F = I - J$. Hence, $I = 9/2$. The given line splits into six components.

10.25. $1 + 3 + 5 + 7 = 16$.

10.26. 2 and 5/2.

10.27. $\omega = g_s eB/2mc$, g_s is the gyromagnetic ratio. $1.76 \cdot 10^{10}$, $2.68 \cdot 10^7$, and $1.83 \cdot 10^7$ s^{-1}, respectively.

10.28. $g_s = 2\pi\hbar\nu_0/\mu_N B = 0.34$; $\mu = g_s I \mu_N = 0.85\mu_N$.

10.29. $\mu = 2\pi\hbar\nu I/B$, whence $\mu_{Li} = 3.26\mu_N$, $\mu_F = 2.62\mu_N$.

10.30. $T_{max} = (\hbar^2/2m)(3\pi^2 n)^{2/3} \approx 25$ MeV, where m is the mass of the nucleon; n is the concentration of protons (or neutrons) in the nucleus.

10.31. $1s^4_{1/2}1p^3_{3/2}$; $1s^4_{1/2}1p^8_{3/2}1p^1_{1/2}$; $1s^4_{1/2}1p^8_{3/2}1p^4_{1/2}1d^9_{5/2}$.

10.32. 5/2 (+); 1/2 (+); 3/2 (+); 7/2 (—); 3/2 (—).

10.33. From the vector model similar to that shown in Fig. 67, we have: $\mu = \mu_s \cos(\mathbf{s}, \mathbf{j}) + \mu_l \cos(\mathbf{l}, \mathbf{j})$. Substituting in the latter equation the following expressions $\mu_s = g_s s \mu_N$; $\mu_l = g_l l \mu_N$; $\cos(\mathbf{s}, \mathbf{j}) = \frac{j^2 + l^2 - s^2}{2sj}$; $\cos(\mathbf{l}, \mathbf{j}) = \frac{j^2 - s^2 - l^2}{2lj}$, where $s = \sqrt{s(s+1)}$; $l = \sqrt{l(l+1)}$; $j = \sqrt{j(j+1)}$, we obtain

$$\mu = g_j j \mu_N;\quad g_j = \frac{g_s + g_l}{2} + \frac{g_s - g_l}{2} \cdot \frac{s(s+1) - l(l+1)}{j(j+1)}.$$

ow it remains to substitute $s = 1/2$ and $j = l \pm 1/2$ into the last xpression.

10.34. In nuclear magnetons:

	$j = l + \frac{1}{2}$	$j = l - \frac{1}{2}$	$s_{1/2}$	$p_{1/2}$	$p_{3/2}$
μ_n	-1.91	$\frac{1.91}{j+2}j$	-1.91	0.64	-1.91
μ_p	$j + 2.29$	$\left(1 - \frac{2.29}{j+1}\right)j$	2.79	-0.26	3.79

10.35. For $f_{5/2}$ $\mu_p = 0.86\mu_N$; for $f_{7/2}$ $\mu_p = 5.79\mu_N$, hence $j = 7/2$.

10.36. **(a)** 2.79 and $-1.91\mu_N$; **(b)** -1.91 and $0.126\mu_N$ (experimental values: 2.98, -2.13, -1.89, and 0.39).

10.37. In accordance with the nuclear shell model it is natural to assume that the unpaired proton of the given nucleus is located on the $2s_{1/2}$ level. In this case the magnetic moment of the nucleus is equal to $2.79\mu_N$ (see the solution of Problem 10.34). If one assumes that this proton is located on the next level $1d_{3/2}$, the magnetic moment becomes equal to $0.124\mu_N$ which drastically differs from the magnitude given in the text of the problem.

11.1. $1 - e^{-\lambda t}$.

11.3. **(a)** 0.78 and 0.084; **(b)** $6.8 \cdot 10^{-5}$; 0.31.

11.4. $9 \cdot 10^{-7}$.

11.5. $0.80 \cdot 10^{-8}$ s^{-1}, 4.0 and 2.8 years.

11.6. $4.1 \cdot 10^3$ years.

11.7. 4 Ci.

11.8. $2 \cdot 10^{15}$ nuclei.

11.9. 0.06 Ci/g.

11.10. 0.05 mg.

11.11. $V = (A/a)\, e^{-\lambda t} = 6$ l.

11.12. Plot the logarithmic count rate vs. time. Extrapolating the rectilinear section corresponding to the longer-life component to $t = 0$, we find the difference curve (in this case, the straight line). The latter corresponds to the other component. From the slopes of the straight lines, we obtain $T_1 = 10$ hours, $T_2 = 1.0$ hour; $N_{10} : N_{20} = 2:1$.

11.13. **(a)** The accumulation rate of radionuclide A_2 is defined by the equation

$$\dot N_2 = \lambda_1 N_1 - \lambda_2 N_2, \quad \text{and} \quad \dot N_2 + \lambda_2 N_2 = \lambda_1 N_{10}\, e^{-\lambda_1 t}.$$

With the initial condition $N_2(0) = 0$, its solution takes the form

$$N_2(t) = N_{10} \frac{\lambda_1}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t}).$$

(b) $t_m = \frac{\ln(\lambda_1/\lambda_2)}{\lambda_1 - \lambda_2}$. **(c)** The ratio N_1/N_2 remains constant provided both N_1 and N_2 depend on time alike. This is possible only when $e^{-\lambda_2 t} \ll e^{-\lambda_1 t}$. The latter inequality is satisfied if λ_1 is appreciably less than λ_2 and the time interval t exceeds considerably the mean lifetime of the more stable nuclide.

11.14. $4.5 \cdot 10^9$ years.

11.15. $\frac{A_{2\,max}}{A_{10}} = \frac{\lambda_2}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t_m} - e^{-\lambda_2 t_m}) = 0.7$, where $t_m = \frac{\ln(\lambda_1/\lambda_2)}{\lambda_1 - \lambda_2}$.

11.16. **(a)** The stable nuclide accumulates according to the equation $\dot N_3 = \lambda_2 N_2$. Substituting into this equation the expression for $N_2(t)$ from the solution of Problem 11.13 and integrating the obtained relation with respect to t, we obtain

$$\frac{N_3}{N_{10}} = \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1}\left(\frac{1 - e^{-\lambda_1 t}}{\lambda_1} - \frac{1 - e^{-\lambda_2 t}}{\lambda_2}\right) = 0.7.$$

(b) $\frac{A}{A_0} = e^{-\lambda_1 t} + \frac{\lambda_2}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t}) = 0.55$, i.e. the activity decreases 1.8 times.

11.17. $N_3(t) = N_{10}\lambda_1\lambda_2\left(\frac{e^{-\lambda_1 t}}{\Delta\lambda_{12}\,\Delta\lambda_{13}} + \frac{e^{-\lambda_2 t}}{\Delta\lambda_{21}\,\Delta\lambda_{23}} + \frac{e^{-\lambda_3 t}}{\Delta\lambda_{31}\,\Delta\lambda_{32}}\right)$, where $\Delta\lambda_{ik} = \lambda_i - \lambda_k$.

11.18. About 0.3 kg.

$4:5:6 = (F+1):(F+2):(F+3)$, where $F = I - J$. Hence, $I = 9/2$. The given line splits into six components.

10.25. $1+3+5+7 = 16$.

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11.1. $1 - e^{-\lambda t}$.

11.3. **(a)** 0.78 and 0.084; **(b)** $6.8\cdot10^{-5}$; 0.31.

11.4. $9\cdot10^{-7}$.

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11.6. $4.1\cdot10^3$ years.

11.7. 4 Ci.

11.8. $2\cdot10^{15}$ nuclei.

11.9. 0.06 Ci/g.

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11.11. $V = (A/a)\,e^{-\lambda t} = 6$ l.

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(b) $t_m = \frac{\ln(\lambda_1/\lambda_2)}{\lambda_1-\lambda_2}$. **(c)** The ratio N_1/N_2 remains constant provided both N_1 and N_2 depend on time alike. This is possible only when $e^{-\lambda_2 t} \ll e^{-\lambda_1 t}$. The latter inequality is satisfied if λ_1 is appreciably less than λ_2 and the time interval t exceeds considerably the mean lifetime of the more stable nuclide.

11.14. $4.5\cdot10^9$ years.

11.15. $\frac{A_{2\,max}}{A_{10}} = \frac{\lambda_2}{\lambda_2-\lambda_1}(e^{-\lambda_1 t_m} - e^{-\lambda_2 t_m}) = 0.7$, where $t_m = \frac{\ln(\lambda_1/\lambda_2)}{\lambda_1-\lambda_2}$.

11.16. **(a)** The stable nuclide accumulates according to the equation $\dot{N}_3 = \lambda_2 N_2$. Substituting into this equation the expression for $N_2(t)$ from the solution of Problem 11.13 and integrating the obtained relation with respect to t, we obtain

$$\frac{N_3}{N_{10}} = \frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1}\left(\frac{1-e^{-\lambda_1 t}}{\lambda_1} - \frac{1-e^{-\lambda_2 t}}{\lambda_2}\right) = 0.7.$$

(b) $\frac{A}{A_0} = e^{-\lambda_1 t} + \frac{\lambda_2}{\lambda_2-\lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t}) = 0.55$, i.e. the activity decreases 1.8 times.

11.17. $N_3(t) = N_{10}\lambda_1\lambda_2\left(\frac{e^{-\lambda_1 t}}{\Delta\lambda_{12}\,\Delta\lambda_{13}} + \frac{e^{-\lambda_2 t}}{\Delta\lambda_{21}\,\Delta\lambda_{23}} + \frac{e^{-\lambda_3 t}}{\Delta\lambda_{31}\,\Delta\lambda_{32}}\right)$, where $\Delta\lambda_{ik} = \lambda_i - \lambda_k$.

11.18. About 0.3 kg.

11.19. **(a)** The activity of either nuclide equals 10 μCi; **(b)** 0.05 mCi.

11.20. **(a)** $4.1\cdot 10^{13}$; **(b)** $2.0\cdot 10^{13}$.

11.21. **(a)** 40 days; **(b)** $M = M_{\text{Te}}\,(e^{-\lambda t} + \lambda t - 1)\,q/\lambda = 1.0$ μg.

11.22. $N_1(t) = \frac{q}{\lambda_1}(1 - e^{-\lambda_1 t})$,

$$N_2(t) = \frac{q}{\lambda_2}\left(1 + \frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_1 t} + \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 t}\right),$$

$$N_3(t) = q\left[t + \frac{\lambda_2(1 - e^{-\lambda_1 t})}{\lambda_1(\lambda_1 - \lambda_2)} + \frac{\lambda_1(1 - e^{-\lambda_2 t})}{\lambda_2(\lambda_2 - \lambda_1)}\right].$$

11.23. $A = q\left(2 + \frac{2\lambda_2 - \lambda_1}{\lambda_1 - \lambda_2} e^{-\lambda_1 t} + \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 t}\right) = 0.4$ Ci.

11.24. **(a)** 1.6%; **(b)** $4\cdot 10^{13}$.

11.25. $Q = T(1 + m_\alpha/M) = 8.5$ MeV; M is the mass of the daughter nucleus; 1.9%; $3.8\cdot 10^5$ m/s.

11.26. **(a)** $Q = N_0 T(1 + m_\alpha/M)(1 - e^{-\lambda t}) = 1.6\cdot 10^4$ kJ, where N_0 is the initial number of nuclei; M is the mass of the daughter nucleus. **(b)** 0.8 mCi.

11.27. 5.40 and 0.82 MeV.

11.28. The energy values: 0, 0.11, 0.24, and 0.31 MeV.

11.29. The energy values: 0, 0.726, 1.673, and 1.797 MeV.

11.30. 29 MeV; $3.6\cdot 10^{-12}$ cm.

11.31. $-dU_{\text{cf}}/dr = F = m_\alpha v^2/r = p_l^2/m_\alpha r^3$, where p_l is the orbital moment. Integrating this expression and taking into account that $p_l = \hbar\sqrt{l(l+1)}$, we get $U_{\text{cf}} = \hbar^2 l(l+1)/2m_\alpha r^2$. The sought ratio is $\frac{U_{\text{cf}}}{U_{\text{C}}} = \frac{\hbar^2 l(l+1)}{4(Z-2)e^2 m_\alpha R} = 1.6\cdot 10^{-2}$, R is the nuclear radius.

11.32. **(a)** Introducing the new variable φ in accordance with the formula $\cos^2\varphi = T/U(r)$, where $U(r)$ is the energy of the Coulomb interaction and r the distance between the α-particle and the daughter nucleus, we obtain after integration:

$$D = \exp\left[-\frac{\varkappa}{\pi\sqrt{T}}(2\varphi_0 - \sin 2\varphi_0)\right],$$

where φ_0 corresponds to the height of the Coulomb barrier The following derivation is obvious.

(b) 3.4 : 1.

11.33. $\lambda_\alpha = 1/\tau\,(1 + N_\gamma/N_\alpha) = 2\cdot 10^7$ s^{-1}.

11.34. $\Gamma_\gamma = \hbar\lambda_\alpha N_\gamma/N_\alpha = 0.9\cdot 10^{-4}$ eV.

11.35. 0.78 MeV.

11.36. $Q = \begin{cases} M_P - M_D \text{ in } \beta^-\text{-decay and } K\text{-capture;} \\ M_P - M_D - 2m_e \text{ in positron decay.} \end{cases}$

11.37. **(a)** 6.0189; **(b)** 21.99444 a.m.u.

11.38. **(a)** impossible; **(b)** possible; **(c)** possible.

11.39. 1.71 MeV, $T_N = Q(Q + 2m_e c^2)/2Mc^2 = 78.5$ eV, Q is the decay energy.

11.40. $p = \sqrt{Q(Q + 2m_e c^2)}/c = 0.94$ MeV/c; Q is the decay energy.

11.41. **(a)** 0.97 MeV and 94 eV; **(b)** 0.32 and 0.65 MeV.

11.42. $\vartheta = \pi - \arccos(p_e/p_\nu) = 110°$, $p_e/p_\nu = \sqrt{T(T + 2mc^2)}/(Q - T)$.

11.43. 1.78 MeV.

11.44. 0.78.

11.45. Level energies: 0, 0.84, 2.65, and 2.98 MeV.

11.46. $T \approx Q^2/2Mc^2 = 9.5$ eV; Q is the energy liberated in this process; M is the mass of the atom; $v = 7.0\cdot 10^3$ m/s.

11.47. 0.32 MeV.

11.48. $T = Q(2\hbar\omega - Q)/2Mc^2 = 6.6$ eV; Q is the energy liberated in this process; M is the mass of the atom; 56 eV.

11.49. 0.41 and 1.25 km/s.

11.50. 26 keV.

11.51. 279 keV.

11.52. 145 keV.

11.53. 566 and 161 keV.

11.54. $1.2\cdot 10^5$ electrons per second.

11.55. $\Delta\hbar\omega/\hbar\omega = E/2Mc^2 = 3.6\cdot 10^{-7}$; M is the mass of the nucleus.

11.56. The probability of such a process will be extremely low because the decrease in the energy of γ-quantum (equal to double energy of the recoil nucleus) is considerably more than the level width Γ.

11.57. $v_{\text{rel}} = \hbar\omega/Mc = 0.22$ km/s; M is the mass of the nucleus.

11.59. $\Gamma \approx 2\hbar\omega v/c = 1\cdot 10^{-5}$ eV; v is the velocity at which the ordinate of the line's contour is equal to half the maximum ordinate; $\tau \approx 0.6\cdot 10^{-10}$ s.

11.60. $v = gl/c = 6.5\cdot 10^{-5}$ cm/s.

11.61. The fractional increase in frequency of γ-quantum "falling" from the height l, $\Delta\omega/\omega = gl/c^2 \geqslant \Gamma/E$; whence, $l_{\text{Fe}} \geqslant 2.8$ km, $l_{\text{Zn}} \geqslant 4.6$ m.

11.62. **(a)** On radiation of γ-quantum the atomic mass decreases by $\delta M = \hbar\omega_0/c^2$, so that its mean kinetic energy, $T = \langle p^2\rangle/2M$, increases by $\delta T = \langle p^2\rangle\,\delta M/2M^2 = \hbar\omega_0\,\langle v^2\rangle/2c^2$. Consequently, the energy of emitted γ-quantum is $\hbar\omega = \hbar\omega_0 - \delta T = \hbar\omega_0(1 - \langle v^2\rangle/2c^2)$.

(b) Assuming $M\langle v^2\rangle = 3kT$, express the frequency of the emitted γ-quantum via the temperature $\omega(T)$. Then find the fractional increase in the frequency of γ-quantum due to the temperature increment δT: $(\delta\omega/\omega_0)_{\text{temp}} = -\frac{3k}{2Mc^2}\delta T$. The gravitational frequency increment of the γ-quantum "falling" from the height l is equal to $(\delta\omega/\omega_0)_{\text{gr}} = gl/c^2$. From the latter expressions, we find $\delta T = 2Mgl/3k = 0.9$ °C.

11.63. $\mu' = -0.15\mu_N$; $B = 3.3\cdot 10^5$ G.

11.64. $\Delta p \approx p(n)\,\Delta n = 0.08$.
11.65. $2.5\cdot 10^2$ and $0.8\cdot 10^2$.
11.66. Using the Gaussian distribution, we find that for $\varepsilon_0 = 5.0$ pulses per minute and $\sigma = \sqrt{100}$, the probability $p_{\varepsilon_0} = 0.035$;

$$P_{\varepsilon > \varepsilon_0} = 2\int_{\varepsilon_0}^{\infty} p(\varepsilon)\,d\varepsilon = 1 - \frac{2}{\sqrt{2\pi}}\int_0^{\varepsilon_0/\sigma} e^{-x^2/2}\,dx = 0.62.$$

11.67. (a) 32%; (b) 4.6%.
11.68. (a) ± 6 pulses per minute; (b) 28 minutes.
11.69. 50 ± 5 pulses per minute.
11.70. Suppose that the radiation produces N_r pulses in the absence of the background. The corresponding relative standard deviation is $\eta_0 = 1/\sqrt{N_r}$. In the presence of the background

$$\eta = \sqrt{N_{rb} + N_b}/(N_{rb} - N_b) = \sqrt{6/N_{rb}},$$

for $N_{rb} = 2N_b$. From the requirement $\eta = \eta_0$, we obtain $N_{rb} = 6N_r$.
11.71. Write the expression for the square of standard deviation of the count rate of the source investigated and its differential:

$$\sigma_r^2 = \frac{n_{rb}}{t_{rb}} + \frac{n_b}{t_b};\ 2\sigma_r\,d\sigma_r = -\frac{n_{rb}}{t_{rb}^2}\,dt_{rb} - \frac{n_b}{t_b^2}\,dt_b.$$

From the condition for the least error ($d\sigma_r = 0$) and the fixed total time, i.e. $dt_{rb} + dt_b = 0$, we get $t_b/t_{rb} = \sqrt{n_b/n_{rb}} \approx 1/2$.

11.72. $t_b = \dfrac{n_b + \sqrt{n_b n_{rb}}}{\eta^2 (n_{rb} - n_b)^2} = 7$ min; $t_{rb} = \dfrac{n_{rb} + \sqrt{n_{rb} n_b}}{\eta^2 (n_{rb} - n_b)^2} = 14$ min.

11.73. The counter is incapable of registering for the time τn in the course of every second. That signifies that $\tau n N$ non-registered particles pass through the counter during that time. Thus, $N = n + \tau n N$, whence $N = 3.3\cdot 10^4$ particles per minute.
11.74. 0.010 and 9%.

11.75. $N_r = \dfrac{n_{rb}}{1 - \tau n_{rb}} - \dfrac{n_b}{1 - \tau n_b} = 5.7\cdot 10^2$ particles per minute.

11.76. $\tau = \dfrac{1}{n_{12}}\left[1 - \sqrt{1 - \dfrac{n_{12}(n_1 + n_2 - n_{12})}{n_1 n_2}}\right] \approx \dfrac{n_1 + n_2 - n_{12}}{2n_1 n_2}$.

The latter equation is valid for sufficiently small τ when $n_1 + n_2$ does not differ much from n_{12}.
11.77. (a) In this case the recording facility will register all pulses from the counter, and $n = N/(1 + \tau N)$.
(b) The number of pulses produced in the counter is $n_1 = N/(1 + \tau_1 N)$. Out of this number the recording facility will register

$$n_2 = n_1/(1 + \tau_2 n_1) = N/[1 + (\tau_1 + \tau_2)N].$$

11.78. $N = n/[1 - (\tau_1 + \tau_2)n] = 6\cdot 10^6$ electrons per second.
11.79. The electromechanical reader registers a pulse from a Geiger-Müller counter in the time interval t, $t + dt$ provided no pulses are fed to the reader in the preceding time interval $t - \tau$, t. The probability of that event is $p = e^{-N\tau}$. Hence, the total number of particles registered by the electromechanical reader is $n = Np = Ne^{-N\tau}$.
11.80. From the condition $dn/dN = 0$, in which the expression for $n(N)$ is taken from the solution of the foregoing problem, we find $N = 1/\tau$; $\tau = 1/en_{\max} = 8$ ms.
11.81. The probability that the pulse from one counter is accompanied by the pulse from the other with time separation $\pm\tau$ is equal to $2\tau n_2$. Consequently, $\Delta n = 2\tau n_1 n_2$.
11.82. $N = \sqrt{\Delta n/2\tau\eta} = 4\cdot 10^5$; η is the registration efficiency.
11.83. We have $0.05 = \sqrt{(n_{rb} + \Delta n_b)\,t}/n_r t$, where $n_{rb} = n_r + \Delta n_b$; Δn_b is the number of coincidences per second caused by the background; $\Delta n_b = 2\tau n_b^2$. Hence, $t = (n_r + 4\tau n_b^2)/0.05^2 n_r^2 = 20$ s.
11.84. $\eta_1\eta_2/(\eta_1 + \eta_2) = 0.03\%$.
12.1. $\vartheta_{\max} = \arcsin\,(m_e/m_\alpha) = 0.5'$.
12.2. The electron acquires the momentum p in the direction perpendicular to the motion direction of the α-particle (Fig. 70).

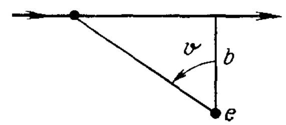


Fig. 70

$$p = \int_{-\infty}^{\infty} f_\perp\,dt = \int_{\pi/2}^{-\pi/2} \frac{qe\cos\vartheta\,d\vartheta}{r^2\dot\vartheta}.$$

Taking into account the law of conservation of angular momentum $r^2\dot\vartheta = \text{const} = -vb$, where the minus sign is due to the fact that $\dot\vartheta < 0$, we obtain

$$p = 2qe/vb;\quad T_e = m_\alpha q^2 e^2/m_e b^2 T_\alpha = 6\ \text{eV}.$$

12.3. Consider the layer of thickness dx perpendicular to the trajectory of α-particle. The number of electrons with aiming parameter in the interval $(b, b + db)$ is $\delta n = n\,dx\,2\pi b\,db$. The energy transferred by the α-particle to these electrons, $\delta E = T_e\,\delta n$, where T_e is the kinetic energy acquired by each electron. After substitution of the expression for T_e (see the answer to the foregoing problem), we get

$$\left|\frac{dE}{dx}\right| = \frac{4\pi q^2 e^2 n}{m_e v^2}\cdot\frac{db}{b}.$$

12.4. 0.17 MeV/cm.
12.5. (a) 23 : 1; (b) 2.4 : 1.
12.6. $p \geqslant 27$ kPa.

12.7. **(a)** $2.8\cdot10^4$; **(b)** $\approx 1/3$.
12.8. 14 mg/cm^2.
12.9. 5.5 MeV.
12.10. 8 μm.
12.11. 75 μm.
12.12. 24 μm.
12.13. Since $-dE/dx = q^2 f(v)$, where $f(v)$ is a function of particle's velocity, the range is

$$R = -\int_v^0 \frac{dE}{q^2 f(v)} = \frac{m}{q^2} F(v),$$

where $F(v)$ is the function of the particle's velocity and properties of the matter. Hence,

$$R_d(v) = \frac{m_d}{m_p} R_p(v); \; R_d(T) = \frac{m_d}{m_p} R_p\left(\frac{m_p}{m_d} T\right).$$

The range of the deuteron is equal to 4.6 cm.
12.14. 42 μm (see the solution of the foregoing problem).
12.15. **(a)** In the C frame, the Rutherford formula takes the form

$$d\sigma = \left(\frac{qe}{\mu v^2}\right)^2 \frac{d\tilde{\Omega}}{4\sin^4(\tilde{\vartheta}/2)} \approx \left(\frac{qe}{m_e v^2}\right)^2 \frac{2\pi \sin\tilde{\vartheta}\, d\tilde{\vartheta}}{4\sin^4(\tilde{\vartheta}/2)},$$

where μ is the reduced mass, m_e is the electron mass. Transform this formula replacing the angular interval $(\tilde{\vartheta}, \tilde{\vartheta} + d\tilde{\vartheta})$ with the corresponding interval of kinetic energies of δ-electron $(T, T + dT)$, taking into account that $\tilde{\vartheta} = \pi - 2\varphi$; $T = \frac{4m_e m}{(m_e + m)^2} T_0 \cos^2\varphi$, where φ is the angle at which the recoil electron moves in the L frame, m and T are the mass and the kinetic energy of the primary particle. The first formula follows from the vector diagram of momenta, the second from the conservation laws of energy and momentum. As $T_0 = mv^2/2$ and $m_e \ll m$, then $d\sigma = \frac{2\pi q^2 e^2}{m_e v^2} \frac{dT}{T^2}$.

(b) $N = n\int d\sigma(T) = \frac{2\pi n q^2 e^2}{m_e v^2}\left(\frac{1}{T_{th}} - \frac{1}{T_{max}}\right)$, where the integration is performed with respect to T from T_{th} to $T_{max} = 2m_e v^2$.
12.16. **(a)** $T_{min} = (m_\alpha/4m_e)\, T_{th} = 20$ MeV; **(b)** from the condition $dN/dv = 0$, we obtain $T_\alpha = (m_\alpha/2m_e)\, T_{th} = 64$ MeV and $N_{max} = \pi n q^2 e^2/T_{th}^2 = 5.1$; **(c)** $q = e$.
12.17. 2.0 MeV/cm; 19 times.
12.18. 114, 62, and 9.8 MeV.
12.19. ≈ 20 MeV.
12.20. ≈ 10 MeV/cm.
12.21. From the formula $T = T_0 e^{-x/l_{rad}}$, we obtain $-\partial T/\partial x = T/l_{rad}$. Comparing this expression with the formula for $(\partial T/\partial x)_{em}$, we find $l_{rad} = \frac{137}{4r_e^2 n Z^2 \ln(183/Z^{1/3})}$; 360 m; 9.8 and 0.52 cm.

12.22. 1.4 cm.
12.23. Finding out that the energy losses of electrons are primarily of radiation nature, we obtain, using formula (12.6), $T_0 = 0.11$ GeV. Here l_{rad} is determined from the formula given in the solution of Problem 12.21.
12.24. The probability of γ-quanta being emitted in the frequency interval $(\omega, \omega + d\omega)$ is equal to $dw = nl\, d\sigma$. Whence,

$$w = \frac{l}{l_{rad}} \int_{0.9}^{0} \frac{d(\hbar\omega)}{\hbar\omega} = 6\cdot10^{-3}.$$

12.25. 0.36 MeV.
12.26. Finding out that these electrons sustain primarily ionization losses, we use the formula defining the range in aluminium; 0.28 cm.
12.27. 0.8 m.
12.28. 0.3 g/cm^2.
12.29. 0.2.
12.30. 1.6 MeV.
12.31. 50; $2.4\cdot10^{-2}$ and $5.7\cdot10^{-3}$ cm.
12.32. $\cos\vartheta = \frac{c'}{v}\left[1 + \frac{\hbar\omega(n^2-1)}{2E}\right] \approx \frac{c'}{v}$, E is the total energy of the particle.
12.33. 0.14 MeV and 0.26 GeV; for muons.
12.34. 0.23 MeV.
12.35. 0.2 MeV.
12.36. 3.2 cm.
12.37. 1.7 mm; 6 times.
12.38. $6.5\cdot10^{-2}$; 5.1 and $4.4\cdot10^3$ cm.
12.39. $d/d_{1/2} \approx (\ln n)/\ln 2 \approx 10$.
12.40. See Fig. 71, where λ_K is the K band absorption edge.
12.41. **(a)** Fe; **(b)** Co.
12.42. About 10 μm.
12.43. **(a)** $\sigma = (8\pi/3)\, r_e^2$; **(b)** 0.3; **(c)** this number is equal to the number of photons scattered within the angular interval which is easily found by means of the formula

$$\cot(\vartheta/2) = (1 + \hbar\omega_0/m_e c^2)\tan\psi,$$

where ϑ is the scattering angle of the photon and ψ is the angle at which the recoil electron moves. In the case of soft X-ray radiation $\hbar\omega_0 \ll m_e c^2$ and $\tan\psi \approx \cot(\vartheta/2)$. Now we can find the angles ϑ_1 and ϑ_2 corresponding to $\psi = \pi/4$ and $\pi/2$ and obtain

$$\eta_e = \frac{\Delta\sigma}{\sigma} = \frac{1}{\sigma}\int_{\vartheta=0}^{\pi/2} d\sigma(\vartheta) = 0.50.$$

12.44. $\sigma/\rho = 0.4Z/A$ cm^2/g, for both cases $\sigma/\rho \approx 0.20$ cm^2/g; the linear scattering coefficients are equal to $1.8\cdot10^{-4}$ and $2.9\cdot10^{-4}$ cm.

12.45. $1.17 \cdot 10^{-22}$ cm²/atom.
12.46. (a) 70 cm²/g; (b) 8.7 : 1.

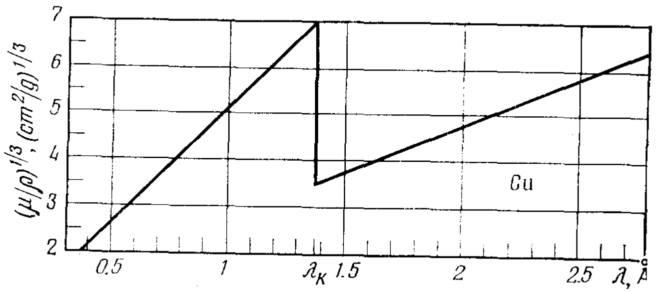


Fig. 71

12.47. Having calculated the mass attenuation coefficient, we find in the tables of the Appendix the corresponding energy of γ-quanta (0.2 MeV) and then, from its magnitude, the mass absorption coefficient. Their difference is equal to $\sigma/\rho \approx 0.095$ cm²/g.

12.48. $J = Ae^{-\mu\Delta r}/4\pi r^2 = 6 \cdot 10^3$ quanta/(cm²·s).

12.49. 1 : 4.7.

12.50. $J = \dfrac{e^{-\mu_2 d} - e^{-\mu_1 d}}{(\mu_1 - \mu_2) d} J_0 = 0.7 J_0$.

12.51. 2.75 b/atom.

12.52. $1.2 \cdot 10^2$ b/atom.

12.53. $l = 1/\mu$; $1.2 \cdot 10^4$; 14 and 6 cm.

12.54. $l = d_{1/2}/\ln 2 = 6.5$ cm.

12.55. $l = 1/n\sigma$, where n is the concentration of nuclei; $l_{tot} = 1.9$ cm; $l_{Comp} = 2.5$ cm; $l_{photo} = 1.5$ cm; $l_{pair} = 1.8$ cm; $l_{tot}^{-1} = l_{Comp}^{-1} + l_{photo}^{-1} + l_{pair}^{-1}$.

12.56. $w_{photo} = \dfrac{\sigma_{photo}}{\sigma_{tot}} (1 - e^{-\mu d}) \approx 0.013$, where $\mu = n\sigma_{tot}$.

12.57. First calculate the total cross-section: 17 b/atom. According to the graph, this value corresponds to two energy values of γ-quanta: 1.75 or 10.25 MeV. Respectively: 0.039 or 0.012 cm²/g.

12.58. (a) $\sigma_T (1 - 2\varepsilon)$ and $\sigma_T \dfrac{3}{8\varepsilon} (\ln 2\varepsilon + 1/2)$; (b) 0.084 cm⁻¹; (c) 0.063 cm²/g.

12.59. $w_{pair} = \sigma_{pair}/2\sigma_{tot} = 0.28$.

12.60. 3.5 mm.

12.61. $w_{pair}/w_{photo+Comp} = 1/(\eta - 1) = 0.37$.

12.62. Use the invariant $E^2 - p^2c^2$, where E and p are the total energy and momentum of the system. Write the invariant in the L and C frames for the threshold values of energy and momentum of γ-quantum: $(\hbar\omega_{th} + Mc^2)^2 - (\hbar\omega_{th})^2 = (M + 2m)^2 c^4$, whence $\hbar\omega_{th} = 2mc^2 (1 + m/M)$. Here m is the mass of either particle in the pair.

12.63. A pair is produced provided the energy of γ-quantum exceed $2mc^2$; m is the particle's mass. Obviously, one can always find a frame in which the energy of the γ-quantum is less than $2mc^2$ and the pair production is therefore impossible. But if this process is impossible in one reference frame, it is impossible in any other ones.

12.64. $T = \dfrac{\alpha^2}{1+\alpha} m_e c^2 = 0.6$ eV, where $\alpha = 2m_e/m_p$.

12.65. 2.1 MeV.

12.66. $P = \dfrac{I}{2.08 \cdot 10^9 e} \cdot \dfrac{p_0 T}{V p T_0} = 0.03$ R/s, where V is the volume, p_0 and T_0 are the normal pressure and temperature.

12.67. The radiation dose rate is the same in both cases: $P = dE/dV = \tau J = 40$ mR/h, where τ is the linear absorption coefficient, J is the flux density. The absorbed dose rate is

$$P' = \frac{1}{\rho} \cdot \frac{\partial E}{\partial V} = \frac{\tau J}{\rho} \approx \begin{cases} 35 \text{ mrad/h, air} \\ 39 \text{ mrad/h, water.} \end{cases}$$

12.68. (a) $D = (1 - e^{-\lambda t}) P_0/\lambda = 1.3p$; λ is the decay constant; (b) 1.2 h.

12.69. 1.8 μR/s.

12.70. 2.5 m.

12.71. $K_\gamma = 192 \sum w_i (\tau/\rho)_i E_i$, R/h, where w_i is the fraction of γ-quanta with energy E_i, MeV; τ/ρ, cm²/g; (a) 18; (b) 1.3; (c) 7 R/h.

12.72. $P = \dfrac{\tau J E}{2\pi l R} \arctan \dfrac{l}{2R} = 2.5$ μR/s.

12.73. $P = 0.25\, \tau A E \ln [1 + (R/h)^2] = 0.18$ R/s.

12.74. $d = \dfrac{1}{\mu} \ln \dfrac{D}{D_{mp}} = 2$ cm; μ is the linear attenuation coefficient.

12.75. Due to spherical symmetry the number of scattered quanta leaving any elementary solid angle will be counterbalanced by quanta getting to the given point as a result of scattering from other solid angles. We can therefore assume that the beam attenuates due to true absorption only. Thus $P = \tau' \dfrac{A\eta E}{4\pi r^2} e^{-\tau \Delta r}$; $\Delta r = 1.7$ cm. Here τ' and τ are the linear attenuation coefficients in air and lead.

12.76. P (s⁻¹) $= 1.14 \cdot 10^{-3} \dfrac{\tau P_0}{\tau' \rho} e^{-\mu l} = 0.07$ s⁻¹. Here τ and τ' are the linear absorption coefficients in lead and air, cm⁻¹; μ is the linear attenuation coefficient in lead, ρ is the density of lead, g/cm³; P_0 is the dose rate, R/s.

12.77. 2.0 m.

12.78. $D = J N_A \sigma T f t / A = 0.09$ rad.

12.79. $7 \cdot 10^7$ particles.

12.80. 0.3 rad = 3 rem.

12.81. $D = (\mu/\rho) J \langle T \rangle t = 5$ rad, where $\mu/\rho = 22/\Delta T_{\beta\,max}^{4/3}$.

13.1. $T = \frac{4m_1 m_2}{(m_1+m_2)^2} T_0 \cos^2 \vartheta = 0.7$ MeV.

13.2. (a) $T_\alpha = \frac{(e\rho B)^2}{m_\alpha c^2}\left(2 + \frac{m_\alpha}{2m_d}\right) = 026.$ MeV;

(b) $T_\alpha = T_d\left[1 + \frac{(m_\alpha - m_d)^2}{4m_\alpha m_d \cos^2 \theta}\right] = 0.6$ MeV.

13.3. $m = m_d/(4\cos^2\vartheta - 1) = m_d/2$, the nucleus of a hydrogen atom.

13.4. The function is not single-valued if the mass of striking particle exceeds the mass of the stationary nucleus (the case (c)). In the cases (a) and (b) $\vartheta_{max} = \pi$, in the case (c) $\vartheta_{max} = \arcsin(m_d/m_\alpha) = 30°$.

13.5. $\frac{\Delta T}{T} = \frac{4m_1m_2}{(m_1+m_2)^2}\sin^2(\tilde{\vartheta}/2) = 0.19$.

13.6. 0.10 MeV.

13.7. 49°.

13.8. (a) $\tan\vartheta = \frac{\sin\tilde{\vartheta}}{m_\alpha/m_{Li} + \cos\tilde{\vartheta}}$; $\vartheta \approx 18°$; (b) $\sin\tilde{\vartheta} = \sin\vartheta\,(\eta\cos\vartheta \pm \sqrt{1-\eta^2\sin^2\vartheta})$, where $\eta = m_\alpha/m_{Li}$. The minus sign in front of the radical has no physical meaning: here $\sin\tilde{\vartheta}$ cannot be negative; $\tilde{\vartheta} = 73°$.

13.9. $T' = T/3 = 0.10$ MeV; $\vartheta_{max} = \arcsin(m_p/m_d) = 30°$.

13.10. $Q = +17.3$ MeV.

13.11. (a) +19.8 MeV; (b) −3.1 MeV; (c) −13.5 MeV; (d) +1.8 MeV.

13.12. 17.00845 a.m.u.

13.13. $v_\alpha = 9.3\cdot10^6$ m/s; $v_{Li} = 5.3\cdot10^6$ m/s.

13.14. Ignoring the momentum of γ-quantum, we get $T \approx (8/9)(\hbar\omega - |Q|) = 115$ keV.

13.15. $E = \hbar\omega - (e\rho B)^2/mc^2 = 2.23$ MeV.

13.16. (a) $Q = \frac{4}{3}T_p - \frac{1}{3}T_d = 4.0$ MeV;

(b) $Q = \frac{18}{17}T_p - \frac{13}{17}T_\alpha - \frac{4}{17}\cos\vartheta\sqrt{T_pT_\alpha} = -1.2$ MeV.

13.17. 5.5 MeV.

13.18. (a) 140.8°; (b) 144.5°.

13.19. Two methods of solution of this problem are given below. 1. Write the laws of conservation of energy and momentum for the threshold value of kinetic energy of the striking particle: $p_m = p_{m+M}$; $T_{th} = |Q| + T_{m+M}$. Solving these equations, we find the sought expression. 2. In the C frame the threshold value of the total kinetic energy of interacting particles is $\tilde{T}_{th} = |Q|$. But $\tilde{T}_{th} = \frac{\mu v^2}{2} = \frac{\mu}{m} T_{th}$. Now the expression for T_{th} is easily obtained.

13.20. (a) 4.4 MeV; (b) 18.1 MeV; (c) 6.2 MeV; (d) 0.

13.21. (a) 1.02 MeV; (b) 3.05 MeV.

13.22. (a) $T_{Be} = \frac{1}{8}|Q| = 0.21$ MeV; (b) $T_O = \frac{3}{76}|Q| = 1.41$ MeV.

13.23. 0.68 MeV.

13.24. $T_{min} = \frac{11}{7}\frac{q_1q_2}{R} \approx 2.8$ MeV, where R is the sum of radii of a Li nucleus and an α-particle. This energy is less than the threshold one ($T_{th} = 4.4$ MeV), i.e. is insufficient to activate the reaction.

13.25. The total energies of the direct and reverse processes are equal in the C frame (see Fig. 37) under the condition $\tilde{T} = \tilde{T}' + |Q|$, Q being the reaction energy (here $Q < 0$). Expressing $\tilde{T}$, $\tilde{T}'$, and $|Q|$ via T, T_d, and T_{th} respectively, we obtain $T_d = (m_B/m_{Be})(T - T_{th}) = 5.7$ MeV.

13.26. $\tilde{p} = \sqrt{2\mu'\left(\frac{\mu}{m}T_m + Q\right)}$, where μ and μ' are the reduced masses of particles before and after the reaction.

13.27. $\tilde{p} = 0.566 p_p$; 0.18 or 0.9 MeV.

13.28. $\tilde{p} = 1.95\, p_d$. From the vector diagram of momenta, we find $\tilde{p}_{\alpha\,max} = \tilde{p} + p_d \frac{m_\alpha}{m_\alpha + m_\alpha}$; $T_{\alpha\,max} = 4.7$ MeV.

13.29. $\tilde{p} = 0.431 p_\alpha$. From the vector diagram of momenta, it follows that

$$\tilde{p}_n = \tilde{p} \pm p_\alpha m_n/(m_n + m_N),$$

where the plus and minus signs refer correspondingly to the maximum and minimum values of neutrons' momentum. Thus, $T_{n\,max} = 5.0$ MeV, $T_{n\,min} = 2.7$ MeV.

13.30. (a) 5.7; 2.9 and 1.5 MeV; (b) the vector diagram of momenta shows that it happens if

$$\tilde{p} \leqslant p_\alpha \frac{m_n}{m_n + m_B}. \text{ Then } T_\alpha \geqslant 4.65 \text{ MeV}.$$

13.31. Ignoring the kinetic energy of slow neutrons we find first the kinetic energy of produced tritium nuclei:

$$T_t = Qm_\alpha/(m_\alpha + m_t) = 2.75 \text{ MeV}.$$

Then we use the vector diagram of momenta.

(a) $\tilde{p} = 1.35\, p_t$; $p_{n\,max} = \tilde{p} + \frac{m_n}{m_n + m_\alpha} p_t = 1.55 p_t$; $T_{n\,max} = 19.8$ MeV.

(b) $\tilde{p} = 1.16 p_t$; $p_{n\,max} = 1.26 p_t$; $T_{n\,max} = 13.1$ MeV.

13.32. From the vector diagram of momenta for the first reaction find the maximum and minimum values of momenta of tritium nuclei: $3.07p$ and $2.21p$, where p is the momentum of incoming neutron. Then from the vector diagrams of momenta for the second reaction (at maximum and minimum values of triton's momentum) find the maximum and minimum values of the momentum of the produced neutron and corresponding values of kinetic energy: 21.8 and 11.0 MeV.

13.33. **(a)** From the vector diagram of momenta it follows that

$$\sin \vartheta_{B\ max} = \frac{10}{9}\frac{p}{p_p} = 0.70;\quad \vartheta_{B\ max} = 44.5°.$$

The angle of emission of the neutron may have any value (from 0 to π).

(b) 46.5° d; 29° (H^3).

13.34. **(a)** First find the angle $\tilde{\vartheta}_0$ in the C frame corresponding to the angle $\vartheta_0 = \pi/2$ in the L frame. From the vector diagram of momenta it follows that $\cos \vartheta_0 = (4/13)\, p_n/\tilde{p} = 0.46$, where p_n is the neutron's momentum in the L frame, $\tilde{p}$ is the momentum of reaction products in the C frame. The sought probability

$$w = \frac{1}{4\pi}\int 2\pi \sin\tilde{\vartheta}\, d\tilde{\vartheta} = \frac{1-\cos\tilde{\vartheta}_0}{2} = 0.27.$$

(b) 137°.

13.35. From the conservation laws of energy and momentum for the threshold value of energy of γ-quantum, we have

$$\hbar\omega_{th} + Mc^2 = \sqrt{M'^2c^4 + (\hbar\omega)^2_{th}},$$

where M' is the sum of rest masses of appearing particles. Hence,

$$\hbar\omega_{th} = \frac{M'^2 - M^2}{2M}c^2 = \left(1 + \frac{|Q|}{2Mc^2}\right)|Q|.$$

13.36. $T_n \approx m_n Q^2/2M^2c^2$, where m_n and M are the masses of the neutron and disintegrating nucleus; Q is the reaction energy. **(a)** 0.68 keV; **(b)** 0.58 keV.

13.37. Using the invariant $E^2 - p^2c^2$, write

$$(\hbar\omega + Mc^2)^2 - (h\omega)^2 = [(m_1 + m_2)c^2 + \tilde{T}']^2,$$

where $\tilde{T}'$ is the total kinetic energy of the reaction products in the C frame. Thus,

$$\tilde{T}' = -(m_1 + m_2)c^2 + \sqrt{M^2c^4 + 2Mc^2\hbar\omega} \approx Q + \hbar\omega.$$

Taking into account that $\tilde{T}' = \tilde{p}^2/2\mu'$, we obtain the sought expression.

13.38. Resorting to the vector diagram of momenta, find the angle $\tilde{\vartheta}_d$ in the C frame corresponding to the angle $\vartheta_d = \pi/2$ in the L frame from the formula $\cos\tilde{\vartheta}_d = \frac{p_\gamma}{\tilde{p}}\frac{m_d}{m_d + m_n}$, where $\tilde{p} = \sqrt{2\mu'(\hbar\omega + Q)}$. The sought probability is equal to

$$w = \frac{1}{4\pi}\int 2\pi \sin\tilde{\vartheta}\, d\tilde{\vartheta} = \frac{1-\cos\tilde{\vartheta}_d}{2} = 0.34.$$

13.39. Suppose $\mathbf{p}'_n$ and $\mathbf{p}_n$ are the momenta of a nucleon arising from its motion within a deuteron and together with a deuteron. Then the nucleon defects through the maximum angle $\theta = \Delta\theta/2$ (from the direction of the primary deuteron beam) under condition that at the stripping moment $\mathbf{p}'_n \perp \mathbf{p}_n$. Therefore $\tan\theta = p'_n/p_n = \sqrt{2T'_n/T_d}$. Now we can find T'_n, the kinetic energy of the internal motion of nucleon within a deuteron. The emerging neutrons possess the kinetic energy

$$T_n = \frac{(\mathbf{p}_n + \mathbf{p}'_n)^2}{2} \approx \frac{T_d}{2} + \frac{\mathbf{p}_n\mathbf{p}'_n}{m},$$

where m is the mass of the nucleon. Thus the maximum spread of neutron energies

$$\Delta T_n = \pm p_n p'_n/m = \pm\sqrt{2T_dT'_n} \approx \pm 27\ \text{MeV}.$$

13.40. $I(^{17}O) = I(^{16}O) + l_n + s_n = 0 + 2 \pm 1/2 = 5/2,\ 3/2$. According to the shell model $I = 5/2$.

13.41. **(a)** The spin of the compound nucleus $\mathbf{I} = \mathbf{s}_p + \mathbf{l} + \mathbf{I}_{Li}$, and parity $P = P_p P_{Li}(-1)^l$. Hence

l	I	P	States of $^8Be^*$
0	2, 1	−1	2^-, 1^-
1	3, 2, 1, 0	+1	3^+, 2^+, 1^+, 0^+

(b) A system of two α-particles has the positive parity since the system is described by an even wave function; consequently, $P_{2\alpha} = P^2_\alpha(-1)^{l_\alpha} = +1$, $l_\alpha = 0, 2, 4, \ldots$. From the law of conservation of angular momentum $l_\alpha = I$, whence $I = 0$ and 2. Thus, the reaction can proceed via branch (1) through two states of the compound nucleus: 2^+ and 0^+, provided $l = 1$. The emission of a dipole γ-quantum is accompanied with the change of parity and change of nuclear spin by unity. But inasmuch as the ^8Be nucleus possesses the spin and parity equal to 0^+ in the ground state the emission of the dipole γ-quantum occurs from the compound nucleus in the state l^- when $l = 0$. The emission of a quadrupole γ-quantum involves no change in parity while changing the nuclear spin by 2. That is why this process proceeds from the state 2^+ of the compound nucleus, when $l = 1$.

13.42. $E = \hbar\omega\,(1 - \hbar\omega/2Mc^2)$.

13.43. $E = E_b + \frac{3}{4} T_p = 21.3$ MeV, E_b is the binding energy of a proton in ^{4}He nucleus.

13.44. $T_{\min} = \frac{10}{9} E_{\text{exc}} = 2.67$ MeV.

13.45. $E_{\text{exc}} = \frac{6}{7} T_0 - \frac{8}{7} T = 0.48$ MeV.

13.46. $T_i = \frac{19}{21} T_0 - \frac{20}{21} E_i^* = 2.5$ and 1.8 MeV.

13.47. $T_n = \frac{17}{16} (E_{\text{exc}} - E_b) = 0.42$; 0.99 and 1.30 MeV. Here E_b is the binding energy of a neutron in a ^{17}O nucleus.

13.48. 16.67; 16.93; 17.49 and 17.71 MeV.

13.49. 2.13; 4.45 and 5.03 MeV.

13.50. $J_{0.84} : J_{1.02} = 1 : 0.8$.

13.51. $\sigma_{ab} = \sigma_a \Gamma_b / \Gamma$.

13.52. $\tau = \tau_n \tau_\alpha / (\tau_n + \tau_\alpha) = 0.7 \cdot 10^{-20}$ s.

13.53. $\tau = 2e/\sigma J = 4 \cdot 10^{10}$ s.

13.54. $4 \cdot 10^2$ neutrons/(cm$^2 \cdot$s).

13.55. $3 \cdot 10^{12}$ neutrons/s; $1.5 \cdot 10^2$ kg.

13.56. (a) $R = \sigma n_0 VJ = 2 \cdot 10^9$ s^{-1}, where σ is the effective reaction cross-section, n_0 is Loschmidt's number.

(b) 0.9 mW.

13.57. $\sigma_2 = \sigma_1 w_2 / w_1 = -0.10$ b, where w is the yield of the reaction.

13.58. $\sigma = w / n_0 d = 0.05$ b, where n_0 is the number of nuclei in 1 cm^3.

13.59. 1.8 b.

13.60. $2 \cdot 10^4$ b.

13.61. $\sigma = \frac{1}{J\tau} \ln \frac{a_1 (1 - a_1')}{a_2 a_2'} = 3.9 \cdot 10^3$ b where τ is the exposure time, a_1 and a_1' is the fractional content of ^{10}B nuclide at the beginning and by the end of the exposure, a_2 is the fractional content of ^{11}B nuclide prior to exposure.

13.62. $w = 1 - e^{-n_0 \sigma d} = 0.8$, where n_0 is the number of nuclei in 1 cm^3 of the target.

13.63. $A = \lambda N = \frac{\ln 2}{T} Jw\tau = 4\,\mu$Ci.

13.64. As a result of the prolonged exposure, the number of radioactive nuclei produced per unit time becomes equal to the number of disintegrating nuclei; $w = Ae^{\lambda\tau}/J \approx 1.5 \cdot 10^{-3}$.

13.65. $\sigma = Ae^{\lambda t}/Jn\ (1 - e^{-\lambda t}) = 0.02$ b, where n is the number of nuclei per 1 cm^2 of the target's surface.

13.66. $w = 1.0 \cdot 10^{-5}$; $\langle\sigma\rangle = w / n_0 L = 0.046$ b where n_0 is the number of nuclei in 1 cm^3 of the target, L is the range of α-particles with the given energy in aluminium.

13.67. 0.54 b.

13.68. $\langle\sigma\rangle = w / n_0 l = 0.10$ b, where n_0 is the number of nuclei in 1 cm^3 of the target, l is the thickness of the target within whose limits the given nuclear reaction is feasible, $l = l_1 - l_2$, l_1 and l_2 are the ranges of α-particles with an energy of 7 MeV and $T_{\text{th}} = 4.39$ MeV, respectively.

13.69. 7.5 mb. See the solution of the foregoing problem.

13.70. $d = -(\ln 0.9)/n\sigma = 1.7$ cm; n is the concentration of Be nuclei.

13.71. The total yield per one particle:

$$w(T) = \int_0^R n_0 \sigma(x)\, dx = \int_0^T n_0 \sigma(T) \frac{dT}{dT/dx}.$$

Differentiating this expression with respect to T, we find

$$\sigma(T) = \frac{1}{n_0} f(T) \frac{dw}{dT}.$$

13.72. Make use of the following relations: $\tilde{p}_1^2 = 2\mu_1 \tilde{T}_1$; $\tilde{p}_2^2 = 2\mu_2 \tilde{T}_2$, and $\tilde{T}_2 = \tilde{T}_1 + Q$, where μ_1 and μ_2 are the reduced masses of the systems $d + d$ and and $n + {}^3$He, respectively; $\tilde{p}_1$, $\tilde{T}_1$ and $\tilde{p}_2$, $\tilde{T}_2$ are the momenta and total kinetic energies of interacting particles in the C frame in the direct and reverse processes, respectively. Using the detailed balancing principle, we obtain

$$2I_{\text{He}} + 1 = \frac{9}{2} \frac{\sigma_1}{\sigma_2} \cdot \frac{m_d^2}{m_n m_{\text{He}}} \cdot \frac{T}{T + 2Q} = 2.0, \text{ whence } I_{\text{He}} = 1/2.$$

13.73. $\sigma_1 = \frac{8}{3} \frac{m_p m_{\text{Be}}}{m_\alpha m_{\text{Li}}} \cdot \frac{T - T_{\text{th}}}{T} \sigma_2 = 2.0\ \mu$b.

13.75. Taking into account that $\tilde{p}_\gamma \approx \hbar\omega/c$ and $\tilde{p}_n^2 = 2\mu(\hbar\omega + Q)$, we get

$$\sigma_2 = \frac{3}{2} \frac{(\hbar\omega)^2}{(\hbar\omega + Q)\, m_n c^2} = 3.6\ \mu\text{b}:$$

$$T_n = 2\,(h\omega + Q) = 0.96 \text{ MeV}.$$

14.1. $T = 2\pi^2 n^2 L^2 m / \alpha^2 = 0.03$ eV.

14.2. $1.6 \cdot 10^2$ s^{-1}; 0.03 μs.

14.3. $\Delta T/T = 2.77 \cdot 10^{-2} \sqrt{T_{\text{eV}}}\,(\Delta\tau/L)$ μs/m; $\Delta T/T = 6.2 \cdot 10^{-2}$; $T_{\max} = 13$ eV.

14.4. Not acceptable.

14.5. 20 m.

14.6. 0.4 and 1.6 eV.

14.7. $\Delta T/T = 2 \cot\vartheta\, \Delta\vartheta \approx 5\%$. Here $\sin\vartheta = \pi\hbar/d\sqrt{2mT}$.

14.8. $\Delta\theta \leqslant 0.1°$.

14.9. $T < \pi^2\hbar^2/2md^2 = 1.8 \cdot 10^{-3}$ eV.

14.10. (a) 5.4 MeV; (b) 6%.

14.11. 0; 6.4 and $11.2 \cdot 10^{-15}$ m.
14.12. $5.0 \cdot 10^{-15}$ m.
14.13. Consider the neutrons with orbital moment l and aiming parameter b_l. The geometric nuclear cross-section can be represented for them as a ring with mean radius b_l. The area of this ring

$$\Delta S_l = \frac{\pi}{2}(b_{l+1}^2 - b_{l-1}^2) = (2l+1)\pi\lambda\!\!\!^{-2}.$$

The highest possible value of l is determined from the condition $b_{l\,max} \leqslant R$, R being the nuclear radius. Hence, $l_{max} \approx R/\lambda\!\!\!^{-}$ and

$$S = \sum_{l=0}^{R/\lambda\!\!\!^{-}} \Delta S_l \approx \pi(R+\lambda\!\!\!^{-})^2; \quad R_{Au} = 2.9 \text{ b}.$$

14.14. $B = \hbar^2 l(l+1)/2mR^2 = 5.3$ MeV, where $l = 3$, m is the neutron mass, R is the nuclear radius. See the solution of Problem 11.31.
14.15. Under the condition $\lambda\!\!\!^{-} \ll R$, we have $\vartheta \propto \lambda\!\!\!^{-}/R = 4.5°$, R being the nuclear radius.
14.16. In this case the interaction of a slow neutron ($l = 0$) with a nucleus of target may induce $(2s+1)(2I+1)$ different ways of formation of the compound nucleus (s is the neutron's spin). Since the degeneracy (statistical weight) of a state with the given J is equal to $2J+1$, the probability of formation of this state is

$$g = \frac{2J+1}{(2s+1)(2I+1)} = \frac{2J+1}{2(2I+1)} = \frac{2}{3}.$$

14.17. $\sigma_{nn} = \sigma_a \Gamma_n/\Gamma$; $\sigma_{n\gamma} = \sigma_a \Gamma_\gamma/\Gamma$ where σ_a is the cross-section of formation of the compound nucleus (see the Breit-Wigner formula).

14.18. $\sigma_{n\gamma} = \sigma_0 \sqrt{\frac{T_0}{T}} \cdot \frac{\Gamma^2}{4(T-T_0)^2+\Gamma^2}$.
14.19. 96 b.
14.20. $\Gamma_{n0}/\Gamma_\gamma = \frac{\sigma_{nn}}{\sigma_{n\gamma}} \sqrt{\frac{T_0}{T}} \approx 0.006$.
14.21. **(a)** From the condition $d\sigma_{n\gamma}/dT = 0$, we get

$$T_{min}^{max} = T_0[0.6 \pm \sqrt{0.16 - 0.05(\Gamma/T_0)^2}].$$

Whence it is seen that $T_{max} \approx T_0$ at $\Gamma \ll T_0$.
(b) $\sigma_0 < \sigma_{max}$ by 1.8%.
(c) $\Gamma/T_0 \geqslant 1.8$.
14.22. $\sigma_{min}/\sigma_0 = 0.87(\Gamma/T_0)^2$, where σ_{min} corresponds to $T_{min} = 0.2T_0$.
14.23. 0.12 eV.
14.25. **(a)** $\sigma_{nn} = 4\pi\lambda\!\!\!^{-2}_0 g(\Gamma_{n0}/\Gamma)^2 = 8 \cdot 10^3 \left(1 \pm \frac{1}{8}\right)$ b. In this case only s neutrons interact with nuclei.

(b) From the formula $\sigma_0 = 4\pi\lambda\!\!\!^{-2}_0 g\Gamma_{n0}/\Gamma$, we find the g factor, whence $J = 4$.
14.26. $\sigma_{nn}/\sigma_{geom} = 4\lambda\!\!\!^{-2}g/R^2 = 3.5 \cdot 10^3$; R is the nuclear radius.
14.27. $\Gamma_{n0} \approx \sigma_0\Gamma/4\pi\lambda\!\!\!^{-2}_0 g \approx 5 \cdot 10^{-4}$ eV.
14.28. $\tau \approx \sigma_0 m T_0/2\pi\hbar g\Gamma_{n0} \approx 4.4 \cdot 10^{-15}$ s; m is the neutron mass.
14.29. $\sigma_{n\gamma}(T) = \frac{2\pi\hbar^2 g}{m\sqrt{T|T_0|}} \cdot \frac{\Gamma_{n0}\Gamma_\gamma}{4(T+|T_0|)^2+\Gamma^2}$,

where m is the neutron mass; Γ_{n0} corresponds to the energy $T = |T_0|$. In the first case $\sigma_{n\gamma} \propto T^{-1/2}$, in the second case $\sigma_{n\gamma} \propto T^{-5/2}$.

14.30. Since $\sigma_{n\gamma} \propto \frac{T^{-1/2}}{4(T-T_0)^2+\Gamma^2}$ $\sigma_{n\gamma}$ is practically proportional to $T^{-1/2}$, if T is small as compared to the bigger of two values T_0 or Γ, and if $\Gamma_n \ll 1$ (in this case Γ is practically independent of T). If $T_0 < 0$, then T must be small as compared to $|T_0|$.

14.31. $\frac{\Gamma'}{\Gamma} = \frac{\sigma'}{\sigma_a} = \frac{\sigma_{tot} - \sigma_{el}}{\sigma_{tot} - 0.44\sigma_{el}} = 0.4$, where σ' is the inelastic scattering cross-section, σ_a is the cross-section of formation of the compound nucleus.
14.32. 0.40 mm.
14.33. About two times.
14.34. Attenuates by a factor of 2.3.
14.35. $J = I_0 e^{-\sigma n(r_2 - r_1)}/4\pi r_2^2 = 5$ neutrons per cm² per second, where n is the concentration of carbon nuclei.
14.36. 5 eV.
14.37. $w = 1 - e^{-\sigma n l} = 1.5\%$ where n is the concentration of nuclei, σ is the cross-section for the energy 10 eV (which is defined through the tabular data: $\sigma = \sigma_0 v_0/v$).

14.38. $w = \frac{\sigma_{n\alpha}}{\sigma_{tot}}(1 - e^{-\sigma_{tot} n l}) = 86\%$, where $\sigma_{tot} = \sigma_{n\alpha} + \sigma_{n\gamma}$; n is the concentration of LiI molecules.
14.39. The decrease in the number of ^{10}B nuclei with time: $-dn = Jn\sigma\,dt$. After integration, we get $n = n_0 e^{-J\sigma t}$, where n_0 is the initial number of nuclei. The fractional decrease in efficiency of the detector is $\Delta w/w = \Delta n/n = 1 - e^{-J\sigma t} \approx J\sigma t = 2.3\%$.
14.40. From the formula $R = JN\langle\sigma\rangle$, where J is the neutron flux and N the number of nuclei per unit area of the target's surface, we obtain

$$\langle\sigma(v)\rangle = \frac{R/N}{J} = \frac{\int \sigma(v) v n(v)\,dv}{\int v n(v)\,dv} = \frac{\sigma_0 v_0}{\langle v\rangle} = \sigma(\langle v\rangle).$$

where it is taken into account that $\sigma v = \sigma_0 v_0$.
14.41. The yield of the reaction is equal to the ratio of the reaction rate to the neutron flux:

$$w = \frac{R_{n\alpha}}{J} = \frac{\int N\sigma(v) v n(v)\,dv}{n\langle v\rangle} = \frac{N\sigma_0 v_0}{\langle v\rangle},$$

where N is the number of nuclei per sq·cm of the target. Thus $\langle v \rangle = 5 \cdot 10^3$ m/s.

14.42. Taking into account that $\sigma v = \sigma_0 v_0$, we obtain the expression giving the number of reactions per 1 s: $R = N\sigma_0 v_0 n$, where N is the number of boron nuclei within the counter's volume; n—concentration of neutrons. **(a)** $2 \cdot 10^6$ cm^{-3}; **(b)** $R = N\sigma_0 v_0 \Phi / \langle v \rangle = 2 \cdot 10^{10}$ s^{-1}.

14.43. Use the symbols $*$ and $\|$ to mark the quantities relating to isotropic and parallel neutron fluxes. Then the rate of the nuclear reaction activated by neutrons falling on 1 cm^2 area of the target's surface at the angles $(\vartheta, \vartheta + d\vartheta)$ to its normal, is $dR_* = dJ_* \sigma n l / \cos\vartheta$, where σ is the reaction cross-section, n is the concentration of nuclei in the target, l is the thickness of the target, dJ_* is the number of neutrons falling on 1 cm^2 area of the target in the angular interval $(\vartheta, \vartheta + d\vartheta)$. Since $dJ_* = a \cos\vartheta \cdot 2\pi \sin\vartheta\, d\vartheta$, where a is the constant determined from the condition of the problem: $J_\| = J_* = 2\int dJ_*(\vartheta) = 2\pi a$, then $R_* = \int dR_*(\vartheta) = 2J_\| \sigma n l = 2R_\|$.

14.44. $\tau = \dfrac{\ln(N_0/N)}{\sigma_0 v_0 n} = 24$ days.

14.45. (a) $A_{max} = \Phi\sigma N_0 = 1.5$ μCi; $N_{max}/N_0 = \Phi\sigma/\lambda = 4 \cdot 10^{-10}$, where N_0 is the number of atoms in the sample;
(b) $t = -\dfrac{1}{\lambda} \ln\left(1 - \dfrac{A}{A_{max}}\right) = 2T = 30$ hours.

14.46. $\Delta t = \dfrac{1}{\lambda} \ln \dfrac{q - A}{q - \eta A} = 1.7$ days; q—$\Phi\sigma n$; n is the number of atoms in 1 g of the foil.

14.47. $A = \Phi N\,[0.69\sigma_1 (1 - e^{-\lambda_1 t}) + 0.31\sigma_2 (1 - e^{-\lambda_2 t})] = 0.2$ Ci/g, where N is the number of atoms in 1 g of copper, σ_1, λ_1, and σ_2, λ_2 are the activation cross-sections and decay constants of ^{63}Cu and ^{65}Cu respectively.

14.48. $\Phi = A e^{\lambda t} / \sigma N_0 (1 - e^{-\lambda t}) = 3 \cdot 10^7$ cm^{-2}·s^{-1}; where N_0 is the number of nuclei in the foil.

14.49. $n = \lambda N / \beta N_0 \sigma_0 v_0 (1 - e^{-\lambda\tau}) = 0.7 \cdot 10^5$ cm^{-3}.

14.50. The saturation activity of the naked foil is $A = A_T + A_{aT}$, where A_T and A_{aT} are the saturation activities for thermal and abovethermal neutrons. Since $R_{Cd} = A/A_{aT}$, then

$$A_{Sat} = A_{aT} = \frac{1}{R_{Cd} - 1} = \frac{N\sigma_0 v_0 n}{R_{Cd} - 1} = 8 \text{ μCi/g},$$

where N is the number of nuclei in 1 g of the foil.

14.51. (a) $\eta = 4\varkappa/(1+\varkappa)^2$; correspondingly 0.89, 0.284, and 0.0167; **(b)** $\eta = \dfrac{2\varkappa}{(1+\varkappa)^2}\,[1 + \varkappa\sin^2\vartheta - \cos\vartheta\sqrt{1 - \varkappa^2\sin^2\vartheta}] = 0.127$; 2/3 and 0.87. Here $\varkappa = 1/A$, A being the mass number of stationary nucleus.

14.52. (a) $T = \dfrac{1 + A^2 + 2A\cos\tilde{\vartheta}}{(1+A)^2}\, T_0$; **(b)** the fraction of neutrons scatterend into the angular interval $(\tilde{\vartheta}, \tilde{\vartheta} + d\tilde{\vartheta})$ is $d\eta = 1/2 \sin\tilde{\vartheta}\, d\tilde{\vartheta}$. These neutrons possess energies lying in the interval dT that can be found from the formula given in the foregoing item **(a)**. Thus, $d\eta = \dfrac{(1+A)^2}{2AT_0}\, dT$ (see Fig. 72), where $\alpha = \left(\dfrac{A-1}{A+1}\right)^2$.

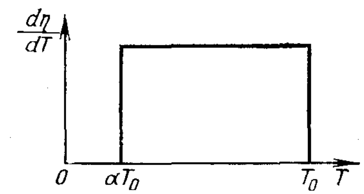


Fig. 72

14.53. $\langle T \rangle = \int\limits_0^\pi T(\tilde{\vartheta})\, d\eta(\tilde{\vartheta}) = \dfrac{1 + A^2}{(1+A)^2}\, T_0 = 0.68$ MeV, where $T(\tilde{\vartheta})$ and $d\eta(\tilde{\vartheta})$ are the expressions cited in the solution of the foregoing problem.

14.54. $\eta = (6A - A^2 - 1)/8A = 0.44$; A is the mass number of the nucleus.

14.55. (a) $\sin 2\vartheta\, d\vartheta$; **(b)** 0.25; **(c)** 45°.

14.56. (a) From the vector diagram of momenta (see Fig. 3), we find for the triangle ABO:

$$\frac{\sin(\tilde{\vartheta} - \vartheta)}{\sin\vartheta} = \frac{p_m}{\tilde{p}} \cdot \frac{1}{A+1}, \quad \text{where } \tilde{p} = \frac{A}{A+1}\, p_m.$$

The remaining part of the proof is obvious.

(b) The fraction of neutrons scattered into the angular interval $(\tilde{\vartheta}_1, \tilde{\vartheta}_2)$ in the C frame is

$$\eta = \frac{1}{2}\int \sin\tilde{\vartheta}\, d\tilde{\vartheta} = \frac{1}{2}(\cos\tilde{\vartheta}_1 - \cos\tilde{\vartheta}_2).$$

In the considered case $\tilde{\vartheta}_2 = \pi$, and the angle $\tilde{\vartheta}_1$ is related to the angle ϑ_1 as shown in the condition of the problem. Thus, $\eta = (A - 1)/2A \approx 0.45$.

(c) According to the condition $\langle\cos\vartheta\rangle = \dfrac{1}{2}\int \cos\vartheta \sin\tilde{\vartheta}\, d\tilde{\vartheta}$. Replacing $\cos\vartheta$ by the expression from the text of the problem and substituting $x = 1 + A^2 + 2A\cos\tilde{\vartheta}$, we obtain the sought result after integration.

14.57. $\langle \cos \vartheta_{BeO} \rangle = \left(\frac{\Sigma_{s\,Be}}{\Sigma_{s\,BeO}} \right) \langle \cos \vartheta_{Be} \rangle + \left(\frac{\Sigma_{s\,O}}{\Sigma_{s\,BeO}} \right) \langle \cos \vartheta_O \rangle = 0.06.$

14.58. **(a)** According to the definition, $\xi = \left\langle \ln \frac{T_0}{T} \right\rangle = \int \ln \frac{T_0}{T}\, d\eta\,(T)$, where $d\eta$ is the probability that the energy of the neutron after its single scattering falls within the interval $(T, T + dT)$. Replacing $d\eta$ with the expression given in the solution of Problem 14.52 and integrating with respect to T from αT_0 to T_0, we obtain the sought formula. When $A \gg 1$, $\xi \approx 2/A$.
(b) In graphite 0.158: in heavy water

$$\langle \xi \rangle = \frac{\xi_D \Sigma_{sD} + \xi_O \Sigma_{sO}}{\Sigma_{s\,tot}} = 0.6.$$

14.59. $z = \frac{1}{\xi} \ln \frac{T_0}{T}$; $2.2 \cdot 10^3$; $1.2 \cdot 10^2$ and 31.

14.60. The neutron moderated down to velocity v experiences $v\,dt/\lambda_s$ collisions per time interval dt. If the mean logarithmic loss of neutron's energy equals ξ, then $-d \ln T = \xi \frac{v\,dt}{\lambda_s}$;

$$t = \frac{\lambda_s \sqrt{2m}}{\xi} \left(\frac{1}{\sqrt{T_t}} - \frac{1}{\sqrt{T_0}} \right) = 5 \cdot 10^{-5}.$$

14.61. $\tau = \frac{\ln (T_0/T_t)}{3\xi \Sigma_s^2 (-\langle \cos \vartheta \rangle)} = 2.6 \cdot 10^2\ \text{cm}^2$; $L = \sqrt{\tau} = 16$ cm.

14.62. Consider a thin spherical layer with radii r and $r + dr$ with the point source of neutrons at its centre. The number of neutrons crossing the given energy level in that layer is equal to $4\pi r^2\ drq$. Then

$$\langle r^2 \rangle = \frac{1}{n} \int r^2 q 4\pi r^2\, dr = 6\tau.$$

14.63. Since the activity A is proportional to the moderation density q, $\ln A$ must be a linear function of r^2. Plotting the graph of this function, we find from the slope of the straight line $(-1/4\tau)$ that $\tau = 3.1 \cdot 10^2\ \text{cm}^2$.

14.64. $J_{res} = \int \sigma(T) \frac{dT}{T} = 0.5\sigma_0$, where $\sigma(T) = \sigma_0 \sqrt{T_0/T}$.

14.65. **(a)** $dw = \Sigma_s e^{-\Sigma_s x}\, dx$; $\lambda_s = \int x\, dw = 1/\Sigma_s$; **(b)** $\langle x^2 \rangle = 2/\Sigma_s^2 = 13\ \text{cm}^2$.

14.66. $\tau = \lambda_a/v = 1/v\Sigma_a = 0.015$ s; $z = \sigma_s/\sigma_a = 1.3 \cdot 10^3$.

14.67. **(a)** $\lambda_{tr} = [\Sigma_s (1 - \langle \cos \vartheta \rangle)]^{-1} = 2.8$ cm; **(b)** 55 cm and 33 m.

14.68. **(a)** Consider the elementary ring layer (see Fig. 54) of volume dV whose points are located at the distance r from the area dS. The number of neutrons scattered in this layer and reaching the area dS is

$$dJ = (\Phi \Sigma_s\, dV)\, (dS \cos \vartheta / 4\pi r^2)\, e^{-\Sigma_s r},$$

where in the first parentheses is the number of scattering collisions per second in the volume dV; in the second parentheses is the probability that a neutron after scattering in the volume dV moves toward the area dS; the exponential function describes the probability that a neutron covers the distance r without collisions. Thus,

$$J = \int \frac{1}{2} \Phi \Sigma_s e^{-\Sigma_s r} \sin \vartheta \cos \vartheta\, d\vartheta\, dr = \frac{\Phi}{4}.$$

(b) Reasoning as in the foregoing item (a) and taking into account that in this case $\Phi = \Phi_0 + (\partial\Phi/\partial n)_0\, r \cos \vartheta$, we obtain

$$J_+ = \frac{\Phi_0}{4} - \frac{1}{6\Sigma_s} \left(\frac{\partial \Phi}{\partial n} \right)_0; \quad J_- = \frac{\Phi_0}{4} + \frac{1}{6\Sigma_s} \left(\frac{\partial \Phi}{\partial n} \right)_0,$$

where J_+ and J_- are the number of neutrons crossing the 1 cm² area per second in the positive and negative directions of the x axis. The resulting neutron flux is $j = J_+ - J_- = -\frac{1}{3\Sigma_s} \left(\frac{\partial \Phi}{\partial n} \right)_0$. In the case of anisotropic scattering in the L frame Σ_s is replaced by Σ_{tr}.

14.69. **(a)** In the case of steady-state distribution the diffusion equation takes the form: $D\Phi'' - \Sigma_a \Phi = 0$. Its solution is $\Phi(x) = ae^{x/L} + be^{-x/L}$, where $L = \sqrt{D/\Sigma_a}$. The constants a and b are found from the boundary conditions: $a = 0$ for Φ must be finite when $x \to \infty$. To determine b, consider the plane parallel to the plane of the source and located at a small distance x from it. The neutron flux across this plane (see the solution of the foregoing problem) is $j = -D \frac{\partial \Phi}{\partial x} = D \frac{b}{L} e^{-x/L}$. When $x \to 0$, $j = n$. Thus, $b = nL/D$. Finally, $\Phi(x) = \frac{nL}{D} e^{-x/L}$.

(b) In this case $\Phi'' + \frac{2}{r} \Phi' - \frac{1}{L^2} \Phi = 0$, where $L^2 = D/\Sigma_a$. After the substitution $\chi = r\Phi$, we obtain the equation $\chi'' - \frac{1}{L^2} \chi = 0$. Its solution is $\chi(r) = ae^{r/L} + be^{-r/L}$, or $\Phi(r) = \frac{a}{r} e^{r/L} + \frac{b}{r} e^{-r/L}$. As in the previous case, $a = 0$. To determine b, surround the source with a small sphere of radius r and find the neutron flux across its surface:

$$j = -D \frac{d\Phi}{dr} = D \frac{b}{r^2} \left(\frac{r}{L} + 1 \right) e^{-r/L}.$$

When $r \to 0$, the total flux $4\pi r^2 j \to n$. Thus, $b = n/4\pi D$ and $\Phi(r) = \frac{n}{4\pi r D} e^{-r/L}$.

(c) $\Phi(r > R) = \frac{nR^2 L}{2D(R+L)} e^{-(r-R)/L}$, where $L = \sqrt{D/\Sigma_a}$.

14.70. Taking into account that in this case $\Phi(r) \sim \frac{1}{r} e^{-r/L}$ (see the solution of the foregoing problem), we get $L = \frac{r_2 - r_1}{\ln(\eta r_2/r_1)} = 1.6$ m.

14.71. Consider the point source of thermal neutrons. Separate a thin spherical layer of radii r and $r + dr$ with the source in the centre. The number of neutrons absorbed in such a layer per second is $dn = \Phi \Sigma_a 4\pi r^2\, dr$. If the source intensity is n, then $\langle r^2 \rangle = \int r^2\, dn/n$, where dn/n is the probability that the emitted neutron, having covered the distance r, gets captured in the layer r, $r + dr$. While integrating, the expression for $\Phi(r)$, given in the solution of Problem 14.69 (b), should be used.

14.72. $w_n = (1-\beta)\beta^{n-1}$; $\langle n \rangle = \frac{\sum n w_n}{\sum w_n} = \frac{\sum n\beta^{n-1}}{\sum \beta^{n-1}} = \frac{1}{n-\beta}$, where the summation is carried out over n running from 1 to ∞. While finding the sum in the numerator, we used the relation

$$1 + 2\beta + 3\beta^2 \ldots = \frac{\partial}{\partial\beta}(\beta + \beta^2 + \ldots) = \frac{\partial}{\partial\beta}\left(\frac{\beta}{1-\beta}\right).$$

14.73. If none of n neutrons, falling on the foil from either side per second penetrated it, $2n$ neutrons would fall on the foil. However, owing to the multiple reflections from nuclei of water molecules the number of neutrons crossing the foil per second must exceed $2n$. Suppose w is the probability of a neutron being absorbed while crossing the foil. Then each of the $2n$ neutrons falling on the foil penetrates it with the probability $1-w$ and reflects back with the probability β; therefore, the probability of the secondary fall of a given neutron on the foil is $\beta(1-w)$ and of the third fall $\beta^2(1-w)^2$, etc. The total number of impacts with allowance made for reflections in the medium is found to be

$$N = 2n[1 + \beta(1-w) + \beta^2(1-w)^2 \ldots] = \frac{2n}{1-\beta(1-w)}.$$

Taking into account that $N/n = 6.9$ and $w = \rho\, d\sigma_a N_A/A$, we obtain $\beta = 0.8$.

14.74. From the solution of Problem 14.68, it follows that the albedo $\beta = J_-/J_+ = [1 + 2D(d\ln\Phi/\partial n)_0]/[1 - 2D(\partial\ln\Phi/\partial n)_0]$, $D = \frac{1}{3\Sigma_{tr}}$, where the gradient of $\ln\Phi$ corresponds to the points lying at the interface between two media. Using this expression, we get

(a) $\beta = (1-2D/L)/(1+2D/L) = 0.935$, $D/L = \sqrt{\sigma_a/3\sigma_s(1-\langle\cos\vartheta\rangle)}$.
(b) $\beta = (1-\alpha)/(1+\alpha)$, where $\alpha = 2D(1/L + 1/R)$.

15.1. (a) $0.8\cdot10^{11}$ kJ, $2\cdot10^6$ kg; (b) $1.0\cdot10^2$ MW; (c) 1.5 kg.

15.2. $3\cdot10^{18}$ s^{-1}; 1.1 MW.

15.3. $Z^2/A \geqslant 15.7 - 367A^{-1.42}$.

15.4. 45.

15.5. $1\cdot10^{16}$ years; $4.5\cdot10^7$ α-decays.

15.6. (a) 196 MeV; (b) 195 MeV.

15.7. $E_{act} = T + E_b = 6.2$ MeV, where T is the kinetic energy of a neutron; E_b is its binding energy in a ^{239}U nucleus.

15.8. 0.65 and 2.0 MeV.

15.9. 4.1 b.

15.10. 90, 84, and 72%.

15.11. 2.28, 2.07, and 2.09.

15.12. 1.33 and 1.65.

15.13. Suppose that the neutron flux of density J_0 falls on the plate. The number of fission neutrons emerging per second in a thin layer of thickness dx and of an area of 1 cm^2 located at the distance x from the surface, is $\nu J e^{-n\sigma_a x} n\sigma_f\, dx$, where n is the concentration of nuclei, σ_a and σ_f are the absorption and fission cross-sections. Integrating this expression with respect to x between 0 and d (the plate's thickness), and equating the result to J_0, we obtain

$$d = -\frac{1}{n\sigma_a}\ln\left(1 - \frac{\sigma_a}{\nu\sigma_f}\right) = 0.20 \text{ mm}.$$

15.14. $W = QJ_0\frac{\Sigma_f}{\Sigma_a}(1 - e^{-\Sigma_a d}) = 0.03$ W/cm^2, where $Q = 200$ MeV, Σ_f and Σ_a are the macroscopic fission and absorption cross-sections.

15.15. $\nu = 1 + \frac{N_B\sigma_B}{N_{Pu}\sigma_{Pu}} = 2.1$, where N is the number of nuclei, σ is the absorption cross-section.

15.16. 9.5%.

15.17. The ratio of the number of neutrons of a certain generation to that of a previous generation is $N = 10^3 k^{i-1} = 1.3\cdot10^5$, where i is the number of generations.

15.18. The number of nuclei fissioned by the end of the nth generation in the process of the chain reaction is $1 + k + k^2 + \ldots + k^n = \frac{k^n - 1}{k-1} = \frac{M}{m}$, where M is the mass of fissioned material and m the mass of the nucleus. Taking into account that $n = t/\tau_f$, where t is the sought time interval and τ_f the time of existence of the neutron from its production till its absorption by a fissionable nucleus, we obtain $t = 0.3$ ms. When $k = 1.01$, $t \approx 0.03$ ms.

15.19. Each fission-triggering neutron produces a certain number of secondary neutrons that is equal to $\frac{2.6}{2}\cdot\frac{0.5}{\text{a few units}} < 1$, i.e., $k < 1$.

15.21. **(a)** $f = \frac{\eta\sigma_1 + (1-\eta)\sigma_2}{\eta\sigma_1 + (1-\eta)\sigma_2 + z\sigma_3} = 0.80$, where η is the fraction of ^{235}U nuclei in the natural uranium, σ_1, σ_2, and σ_3 are the absorption cross-sections of ^{235}U, ^{238}U, and carbon; **(b)** 4.8%.

15.22. 0.74.

15.23. $k = \varepsilon p f \eta = 1 \cdot 0.744 \cdot 0.835 \cdot 1.335 = 0.83$.

15.24. $1.0 \cdot 10^{12}$ neutrons/(cm^2·s); $4.6 \cdot 10^6$ cm^{-3}.

15.25. The accumulation rate of n-active nuclei with n-decay constant λ is $\dot{N} = qw - \lambda N$, where q is the number of fissions per second, w is the yield of n-active nuclei (of interest to us) per fission (which is the yield of delayed neutrons per fission). Integrating this equation with respect to time from 0 to τ, we get the number of n-active nuclei by the end of the exposure. After the time interval t following the end of the exposure, the number of these nuclei equals $N' = \frac{qw}{\lambda}(1 - e^{-\lambda\tau})\,e^{-\lambda t}$; $w = \frac{Ae^{\lambda t}}{q(1-e^{-\lambda\tau})} = 6 \cdot 10^{-4}$. Here $A = \lambda N$, $q = In\sigma_f$, where n is the number of nuclei in the foil.

15.26. $\tau = (v\Sigma_a)^{-1} \approx 0.7$ ms, where $v = 2200$ m/s, $\Sigma_a = 6.3 \cdot 10^{-3}$ cm^{-1}.

15.27. The mean delay time $\Delta\tau = \frac{\Sigma w_i \tau_i}{2.47 + \Sigma w_i} = 0.083$ s; $\tau_i = T_i/\ln 2$.

15.28. The increase in the number of neutrons during the lifetime of one generation is equal to $n(k-1)$. From the equation $dn/dt = n(k-1)/\tau$, we obtain $n = n_0 e^{(k-1)t/\tau}$, whence $T = 10$ s.

15.29. **(a)** $3.4 \cdot 10^8$ kJ; **(b)** $0.9 \cdot 10^8$ kJ; **(c)** $2.7 \cdot 10^8$ kJ. For uranium $0.8 \cdot 10^8$ kJ.

15.30. About 10^6 years.

15.31. $2.5 \cdot 10^9$ kJ.

15.32. Recalling that the C frame for interacting nuclei practically coincides with the L frame and that the kinetic energy of relative motion $\tilde{T} \ll Q$ (at 10^8 K $\tilde{T} \approx 10$ keV and Q is about 10 to 20 MeV), we obtain: **(a)** $E_n/Q \approx m_\alpha/(m_\alpha + m_n) = 0.8$; **(b)** $E_n/(Q_1 + Q_2) = 0.34$.

15.33. $\eta = (Q_{dt} + Q_{n\text{Li}})/0.2Q_{dt} = 6.4$.

15.34. $\theta \approx e^2/R = 7 \cdot 10^2$ keV.

15.35. **(a)** *Instruction.* In the general case in the initial formula for D one should replace m by the reduced mass μ of the interacting particles, and T by the kinetic energy of their relative motion $\tilde{T}$. Then see the solution of Problem 11.32. **(b)** $D \approx \exp(-31.3/\sqrt{\theta_{\text{keV}}})$; 10^{-14} and $5 \cdot 10^{-5}$.

15.36. $3 \cdot 10^{-12}$ and $6 \cdot 10^{-4}$ b.

15.37. $w = \frac{1}{2} nfQ_{dd} = 0.3$ W/cm^3; $\theta \propto 10$ keV.

15.38. The reaction rate is proportional to the product $nv\sigma$, where v is the relative motion velocity of interacting deuterons. In its turn, $nv\sigma \propto e^{-\tilde{T}/\theta} \times e^{-31.3/\sqrt{\tilde{T}}}$, where $\tilde{T}$ and θ, keV, θ being the plasma temperature. This expression has a maximum at $\tilde{T}_m = 6.25\theta^{2/3} = 10$ keV.

15.39. **(a)** $\tau = 1/n\langle\sigma v\rangle$; $6 \cdot 10^6$ and $1 \cdot 10^3$ s; **(b)** $R = \frac{1}{2} n^2 \langle\sigma v\rangle$; correspondingly, $0.8 \cdot 10^8$ and $5 \cdot 10^{11}$ cm^{-3}·s^{-1}; $5 \cdot 10^{-5}$ and 0.3 W/cm^3.

15.40. **(a)** $R = \frac{1}{2} n_d^2 \langle\sigma v\rangle_{dd} + n_d n_t \langle\sigma v\rangle_{dt}$; correspondingly, $1{,}5 \cdot 10^8$ and $1.1 \cdot 10^{12}$ cm^3·s^{-1}; **(b)** $2.2 \cdot 10^{-4}$ and 2.0 W/cm^3.

15.41. $n_t/n_d = 1 - \alpha$, where $\alpha = \langle\sigma v\rangle_{dd} Q_{dd}/\langle\sigma v\rangle_{dt} Q_{dt}$. At both temperatures $\alpha \ll 1$, and therefore $n_t \approx n_d$; $w_{\max} = \frac{n^2 \langle\sigma v\rangle_{dt} Q_{dt}}{2(2-\alpha)} \approx \frac{1}{4} n^2 \langle\sigma v\rangle_{dt} Q_{dt}$; correspondingly, $4.5 \cdot 10^{-3}$ and 43 W/cm^3. The contribution of dd reaction is negligible.

15.42. The problem is reduced to the solution of the equation $\theta e^{30/\theta^{1/3}} = 3.065 \cdot 10^9/w^{3/2}$, where θ, keV; w, W/cm^3. Solving by inspection, we find $\theta \approx 3$ keV.

15.43. $r = 6\sigma T^4/n^2 \langle\sigma v\rangle_{dd} Q_{dd}$, where σ in the numerator is the Stefan-Boltzmann constant. From the condition $dr/d\theta = 0$ where $\theta = kT$, we get $\theta_m = 2.5$ keV; $r_{\min} = 3 \cdot 10^6$ km.

15.44. $40 < \theta < 45$ keV.

15.45. **(a)** In the steady-state case, we have the system of two equations: $q = n_d^2 \langle\sigma v\rangle_{dd} + n_d n_t \langle\sigma v\rangle_{dt}$, $\frac{1}{4} n_d^2 \langle\sigma v\rangle_{dd} = n_d n_t \langle\sigma v\rangle_{dt}$. Here $n_t = \frac{1}{4} n_d \langle\sigma v\rangle_{dd}/\langle\sigma v\rangle_{dt} = 4 \cdot 10^{12}$ cm^{-3}; $q = \frac{5}{4} n_d^2 \langle\sigma v\rangle_{dd} = 1 \times 10^{12}$ cm^{-3}·s^{-1}.

(b) $w = \frac{1}{2} n_d^2 \langle\sigma v\rangle_{dd} \left(Q_{dd} + \frac{1}{2} Q_{dt}\right) = 24$ W/cm^3; $n_d = 1.0 \cdot 10^{16}$ cm^{-3}.

15.46. From the equation $m_e\ddot{x} = -eE = -4\pi ne^2 x$, we obtain $\omega_0 = \sqrt{4\pi ne^2/m_e}$.

15.47. **(a)** See the solution of Problem 9.23.

(b) The equation of the wave is $E = E_0 e^{-i(\omega t - kx)}$, where $k = 2\pi/\lambda$. When $\omega < \omega_0$, the refractive index $n = \sqrt{\varepsilon} = i\varkappa$ and $k = 2\pi n/\lambda_0 = i\frac{2\pi\varkappa}{\lambda_0}$, where λ_0 is the wavelength of the wave in vacuum. In this case $E = E_0 e^{-2\pi\varkappa/\lambda_0 x} \cos\omega t$, i.e. the standing wave sets up whose amplitude falls off exponentially.

(c) $n_e = \pi c^2 m_e/e^2\lambda_0^2 = 5 \cdot 10^{12}$ cm^{-3}.

15.48. $N = \frac{1}{4} n_d^2 \langle\sigma v\rangle_{dd} V = 5 \cdot 10^{11}$ s^{-1}, where $n_d = n_e = \pi m_e c^2/e^2\lambda_0^2$.

15.49. Poisson's equation takes the following form, when written in spherical coordinates $\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\varphi) = -4\pi e(n_i - n_e)$. Since $n_e = ne^{+e\varphi/\theta} \approx n(1 + e\varphi/\theta)$; $n_i = ne^{-e\varphi/\theta} \approx n(1 - e\varphi/\theta)$, the initial equation can be written as

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\varphi)=\alpha^2\varphi;\quad \alpha^2=8\pi ne^2/\theta.$$

Introducing the new function $f=r\varphi$, we obtain the equation $f''=\alpha^2 f$, whose solution is $f\propto e^{-\alpha r}$, i.e. $\varphi\sim\frac{1}{r}e^{-\alpha r}$. Thus, $\varphi\sim\frac{1}{r}e^{-r/d}$, $d=1/\alpha=\sqrt{\theta/8\pi ne^2}$.

15.50. $1.7\cdot10^{-3}$ cm; $2\cdot10^{7}$.

15.51. $\sigma=\pi e^4/4T^2=1.6\cdot10^{-20}$ cm^2.

15.52. $\vartheta_{\min}=\sqrt{8\pi n}\,e^3\theta^{3/2}\approx(6\cdot10^{-3})''$; 15.8.

15.53. (a) $\sigma=\frac{\pi e^4}{\theta^2}\ln\frac{2}{\vartheta_{\min}}=1\cdot10^{-18}$ cm^2, $\vartheta_{\min}=e^3\sqrt{8\pi\, n}/\theta^{3/2}$;
(b) $l_{ei}=1/n_i\sigma\approx10$ m; $\tau_{ei}=l_{ei}/v_{\mathrm{pr}}=0.5$ μs; $\nu_{ei}=1/\tau_{ei}=2\cdot10^6$ s^{-1}.

15.54. $w=5\cdot10^{-31}n_en_i\sqrt{\theta_e}$ W/cm^{-3}.

15.55. (a) Integrating the equation $dE_i=w_{ei}\,dt$ and taking into account that $E_i=\frac{3}{2}n_i\theta_i$, we get $\theta_i=\theta_e\,[1-\exp(-\alpha nt/\theta_e^{3/2})]$; $\alpha=7.09\cdot10^{-13}$, where n, cm^{-3}; t, s; θ_e, keV. Thus, $t\approx1.6$ ms.
(b) The problem is reduced to the integration of the equation $dE_e=-w_{ei}\,dt$. It should be noted that $E_e=\frac{3}{2}n_e\theta_e$ and $d\theta_i=-d\theta_e$, as $\theta_e+\theta_i=\mathrm{const}=2\theta$, where θ is the mean plasma temperature equal to $\theta_{e0}/2$ in our case. Introducing the variable $y=\sqrt{\theta_e/\theta}$, we obtain

$$t=\frac{\theta^{3/2}}{\alpha n}\int\limits_{y_0}^{y}\frac{y^4\,dy}{y^2-1}=\frac{\theta^{3/2}}{\alpha n}\left[\frac{1}{2}\ln\frac{y+1}{y-1}-y-\frac{1}{3}y^3\right]_{y_0}^{y}=0.8\ \text{ms},$$

where $\alpha=7.09\cdot10^{-13}$; θ, keV; n, cm^{-3}.

15.56. The Lorentz force acting on 1 cm^2 of the cylindrical layer r, $r+dr$ is $\delta p=df/dS=\frac{1}{c}j_rB_r\,dr$. Noting that $B_r=2I_r/cr$, where I_r is the electric current flowing inside the cylinder of radius r, and that $j_r=dI_r/2\pi r\,dr$, we obtain $\delta p=I_r\,dI_r/\pi c^2r^2$. Integrating this expression and noting that the variable r can be replaced by r_0, we find $p=I_0^2/2\pi c^2r_0^2=B_0^2/8\pi$.

15.57. $3.1\cdot10^{15}$ cm^{-3}; 10 MPa; 2.7 W/cm^3.

15.58. $\frac{4}{3}\ln\frac{r_2}{r_1}$.

15.59. (a) From the condition that the magnetic pressures developed on the inside and outside surfaces of the plasma layer are equal, we have $B_2=B_1-B_2$ and $I_1=2I_2$.
(b) $I_1=4cr_0\sqrt{\pi n\theta}=5.4\cdot10^5$ A.

15.60. (a) Proceeding from the basic equation $\nabla p=[\mathbf{jB}]/c$, we have $-\partial p/\partial r=jB_\varphi/c$, where $B_\varphi=2I_r/cr$, I_r is the current flowing inside the cylinder of radius r. Taking into account that $j=dI_r/2\pi r\,dr$, we get: $-r^2\,dp=I_r\,dr/\pi c^2$. Integrating this equation (the left side, by parts) and noting that $p\,(r_0)=0$, we find $\langle p\rangle=I_0^2/2\pi c^2r_0^2$.
(b) As $p=2n\theta$, then $\theta=I_0^{2\,2}/4c^2N=2$ keV.

15.61. (a) The problem is reduced to the integration of the equation $-\partial p/\partial r=jB/c$, where $p=2n\theta$, $B=2\pi rj/c$. Besides, it should be noted that $\theta=I^2/4c^2N$. The mean value of n^2 is $\langle n^2\rangle=\frac{4}{3}\,(N/\pi r_0^2)^2$.
(b) $1.3\cdot10^6$ A. In calculations one should use the formula $w_{\mathrm{rad}}=4.8\cdot10^{-31}\,\langle n^2\rangle\sqrt{\theta_{\mathrm{keV}}}$ W/cm^3, where the value of $\langle n^2\rangle$ is taken from the solution of the foregoing point (a). The corresponding temperature is equal to 5 keV.

15.62. Noting that $B=4\pi I/cl$, the concentration of nuclei at the maximum compression $n=2n_0\,(r_0/r)^2$, and $B^2/8\pi=2n\theta$, we obtain $\theta\approx\pi\,(rI/clr_0)^2/2n_0\approx10$ keV.

15.63. $t\propto r_0^24\pi\sigma(\theta)/c^2\propto10^{-3}$ s, where $\theta=I_0^2/4c^2N=0.39$ keV.

15.64. 0.8 ms.

15.65. At the narrowest point of the filament, where its radius decreases by Δr, the magnetic field grows by $\Delta B_z=-2B_z\,\Delta r/r$, since the B_z field is "frozen" and its magnetic flux does not change. The intrinsic magnetic field B_I' induced by the current I also increases by the value of $\Delta B_I=-B_I\,\Delta r/r$ at that location. The gas-kinetic pressure, however, does not vary because the plasma is free to flow out of this region in both directions. To counterbalance the instability, it is necessary that $\Delta\,(B_z^2/8\pi+p)>\Delta\,(B_I^2/8\pi)$, whence $B_I<\sqrt{2}B_z$; $B_z>\sqrt{2}I/cr=14$ kG.

15.67. Denote the combined stretching force per unit length of the turn by f. Then $2\pi Rf\,\Delta R=\Delta\,(LI^2/2c^2)+p\,\Delta V=2\pi^2r^2\,\Delta R$, $p=I^2/2\pi c^2r^2$ (from the equilibrium condition involving the cross-sectional radius r). Thus, $f=\left(\ln\frac{8R}{r}-\frac{1}{2}\right)I^2/c^2R$. In the considered case one can disregard the changes in f since R varies insignificantly. Therefore, $\ddot R=\mathrm{const}$ and $t\approx\sqrt{2a/\ddot R}=\sqrt{2\pi r^2a\rho/f}=0.7$ μs, where ρ is the plasma density.

15.68. In the solution of the foregoing problem, the expression is found for the stretching force f per unit length of the turn. This force must be counterbalanced by the Ampere force acting on the current I and developed by the field B_z; when reduced to a unit length of the turn, it becomes equal to $f=IB_z/c$. Thus, $B_z=\left(\ln\frac{8R}{r}-1\right)I/cR=0.11$ kG.

15.69. From the condition $2\pi R\leqslant2\pi rB_\varphi/B_I$, where $B_I=2I/cr$, we get $I\leqslant cr^2B_\varphi/2R=2$ kA.

16.1. $p=\sqrt{T(T+2m)}$; 1.7; 1.1 and 1.0 GeV/c.

16.2. Resorting to the invariance of the expression E^2-p^2, write $(T+2m)^2-p^2=[2(\tilde{T}+m)]^2$, where the left-hand side of the equation refers to the L frame and the right-hand side to the C frame. Taking into account that $p^2=T(T+2m)$, we obtain $\tilde{T}=2m(\sqrt{1+T/2m}-1)$; $\tilde{p}=\sqrt{mT/2}$; $\beta_c=\sqrt{T/(T+2m)}$.

16.3. From the expression $E^2-p^2=\text{inv}$, we have $(T'+2m)^2-p^2=[2(T+m)]^2$; $T'=2T(T+2m)/m=2.0\cdot10^3$ GeV.

16.4. (a) $\tilde{T}=\sqrt{(m_1+m_2)^2+2m_2T}-(m_1+m_2)$; (b) $\tilde{p}=\sqrt{\frac{m_2^2T(T+2m_1)}{(m_1+m_2)^2+2m_2T}}$; $\tilde{E}_{1,2}=\sqrt{\tilde{p}^2+m_{1,2}^2}$.

16.5. $\tilde{T}_{1,2}=\tilde{T}(\tilde{T}+2m_{2,1})/2(m_1+m_2+\tilde{T})$.

16.6. From the formula for $\tilde{p}_x$ and the equality $\tilde{p}_y=p_y$ (see Fig. 41) follows Eq. (16.3) for $\tan\tilde{\vartheta}$.

16.7. (a) From the expression for $\tan\tilde{\vartheta}$, it follows:

$$\tan\vartheta_1=\frac{\sqrt{1-\beta_c^2}\sin\tilde{\vartheta}}{\cos\tilde{\vartheta}+\beta_c\tilde{E}_1/\tilde{p}};\quad \tan\vartheta_2=\frac{\sqrt{1-\beta_c^2}\sin(\pi-\tilde{\vartheta})}{\cos(\pi-\tilde{\vartheta})+\beta_c\tilde{E}_2/\tilde{p}}.$$

It remains to take into account that $\tilde{p}/\tilde{E}_1=\tilde{p}/\tilde{E}_2=\beta_{1,2}=\beta_c=\sqrt{T/(T+2m)}$.

(b) From the formula $\cot\vartheta_1\cot\vartheta_2=\alpha$, where $\alpha=1+T/2m$, we obtain $\theta=\vartheta_1+\vartheta_2=\vartheta_1+\operatorname{arccot}\frac{\alpha}{\cot\vartheta_1}$. From the condition $\partial\theta/\partial\vartheta_1=0$, we find $\cot\vartheta_1=\sqrt{\alpha}$, and therefore $\cot\vartheta_2=\sqrt{\alpha}$ as well. The angle θ is thus minimal for the symmetric divergence of the particles provided their masses are equal: $\vartheta_1=\vartheta_2=\vartheta_{min}/2$. Thus, $\cos(\theta_{min}/2)=\sqrt{1+T/2m}$; $\theta_{min}=53°$.

(c) $T=1.37$ GeV; $T_1=0.87$ GeV; $T_2=0.50$ GeV.

16.8. From the conservation laws of energy and momentum, we have: $T=T_1+T_2$; $p_2^2=p_1^2+p^2-2pp_1\cos\vartheta_1$, where T and p are the kinetic energy and momentum of the striking particle. Taking into account that $p=\sqrt{T(T+2m_1)}$, express $\cos\vartheta_1$ in terms of T_1. From the condition $d\cos\vartheta_1/dT_1=0$, we get the value of T_1 corresponding to the maximum value of the angle ϑ_1. Substituting this value into the expression for $\cos\vartheta_1$, we find the sought relationship.

16.9. $T_e=\frac{2m_eT(T+2m_\mu)}{(m_\mu+m_e)^2+2m_eT}=2.8$ MeV.

16.10. (a) From the expression for $\tan\tilde{\vartheta}$, it follows that $\tan\vartheta=\frac{\sqrt{1-\beta_c^2}\sin\tilde{\vartheta}}{\cos\tilde{\vartheta}-\beta_c\tilde{E}/\tilde{p}}=\sqrt{1-\beta_c^2}\tan(\tilde{\vartheta}/2)$, for $\tilde{p}/\tilde{E}=\beta_c$. It remains to take into account that $\beta_c=\sqrt{T/(T+2m)}$.

(b) Since $\sigma(\vartheta)\sin\vartheta\,d\vartheta=\tilde{\sigma}(\tilde{\vartheta})\sin\tilde{\vartheta}\,d\tilde{\vartheta}$, then $\tilde{\sigma}(\tilde{\vartheta})=\sigma(\vartheta)\frac{d\cos\vartheta}{d\cos\tilde{\vartheta}}$. Using the formula of the foregoing point (a), express $\cos\tilde{\vartheta}$ in terms of $\cos\vartheta$, find the derivative $d\cos\tilde{\vartheta}/d\cos\vartheta$, and substitute it into the expression for $\tilde{\sigma}(\tilde{\vartheta})$.

(c) Calculate $\tilde{\sigma}(\tilde{\vartheta})$ corresponding to the angles ϑ_1 and ϑ_2; 5.5 and 3.2 mb/sr. It is obvious now that the scattering in the C frame is anisotropic.

16.11. (a) From the condition $\sigma(T)\,dT=\tilde{\sigma}(\tilde{\vartheta})\,d\tilde{\Omega}$, where T is the kinetic energy of the scattered proton, $d\tilde{\Omega}$ is the solid angle element in the C frame, we obtain $\sigma(T)=\tilde{\sigma}(\tilde{\vartheta})\,d\tilde{\Omega}/dT$. On the other hand, from the Lorentz transformation (16.3), it follows that

$$\frac{dT}{d\tilde{\Omega}}=\frac{dE}{d\tilde{\Omega}}=\frac{dE}{2\pi d\cos\tilde{\vartheta}}=\frac{\beta_c\tilde{p}}{2\pi\sqrt{1-\beta_c^2}}=\frac{T_0}{4\pi}.$$

Here we took into account that $\beta_c=\sqrt{T_0/(T_0+2m)}$ and $\tilde{p}=\sqrt{mT_0/2}$

(b) $\frac{dw}{dT}=\frac{dw}{d\tilde{\Omega}}\frac{d\tilde{\Omega}}{dT}=\frac{1}{T_0}=\text{const}$. Here we took into account that in the case of isotropic scattering in the C frame the relative number of protons scattered into the solid angle $d\tilde{\Omega}$ is $dw=d\tilde{\Omega}/4\pi$.

16.12. $\cos\theta=\frac{1}{2n}\left[\frac{T}{T+2m_e}(1+n)^2-1-n^2\right]$; $\theta=120°$.

16.13. $\sigma(E_\gamma)=\frac{\partial\sigma}{\partial E_\gamma}=\frac{\partial\sigma}{\partial\tilde{\Omega}}\cdot\frac{\partial\tilde{\Omega}}{\partial E_\gamma}=2\pi\tilde{\sigma}(\tilde{\vartheta})\left|\frac{\partial\cos\tilde{\vartheta}}{\partial E_\gamma}\right|$, where $\partial\tilde{\Omega}$ is the solid angle element in the C frame. From the formula $E_\gamma=\frac{\tilde{E}_\gamma+\beta_c\tilde{p}_\gamma\cos\tilde{\vartheta}}{\sqrt{1-\beta_c^2}}$, we obtain $\frac{\partial E_\gamma}{\partial\cos\tilde{\vartheta}}=\frac{\beta_c\tilde{p}_\gamma}{\sqrt{1-\beta_c^2}}=\frac{p}{2}$. Here we took into account that $\beta_c=p/(E_e+m_e)$ and $\tilde{p}=\tilde{E}_\gamma=\tilde{E}_e=m_e/\sqrt{1-\beta_c^2}$. Thus, $\sigma(E_\gamma)=4\pi\tilde{\sigma}(\tilde{\vartheta})/p$.

16.14. 0.32 GeV (see the solution of Problem 12.62).

16.15. Using the invariance of E^2-p^2, write this expression in the L and C frames for the threshold value of energy of a particle with mass m: $(T_{m\,\text{th}} + m + M)^2 - p^2_{m\,\text{th}} = (\Sigma\, m_i)^2$, where $p^2_{m\,\text{th}} = T_{m\,\text{th}}\,(T_{m\,\text{th}} + 2m)$. From this, we find the sought expression.

16.16. (1) 0.20 GeV; (2) 0.14 GeV; (3) 0.78 GeV; (4) 0.91 GeV; (5) 1.38 GeV; (6) 1.80 GeV; (7) $6m_p = 5.63$ GeV; (8) 7.84 GeV.

16.17. **(a)** 1.36 MeV; **(b)** 197 MeV.

16.18. **(a)** From the condition that in the C frame the total energies of both processes have the same value, we can write (using the invariant $E^2 - p^2$): $(m_a + m_A + T_a)^2 - T_a\,(T_a + 2m_a) = (m_b + m_B + T_b)^2 - T_b\,(T_b + 2m_b)$.

Hence, $T_b = \frac{m_A}{m_B}\,(T_a - T_{a\,\text{th}})$.

(b) $T_\pi = \hbar\omega - m_\pi\left(1 + \frac{m_\pi}{2m_p}\right) = 50$ MeV.

16.19. From the expression for $\sin\vartheta_{\max}$, where $M = \sqrt{2m\,(T+2m)}$, we find $\vartheta_{\max} = 10.5°$.

16.20. Using the invariant $E^2 - p^2$, we obtain $M = \sqrt{(m_p+m_\pi)^2 + 2m_pT}$; 1.24, 1.51, and 1.69 GeV.

16.21. 19.5 MeV; 193 MeV/c.

16.22. 52.4 MeV; 53 MeV/c.

16.23. $E_\Lambda = m_\Lambda - (Q + m_p + m_\pi) = 2.8$ MeV.

16.24. $\tan\vartheta_\mu = \frac{E_\nu}{\sqrt{T_\pi\,(T_\pi + 2m_\pi)}}$, where $E_\nu = \frac{m_\pi^2 - m_\mu^2}{2\,(m_\pi + T_\pi)}$; $\vartheta_\mu \approx 10°$.

16.25. **(a)** From the conservation laws of energy and momentum, we find: $\sin(\theta/2) = m_\pi/2\sqrt{E_1E_2}$, where E_1 and E_2 are the energies of γ-quanta. It is seen from this expression that θ reaches the minimum value when $E_1 = E_2 = (m_\pi + T_\pi)/2 = m_\pi$. Hence, $\theta_{\min} = 60°$.

(b) At $\theta = \pi$, the energy of one γ-quantum is the highest and of the other the lowest. In this case $\sin(\pi/2) = m_\pi/2\sqrt{E_1\,(E - E_1)}$, where E is the total energy of the pion. Thus, $E = \frac{1}{2}\,[m_\pi + T_\pi \pm \sqrt{T_\pi\,(T_\pi + 2m_\pi)}] = 252$ and 18.1 MeV.

16.26. **(a)** $T = (m_K - 2m_\pi)\,m_K/2m_\pi = 0.42$ GeV; **(b)** $\cos\theta = \frac{2T\,(T + 2m_K)}{(T + m_K)^2 - 4m_\pi^2} - 1$; $\theta = 103°$.

16.27. From the conservation laws of energy and momentum, we get: $m^2 = m_\Sigma^2 + m_\pi^2 - 2\,[\sqrt{(m_\Sigma^2 + p_\Sigma^2)\,(m_\pi^2 + p_\pi^2)} - p_\Sigma p_\pi\cos\vartheta]$. Whence $m = 0.94$ GeV (a neutron); $Q = 0.11$ GeV.

16.28. Since $\theta \neq \pi$, the decay must have occurred when the particle was moving. From the conservation laws of energy and momentum, we obtain: $m^2 = m_p^2 + m_\pi^2 + 2[\sqrt{(m_p^2 + p_p^2)\,(m_\pi^2 + p_\pi^2)} - p_pp_\pi\cos\theta]$. Whence $m = 1115$ MeV (a Λ-hyperon).

16.29. *Instruction.* In the C frame, $M = \tilde{E} = \sqrt{m_1^2 + \tilde{p}^2} + \sqrt{m_2^2 + \tilde{p}^2}$.

16.30. We proceed from the relations $p_{1x} = (\tilde{p}_{1x} + \beta\tilde{E}_1)/\sqrt{1-\beta^2}$ and $p_{1y} = \tilde{p}_{1y}$. Noting that $\tilde{p}_{1x}^2 + \tilde{p}_{1y}^2 = \tilde{p}^2$, we obtain the equation of an ellipse: $\frac{p_{1y}^2}{b^2} + \frac{(p_{1x} - \alpha_1)^2}{a^2} = 1$, where $b = \tilde{p}$; $a = \tilde{p}/\sqrt{1-\beta^2}$; $\alpha_1 = \tilde{E}_1\beta/\sqrt{1-\beta^2} = \tilde{E}_1 f/b$. The focal length is $f = \sqrt{a^2 - b^2} = \tilde{p}\beta/\sqrt{1-\beta^2}$. The line segment $\alpha_2 = p_M - \alpha_1 = p_M - \frac{f}{b}\,(M - \tilde{E}_2) = \frac{f}{b}\,\tilde{E}_2$. It is easy to see that $\alpha_1 \geqslant f_1$ and $\alpha_2 \geqslant f$, with the sign "equals" being valid only for the particles with zero rest mass.

16.31. **(a)** $p_\pi = \sqrt{3}\,m_\pi$; $\beta = \sqrt{3}/2$; $\tilde{p} = m_\pi/2$. The parameters of the ellipse: $a = 2b = p_\pi/\sqrt{3}$; $\alpha_1 = \alpha_2 = f = p_\pi/2$; **(b)** $p_\pi = m_\pi\sqrt{5/2}$; $\beta = \sqrt{5/3}$; $\tilde{p} = (m_\pi^2 - m_\mu^2)/2m_\pi = 0.19p_\pi$. The parameters of the ellipse: $b = 0.19p_\pi$; $a = 0.29p_\pi$; $f = 0.21p_\pi$; $\alpha_\nu = f$; **(c)** this case can be treated as a decay of a particle whose rest mass equals the total energy of interacting particles in the C frame: $M = \tilde{E} = \sqrt{(2m_p)^2 + 2m_pT} = \sqrt{6}\,m_p$; $p_p = \sqrt{3}\,m_p$. The parameters of the ellipse: $b = p_p/\sqrt{6}$; $a = p_p/2$; $f = p_p/2\sqrt{3}$; $\alpha_1 = \alpha_2 = a$; **(d)** as in the previous case, we have: $M = \tilde{E} = \sqrt{11}\,m_p$; $p_d = \sqrt{5}\,m_p$. The parameters of the ellipse: $b = p_d/\sqrt{11}$; $a = 4p_d/11$; $f = p_d\sqrt{5/11}$; $\alpha_p = a$; **(e)** $b = 0.224p_p$; $a = 0.275p_p$; $f = 0.1585p_p$; $\alpha_\pi = 0.1685p_p$.

16.32. **(a)** About 20°; **(b)** assuming $\sin\vartheta_{\max} = 1$, we obtain $T_{\pi\,\text{th}} \geqslant (m_\pi - m_\mu)^2/2m_\mu = 5.5$ MeV.

16.33. **(a)** From the formulas $\tilde{p}\cos\tilde{\vartheta} = \frac{p\cos\vartheta - \beta E}{p\sin\vartheta}$ and $\tilde{p}\sin\tilde{\vartheta} = p\sin\vartheta$ (see Fig. 41), we have $\tan\tilde{\vartheta} = \frac{\sin\vartheta\,\sqrt{1-\beta^2}}{\cos\vartheta - \beta}$, where we took into account that $E/p = 1$, for the neutrino's rest mass is zero. The rest of the proof is obvious.

(b) As $\sigma(\vartheta)\sin\vartheta\,d\vartheta = \tilde{\sigma}(\tilde{\vartheta})\sin\tilde{\vartheta}\,d\tilde{\vartheta}$, then $\sigma(\vartheta) = \tilde{\sigma}(\tilde{\vartheta})\,\frac{d\cos\tilde{\vartheta}}{d\cos\vartheta}$. Now we have only to find the derivative by means of the formula given in the point **(a)** of this problem.

(c) $w = (1 + \beta)/2 = 0.93$.

16.34. **(a)** The narrow maximum belongs to reaction branch (1), the broad one to reaction branch (2).

(b) Disregarding the momentum of a π^--meson, write the laws of conservation of total energy and momentum for branch (1): $m_\pi +$

$+ m_p = E_n + E_\gamma$; $p_n = p_\gamma$. Whence $m_\pi = \sqrt{m_n^2 + E_\gamma^2} + E_\gamma - m_p = 0.14$ GeV.

(c) From the spectral characteristics of γ-quanta emerging in the decay of π^0-meson (the broad maximum), it follows that π^0-mesons disintegrate in flight (otherwise, monochromatic γ-quanta would be emitted). From the conservation laws of energy and momentum, it follows that $m_\pi = 2\sqrt{E_1 E_2} = 135$ MeV.

16.35. If the process goes via the bound state ρ (in two stages), then in the reference frame fixed to the ρ-particle the rest mass of the ρ-particle equals the sum of the total energies of the particles into which it decays:

$$\tilde{E}_0 = \tilde{E}_{\pi^+} + \tilde{E}_{\pi^-} = \tilde{E}_\rho = M_\rho; \quad \tilde{p} = \tilde{p}_{\pi^+} + \tilde{p}_{\pi^-} = 0.$$

Using the invariance of the expression $E^2 - p^2$ on transition from the C to L frame, we obtain for the pions: $E^2 - p^2 = \tilde{E}_0^2$, where $E = E_{\pi^+} + E_{\pi^-}$; $p = |\mathbf{p}_{\pi^+} + \mathbf{p}_{\pi^-}|$.

If the reaction proceeded only via the bound state, we would obtain the same value of $E^2 - p^2$ in any case considered. But if the reaction goes partially via the bound state, the value of $E^2 - p^2$ varies from case to case and exhibits the maximum which proves the existence of the resonance, or the bound state.

16.36. Using the $E^2 - p^2$ invariant, find the total energy of interacting particles in the C frame: $\tilde{E} = \sqrt{(m_K + m_p)^2 + 2m_p T_K}$. The total energy of the resonance (the Y^*-particle) in the C frame is $\tilde{E}_Y = \tilde{E} - \tilde{E}_\pi = \tilde{E} - (\tilde{T}_\pi + m_\pi)$ and $M_Y = \sqrt{\tilde{E}_Y^2 - \tilde{p}_Y^2} = 1.38$ GeV, where $\tilde{p}_Y^2 = \tilde{p}_\pi^2 = \tilde{T}_\pi(\tilde{T}_\pi + 2m_\pi)$. The decay energy is equal to 125 MeV.

16.37. (a) $\tau_0 = \tau\sqrt{1-\beta^2} = \tau m_\mu/(m_\mu + T) = 2.2$ μs.

(b) $\tau_0 = l\sqrt{1-\beta^2}/\beta c = l m_\pi/pc = 2.5 \cdot 10^{-8}$ s.

16.38. $w = 1 - e^{-t/\tau} = 0.43$, where t is the flight time, τ is the mean lifetime of the moving meson.

16.39. From the condition $\mu_p = \alpha\mu_N + (1-\alpha)\mu_\pi$, where α is the fraction of time during which the proton possesses the properties of "the ideal proton", we find $\alpha \approx \frac{2}{3}$. Here we took into account that $\mu_\pi/\mu_N = m_p/m_\pi$.

16.40. $2s_\pi + 1 = \frac{4}{3}\frac{\sigma_{pp}}{\sigma_{\pi d}}(\tilde{p}_p/\tilde{p}_\pi)^2 = 1.05$; $s_\pi = 0$. The proton's momentum $\tilde{p}_p$ in the C frame is found by means of the $E^2 - p^2$ invariant on transition from the L to C frame; $\tilde{p}^2 = m_p T_p/2$. The momentum $\tilde{p}'_\pi$ of the pion in the reverse process can be found from Eq. (16.5), considering this process as a decay of the system with rest mass M equal to the total energy $\tilde{E}'_{\pi d}$ of the interacting particles in the C frame. In accordance with the detailed balancing principle $\tilde{E}'_{\pi d} = \tilde{E}_{pp}$, so that $M = \tilde{E}_{pp} = \sqrt{\tilde{E}_{pp}^2 - p^2} = \sqrt{2m_p(T_p + 2m_p)}$.

16.41. For the γ-quantum, $2s_\gamma + 1 = 2$ in accordance with two possible polarizations, so that $\sigma_{\pi p} = 2\sigma_{\gamma p}(\tilde{p}_\gamma/\tilde{p}'_\pi)^2 = 0.6$ mb. Here $\tilde{p}_\gamma$ is found by means of the $E^2 - p^2$ invariant and the momentum $\tilde{p}'_\pi$ in the reverse process is found from the condition of equality of total energies in both processes in the C frame ($\tilde{E}_{\gamma p} = \tilde{E}_{\pi p}$):

$$\tilde{p}^2 = \frac{m_p}{m_p + 2E_\gamma}E_\gamma^2; \quad \tilde{p}'^2 = \frac{(2m_p E_\gamma - m_\pi^2)^2 - 4m_p^2 m_\pi^2}{4m_p(m_p + 2E_\gamma)}.$$

16.42. Forbidden are reactions 1, 3, 5, and 6.

16.43. (2) and (6).

16.44. (a) Branch (2) is forbidden in terms of energy; **(b)** Branch (1) is forbidden for $|\Delta S| = 2$.

	nn	pp	np	π^+p	π^-p	π^0p	π^+n	π^-n	π^0n
T_z	−1	+1	0	+3/2	−1/2	+1/2	+1/2	−3/2	−1/2
T	1	1	1; 0	3/2	3/2; 1/2	3/2; 1/2	3/2; 1/2	3/2	3/2; 1/2

16.46. (a) The system can possess $T = 1$ or 0. ^3P: $(-1)^{1+1+T} = -1$, $T = 1$; ^3D: $(-1)^{2+1+T} = -1$, $T = 0$.

(b) The system can possess $T = 2$ or 1 ($T = 0$ is out since $T_z = +1$). ^1P: $(-1)^{1+0+T} = +1$, $T = 1$, ^1D: $(-1)^{2+0+T} = +1$, $T = 2$.

(c) Here $T = 0$, 1, and 2. In ^1P states $T = 1$; in ^1D states $T = 2$ and 0.

16.47. (a) Write out all possible reactions of this type:

			π^+	π^-	π^0
(1) $\tilde{p}p \to \pi^+\pi^-$,	(1′) $\tilde{n}n \to \pi^-\pi^+$	. . . σ_1	2	2	—
(2) $\tilde{p}p \to \pi^0\pi^0$,	(2′) $\tilde{n}n \to \pi^0\pi^0$	. . . σ_2	—	—	4
(3) $\tilde{p}n \to \pi^-\pi^0$,	(3′) $\tilde{n}p \to \pi^+\pi^0$	. . . σ_3	1	1	2

All of these reactions have three different cross-sections σ_1, σ_2, and σ_3. The cross-sections σ_1 of processes 1 and 1′ are equal due to the charge symmetry. Just as equal are the cross-sections σ_2 of processes 2 and 2′ and cross-sections σ_3 of processes 3 and 3′.

Let us construct the Table with the numbers a_i for the produced pions of either sign in the reactions with different cross-sections σ_i. The total number of pions of either sign is equal to the sum $\Sigma a_i\sigma_i$. Since the emerging particles are non-polarized, the number of pions of either sign must be the same, and therefore

$$2\sigma_1 + 1\sigma_3 = 2\sigma_1 + 1\sigma_3 = 4\sigma_2 + 2\sigma_3, \quad \text{or} \quad 2\sigma_1 = 4\sigma_2 + \sigma_3.$$

(b) In this case, as one can easily see, the analysis of the direct reactions does not provide the sought relation. Therefore, having written all reactions of this type, we compile the table for the *reverse* processes:

	π^+	π^-	π^0
(1) $\pi^+ n \to \Lambda K^+$, $\pi^- p \to \Lambda K^0$ . . . σ_1	1	1	—
(2) $\pi^0 p \to \Lambda K^+$, $\pi^0 n \to \Lambda K^0$ . . . σ_2	—	—	2

From the absence of polarization in the reverse processes, we obtain using this table $\sigma_1 = 2\sigma_2$.

(c) In this case, having written all reactions of this type, we compile the table for the direct and reverse processes:

	Σ^+	Σ^-	Σ^0	π^+	π^-	π^0
(1) $\pi^+ p \to \Sigma^+ K^+$, $\pi^- n \to \Sigma^- K^0$ . . . σ_1	1	1	—	1	1	—
(2) $\pi^0 p \to \Sigma^0 K^+$, $\pi^0 n \to \Sigma^0 K^0$ . . . σ_2	—	—	2	—	—	2
(3) $\pi^0 p \to \Sigma^+ K^0$, $\pi^0 n \to \Sigma^- K^+$ . . . σ_3	1	1	—	—	—	2
(4) $\pi^- p \to \Sigma^- K^+$, $\pi^+ n \to \Sigma^+ K^0$ . . . σ_4	1	1	—	1	1	—
(5) $\pi^- p \to \Sigma^0 K^0$, $\pi^+ n \to \Sigma^0 K^+$ . . . σ_5	—	—	2	1	1	—

From the condition of absence of polarization for the direct and reverse processes, we obtain $\sigma_1 + \sigma_3 + \sigma_4 = 2(\sigma_2 + \sigma_5)$, $\sigma_1 + \sigma_4 + \sigma_5 = 2(\sigma_2 + \sigma_3)$, whence $\sigma_3 = \sigma_5$; $\sigma_1 + \sigma_4 = 2\sigma_2 + \sigma_3$.

(d) Assuming the τ-particles to be non-polarized in terms of isotopic spin, write all possible decay reactions of this type and compile the corresponding table for these processes:

	π^+	π^-	π^0
(1) $\tau^+ \to \pi^+\pi^0\pi^0$, $\tau^- \to \pi^-\pi^0\pi^0$ . . . w_1	1	1	4
(2) $\tau^- \to \pi^-\pi^+\pi^-$, $\tau^+ \to \pi^+\pi^-\pi^+$ . . . w_2	3	3	—
(3) $\tau^0 \to \pi^0\pi^0\pi^0$, ... w_3	—	—	3
(4) $\tau^0 \to \pi^0\pi^-\pi^+$, ... w_4	1	1	1

Whence $w_1 + 3w_2 + w_4 = 4w_1 + 3w_3 + w_4$, or $w_1 + w_3 = w_2$.

(e) Write the hypothetical reaction branches for a w^0-quasiparticle decaying into three pions, indicating the probability of each branch, and compile the corresponding table:

	π^+	π^-	π^0
$w^0 \nearrow \pi^0\pi^+\pi^-$ . . . . . . w_1	1	1	1
$w^0 \searrow 3\pi^0$ . . . . w_2	—	—	3

From the condition of absence of polarization in the produced pions, we obtain $w_1 = w_1 + 3w_2$, whence $w_2 = 0$.

16.48. (a) and **(b)** $\Delta T_z = 0$ and $\Delta S = 0$; thus, the interaction is strong and $\Delta T = 0$ for it; **(c)** the isotopic spin T of the system $(\pi^0\pi^+)$ is equal to 2 and 1. From the generalized Pauli principle, it follows that $(-1)^{l+s+T} = (-1)^{l+T} = +1$. According to the law of conservation of the angular momentum l must be equal to zero. Whence $(-1)^T = +1$, $T = 2$. Thus, $\Delta T = 3/2$, $\Delta T_z = 1/2$; **(d)** the projection of the isotopic spin of the system $2\pi^0 T_z = 0$. Of all possible values of isotopic spin (2, 1, and 0), only 0 and 2 are realized, since according to the generalized Pauli principle $(-1)^{l+O+T} = +1$. From the law of conservation of angular momentum, it follows that $l = 0$. Thus, T must be even, i.e. 0 or 2. Consequently, ΔT is equal to 1/2 or 3/2.

16.49. From the laws of conservation of parity P and angular momentum, we have

$$P_\pi P_d (-1)^{l_\pi} = P_n^2 (-1)^{l_n}, \quad \text{whence} \quad P_\pi = (-1)^{l_n};$$

$$s_\pi + s_d + l_\pi = 2s_n + l_n, \quad \text{whence} \quad 1 = s_n + s_{n'} + l_n.$$

If the neutrons were produced in the s state ($l_n = 0$), they would possess, according to the Pauli principle, the opposite spins; in this case, however, the total moment would be equal to 0, which is impossible. When $l_n = 1$ (p-state), the momentum conservation law is satisfied: $1 = 1/2 - 1/2 + 1$. The other values of l_n are not suitable. Thus, $P_\pi = (-1)^{l_n} = -1$.

16.50. (a) It follows from the generalized Pauli principle that $(-1)^{l+0+T} = +1$. Besides, taking into account the law of conservation of isotopic spin in strong interactions, we find $T = 1$ and l which is equal to 1, 3, 5, . . . From the law of conservation of angular momentum, we obtain for the spin of ρ-particle $I_\rho = l = 1, 3, 5, \ldots$ From the experiment, we have $I_\rho = 1$.

(b) $\rho^+ \to \pi^+ + \pi^0$; $\rho^0 \to \pi^+ + \pi^-$; $\rho^- \to \pi^- + \pi^0$. The decay $\rho^0 \to 2\pi^0$ is forbidden because in this case l must be even (the wave function is symmetric for the particles are indistinguishable); yet l cannot be even owing to the law of conservation of angular momentum (as it was shown in the solution of the foregoing item (a), the spin of the ρ-particle is odd, or, to be more precise, $I_\rho = 1$).

16.51. (a) $q_1q_1q_2$; $q_1q_2q_2$; $q_1q_1q_3$; $q_2q_2q_3$; $q_1q_2q_3$; $q_2q_3q_3$. **(b)** $q_1\tilde{q}_2$, $\tilde{q}_1q_2$, $q_1\tilde{q}_3$, $\tilde{q}_1q_3$, $q_2\tilde{q}_3$.

(c) The magnetic moments of quarks q_1 and q_2, of which a neutron $n\,(q_1q_2q_2)$ and proton $p\,(q_1q_1q_2)$ are composed, are equal to $\mu_1 = \frac{2}{3}\mu_0$ and $\mu_2 = -\frac{1}{3}\mu_0$, where μ_0 is a certain constant. Allowing for the probability of possible states, we can find the magnetic moments of a neutron and proton (in units of μ_0):

$$\mu_n = \frac{2}{3}\left(-\frac{2}{3} - \frac{1}{3} - \frac{1}{3}\right) + \frac{1}{3}\left(\frac{2}{3} - \frac{1}{3} + \frac{1}{3}\right) = -\frac{2}{3};$$

$$\mu_p=\frac{2}{3}\times\left(\frac{2}{3}+\frac{2}{3}+\frac{1}{3}\right)+\frac{1}{3}\left(\frac{2}{3}-\frac{2}{3}-\frac{1}{3}\right)=1.$$

Thus, $\mu_n/\mu_p=-2/3=0.667$ (cf. the experimental value -0.685).

17.1. $T=\frac{m}{2}\left(\frac{9e}{2m}l^2\dot{E}\right)^{2/3}=18$ eV.

17.2. Integrate the relativistic equation of motion: $\frac{d}{dt}\left(\frac{mv}{\sqrt{1-(v/c)^2}}\right)=eE$; $v=\frac{c\eta t}{\sqrt{1+\eta^2t^2}}$, $\eta=eE/mc$. Thus $T=mc^2(\sqrt{1+\eta^2\tau^2}-1)=2.5$ MeV; 2.5 m.

17.3. **(a)** $E=\frac{4T\sin^2\alpha_0}{ex}\left(\frac{y}{x}-\cot\alpha_0\right)=0.31$ kV/cm; **(b)** $\cot\alpha_0=2y/x-\cot\alpha$, $\alpha_0=103°$; $E=0.22$ kV/cm; **(c)** $t^2=\frac{2m\eta}{E}\left(1\pm\sqrt{1-\frac{x^2+y^2}{\eta^2}}\right)$, whence $\eta=y+\frac{2T}{eE}$; hence $t=0.27$ or 0.10 μs.

17.4. **(a)** $\tan\alpha=\frac{eEa}{2T}$, $\alpha\approx6°$; $\delta\left(\frac{a}{2}+b\right)\tan\alpha=2.5$ cm;

(b) $\tan\alpha=\frac{e\dot{E}a^2}{2m}\left(\frac{m}{2T}\right)^{3/2}$; $\alpha\approx3°$; $\delta=\left(\frac{a}{3}+b\right)\tan\alpha=1.2$ cm;

(c) $\tan\alpha=\frac{eE_0}{mv\omega}\left(1-\cos\frac{\omega a}{v}\right)$, $\alpha=7°$; $\sigma=\frac{eE_0}{m\omega^2}(\omega\tau-\sin\omega\tau)+b\tan\alpha=2.8$ cm, where v is the initial velocity; $\omega=2\pi\nu$; $\tau=a/v$.

17.5. $e/m=2(\nu_2-\nu_1)^2l^2/V=5\cdot10^{17}$ CGSE units/g.

17.6. $T_p=1.2$ MeV; $T_e=mc^2(\sqrt{1+(e\rho B/mc^2)^2}-1)=1.1$ MeV.

17.7. $\sin\alpha=\frac{eBa}{c\sqrt{2mT}}$, $\alpha=7°$; $\delta=\frac{v}{\omega}(1-\eta)+\frac{\omega}{v}\frac{ab}{\eta}$, where $\eta=\sqrt{1-(a\omega/v)^2}$; $\omega=eB/mc$; in the given case $(a\omega/v)^2\ll1$, therefore $\delta=\frac{eBa}{c\sqrt{2mT}}\left(\frac{a}{2}+b\right)=3$ cm.

17.8. **(a)** $\Delta z=\frac{2\pi v}{\omega}\cos\alpha=8$ cm; **(b)** $r=2\rho\left|\sin\frac{\varphi}{2}\right|=1.5$ cm. Here v is the velocity; $\omega=eB/mc$; $\rho=(v/\omega)\sin\alpha$; $\varphi=\omega l/v\cos\alpha$.

17.9. **(a)** $\frac{e}{m}=\frac{8\pi^2c^2V}{l^2(B_2-B_1)^2}=5.3\cdot10^{17}$ CGSE units/g; **(b)** 32 G.

17.10. $T=mc^2(\sqrt{1+(eBl/2\pi mc^2\cos\alpha)^2}-1)=0.24$ MeV.

17.11. **(a)** $T=mc^2\left(\sqrt{1+(l^2+\pi^2R^2)\left(\frac{eB}{2\pi mc^2}\right)^2}-1\right)=0.32$ MeV.

(b) $B=\frac{2\pi n}{el}\sqrt{\frac{T(T+2mc^2)}{1+(n\pi R/l)^2}}=\begin{cases}335\text{ G for } n=1,\\ 642\text{ G for } n=3.\end{cases}$ $\tan\alpha=n\pi R/l$, $n=1, 3, 5, \ldots$; 25° and 55°.

17.12. **(a)** $\frac{\delta x}{\delta A}=\frac{1}{2}\frac{x}{A}=0.85$ mm/a.m.u.;

(b) $\frac{\delta x}{\delta T}=\frac{T+mc^2}{T(T+2mc^2)}x=0.3$ mm/keV.

17.13. $\frac{\delta\alpha}{\delta A}=\frac{\sin\varphi}{2A}=0.62$ deg/a.m.u.

17.14. **(a)** A particle moving along a non-circular trajectory $\ddot{r}-r\dot{\varphi}^2=-eE(r)/m$. Taking into account that $r^2\dot{\varphi}=\text{const}=r^2\dot{\varphi}_0$ and $\dot{\varphi}_0=v\cos\alpha/r_0\approx v/r_0=\omega_0$, for $\alpha\ll1$, we obtain $\dot{\varphi}=(r_0/r)^2\omega_0$. Then substitute this expression into the initial equation of motion.

Introduce the parameter $\delta\ll1$, describing the deviation of r from r_0, according to the formula $r=r_0(1+\delta)$. After the appropriate transformations with due regard for $\delta\ll1$, we obtain $E=E_0(1-\delta)$ and $\ddot{\delta}+2\omega^2\delta=0$, $\omega_0=v/r_0=\sqrt{eE_0/mr_0}$ where E_0 is the field strength at $r=r_0$. The solution of this equation is $\delta=\delta_m\times\sin(\varphi\sqrt{2})$, $\varphi=\omega_0t$, where we took into account that $\delta(0)=0$. From the requirement $\delta(\Psi)=0$, we find $\Psi=\pi/\sqrt{2}$.

(b) Consider the two ions leaving the point A (see Fig. 48) along the normal of the radius vector with velocities v and $v(1+\eta)$, where $\eta\ll1$. If the first ion moves along the circle of radius r_0 and its motion is thus described as $r^2\dot{\varphi}=r_0v$, then for the second ion the equation $r^2\dot{\varphi}=r_0v(1+\eta)$ is valid. Substituting the latter expression into the initial equation of motion and taking into account that $r=r_0(1+\delta)$, where $\delta\ll1$, we obtain $\ddot{\delta}+2\omega_0^2\delta=2\omega_0^2\eta$; $\delta=\eta(1-\cos\sqrt{2}\varphi)$; $\varphi=\omega_0t$. When $\varphi=\pi/\sqrt{2}\,\delta=2\eta$, or $\Delta r/r_0=2\Delta v/v$.

17.15. **(a)** The motion of the particle in the horizontal plane is described by the equation:

$$\ddot{r}-\frac{v^2}{r}=-\frac{ev}{mc}B(r),\quad e>0.$$

Here we took into account that $B_z=-B(r)$ and also the fact that in the case of motion along the trajectory which only insignificantly differs from the equilibrium one, $\dot{r}\ll v$ and therefore $r\dot{\varphi}\approx v$.

Introduce the parameter $\delta\ll1$ describing the deviation of r from r_0, according to the formula $r=r_0(1+\delta)$. Then $\ddot{r}=r_0\ddot{\delta}$ and $B(r)=B_0(r_0/r)^n\approx B_0(1-n\delta)$, where B_0 is the induction of the magnetic field when $r=r_0$. Substituting these expressions into the initial equation of motion and taking into account that $\delta\ll1$, we obtain: $\ddot{\delta}+\omega_0^2(1-n)\delta=0$; $\omega_0=v/r_0=eB_0/mc$. The solution of this equation is $\delta=\delta_m\sin(\varphi\sqrt{1-n})$, $\varphi=\omega_0t$, where the initial condition $\delta(0) = 0$ is taken into account. From the requirement $\delta(\Psi) = 0$, we find $\Psi = \pi/\sqrt{1-n}$.

(b) The motion of the particle in the vertical plane is described by the equation: $\ddot{z} = -\frac{ev}{mc} B_r$, $e > 0$. Here we also took into account that $r\dot{\varphi} \approx v$. When the deviations z from the symmtry plane are small, $B_r \approx \left(\frac{\partial B_r}{\partial z}\right)_0 z$. Since rot $\mathbf{B} = 0$, $\frac{\partial B_r}{\partial z} = \frac{\partial B_z}{\partial r} \approx \frac{\partial B}{\partial r}$ and $B_r = n\frac{B_0}{r_0} z$. Consequently, the equation of motion takes the form $\ddot{z} + n w_0^2 z = 0$, $\omega_0 = v/r_0 = eB_0/mc$. Its solution is $z = z_m \sin(\varphi\sqrt{n})$, $\varphi = \omega_0 t$. It can be seen that at $n = 1/2$ both deviations, δ and z, turn to zero when $\varphi = \pi\sqrt{2}$,

(c) The reasoning is similar to that given in the solution of the point **(b)** of the foregoing problem. Assuming $\Delta v/v = \eta$, we obtain

$$\ddot{\delta} + (1-n)\omega_0^2\delta = \omega_0^2\eta;\quad \delta = \frac{\eta}{1-n}(1-\cos\varphi\sqrt{1-n}),\quad \varphi = \omega_0 t.$$

When $n = 1/2$, $\varphi = \pi\sqrt{2}$ and $\delta = 4\eta$, or $\Delta r/r_0 = 4\Delta v/v$.

17.16. 42 keV.

17.17. **(a)** $v = \frac{r_1}{r_2}\cdot\frac{cE}{B}$; $\frac{e}{m} = \frac{r_1}{r_2}\cdot\frac{c^2E}{B^2}$, where $E = \frac{V}{r_1\ln(R_2/R_1)}$;

(b) $v = \frac{cE}{B}$; $\frac{e}{m} = \frac{2\delta}{\delta^2 + l^2}\cdot\frac{c^2E}{B^2}$.

17.18. $B_{\lim} = \frac{2cr_2}{r_2^2 - r_1^2}\sqrt{\frac{2mV}{e}} = 0.05$ kG.

17.19. $V_{\lim} = \frac{2e}{mc^4}\left(I\ln\frac{r_2}{r_1}\right)^2 = 4$ V.

17.20. The equations of motion of the particle: $\ddot{x} = -\omega\dot{z}$, $\ddot{y} = a$, $\ddot{z} = \omega\dot{x}$, where $a = eE/m$, $\varphi = eB/mc$. Their solutions are $x = \frac{v}{\omega}\sin\omega t$; $y = \frac{a}{2}t^2$; $z = \frac{v}{\omega}(1-\cos\omega t)$.

17.21. $\tan\sqrt{\frac{e}{m}\frac{y}{A}} = \frac{z}{l}$, where $A = \frac{2c^2E}{B^2}$; when $z \ll l$, $y = A\frac{m}{e}\left(\frac{z}{l}\right)^2$.

17.22. **(a)** The equations of motion of the particle are: $\ddot{x} = \omega\dot{y}$; $\ddot{y} = a - \omega\dot{x}$, where $\omega = eB/mc$, $a = eE/m$. Their solutions are $x = \frac{a}{\omega^2}(\omega t - \sin\omega t)$; $y = \frac{a}{\omega^2}(1-\cos\omega t)$. This is the equation of a cycloid

(Fig. 73). The particle's motion can be visualized as a rotation angular velocity ω along the circle whose centre is displaced with constant velocity a/ω; **(b)** $8a/\omega^2$; **(c)** $\langle\dot{x}\rangle = a/\omega$.

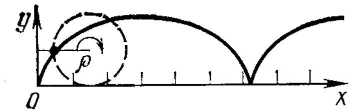


Fig. 73

17.23. $x = \frac{a}{\omega^2}\left[\omega t - \left(1 - \frac{\omega\dot{x}_0}{a}\right)\sin\omega t + \frac{\omega\dot{y}_0}{x}(1-\cos\omega t)\right]$; $y = \frac{a}{\omega^2}\left[\frac{\omega\dot{y}_0}{a}\sin\omega t + \left(1 - \frac{\omega\dot{x}_0}{a}\right)(1-\cos\omega t)\right]$, where $a = eE/m$; $\omega = eB/mc$.

(a) $x = \frac{a}{\omega^2}\left(\omega t - \frac{1}{2}\sin\omega t\right)$; $y = \frac{a}{2\omega^2}(1-\cos\omega t)$.

(b) $x = \frac{a}{\omega^2}(\omega t - 2\sin\omega t)$; $y = 2\frac{a}{\omega^2}(1-\cos\omega t)$.

(c) $x = \frac{a}{\omega^2}(1 + \omega t - \sin\omega t - \cos\omega t)$; $y = \frac{a}{\omega^2}(1 + \sin\omega t - \cos\omega t)$.

(d) $x = \frac{a}{\omega^2}(1 + \omega t - \cos\omega t)$; $y = \frac{a}{\omega^2}\sin\omega t$. The corresponding curves (trochoids) are shown in Fig. 74.

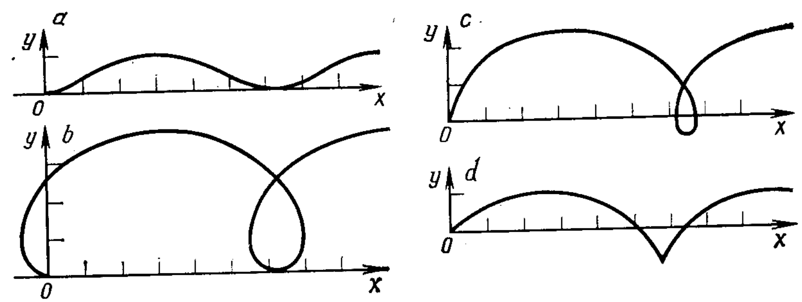


Fig. 74

17.24. From the equations of motion derived in the solution of the foregoing problem, it follows that the equation $y(\omega t) = 0$ possesses the roots of two types, one of which depends on the initial conditions and the order does not. We are interested in the latter type of roots: $\omega t_n = 2\pi n$, n is an integer. For $n = 1$, we have $x_1 = \frac{a}{\omega^2}\omega t_1 = 2\pi\frac{a}{\omega^2}$.

17.25. $x=\frac{a}{2\omega^2}(\sin\omega t-\omega t\cos\omega t)$; $y=\frac{a}{2\omega^2}\omega t\sin\omega t$, where $a=\frac{eE_0}{m}$. The trajectory has the shape of an unwinding spiral.

17.26. (a) Use the equations $m\ddot{r}=eE_r$, $mv^2/2=-eV_0$, where the minus sign in the latter equation is due to the fact that $V_0<0$. Express E_r as a derivative of the potential on the axis with respect to z. To do this, separate a small imaginary cylinder in the vicinity of z axis (Fig. 75) with the height δz and radius r. From Gauss's theorem, $\oint E_n\,dS=0$, it follows:

$$2\pi r\delta z E_r+\pi r^2\frac{\partial E_z}{\partial z}\delta z=0;$$

$$E_r=-\frac{r}{2}\cdot\frac{\partial E_z}{\partial z}\approx\frac{r}{2}V_0''.$$

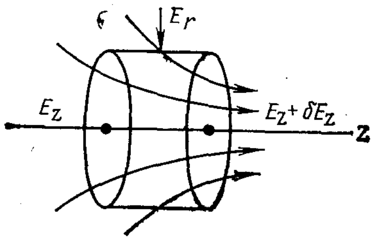


Fig. 75

Pass from differentiating with respect to time in equations of motion to differentiating with respect to z:

$$\ddot{r}=v\frac{\partial}{\partial z}(r'v)=v^2r''+vv'r'=\frac{e}{m}\cdot\frac{r}{2}V_0''.$$

Now it remains to take into account that $v^2=-2eV_0/m$ and $vv'=-eV_0'/m$.

(b) The particles with different values of e/m will move along the identical trajectories under the same initial conditions since the trajectory equation does not contain the quantity e/m. In the second case the trajectory also increases n-fold, retaining its form, which follows from the linearity of the equation with respect to r and its derivatives.

17.27. (a) Transform the equation given in the foregoing problem so that it can be easily integrated. To do this, divide all the terms by $\sqrt{V_0}$ and transfer the last term to the right-hand side: $\frac{\partial}{\partial z}(r'\sqrt{V_0})=-\frac{r}{4}V_0^{-1/2}V_0''$. Integrate this equation with respect to z between the points z_1 (1) and z_2 (2), taking into account that within these limits $r\approx r_0=\text{const}$:

$$(r'\sqrt{V_0})_2-(r'\sqrt{V_0})_1=-\frac{r_0}{4}\int_1^2 V_0^{-1/2}V_0''\,dz.$$

Inasmuch as the field is practically absent outside the lens, the value of the integral will not change if the integration limits are replaced by $-\infty$ and $+\infty$. Finally, it should be taken into account

that $r_1'=r_0/(-s_1)$ and $r_2'=-r_0/s_2$ (see Fig. 52).

(b) When $s_1=-\infty$, $s_2=f_2$ and $\frac{n^2}{f_2}=\frac{1}{4}\int_{-\infty}^{\infty}V_0^{-1/2}V_0''\,dz=\frac{1}{4}\times\left(V_0^{-1/2}V_0'\Big|_{-\infty}^{\infty}+\frac{1}{2}\int_{-\infty}^{\infty}V_0^{-3/2}V_0'\,dz\right)$, where $V_0'(\pm\infty)=0$, since the field is absent outside the lens.

17.28. Use the equations given in the introduction to this chapter (in our case $E_r=E_z=0$).

First of all, express B_z and B_r in terms of the induction on the axis. B_z differs only slightly from B_0, and we can therefore assume that $B_z=B_0$. To determine B_r, we proceed as in the solution of Problem 17.26, when we derived the component E_r. This way we obtain: $B_r=-\frac{r}{2}\cdot\frac{\partial B_z}{\partial z}\approx-\frac{r}{2}B_0$. Now we integrate the second equation of motion from 0 to z, taking into account that the particle leaves the source located on the axis ($r_0^2\dot{\varphi}_0=0$):

$$r^2\dot{\varphi}=-\frac{e}{mc}\int_0^z r\left(\frac{r}{2}B_0'+r'B_0\right)dz=-\frac{e}{mc}\int d\left(\frac{r^2B_0}{2}\right)=$$

$$=-\frac{e}{2mc}r^2B_0.$$

Since $\dot{\varphi}=v\varphi'$, we obtain $\varphi'=-\alpha B_0$. Now substitute the latter expression for $\dot{\varphi}$ into the first equation of motion. Taking into account that $\ddot{r}=v^2r''$, we obtain after simple transformations the second equation given in the text of the problem.

17.29. Integrate the equation for $r(z)$ with respect to z between points 1 and 2 (see Fig. 52), noting that within these limits $r\approx r_0=\text{const}$:

$$r_2'-r_1'=-\left(\frac{e}{2mcv}\right)^2 r_0\int_1^2 B_0^2(z)\,dz.$$

Since (according to the condition) there is no field outside the lens, the integral will keep its value if the limits of integration are replaced by $-\infty$ and $+\infty$. Besides, taking into account that $r_1'=r_0/(-s_1)$ and $r_2'=-r_0/s_2$, and putting s equal to $-\infty$ (with $s_2=f$), we obtain the sought expression; **(a)** $f=8\sqrt{\frac{2}{\pi}}\frac{mc^2va}{eA^2}$; **(b)** $f=\frac{16c^4RV}{3\pi^{3/2}}=0.5$ m. Integrating the equation for φ' (see the foregoing problem), we get: $\Delta\varphi=\pi I/\sqrt{2e/mV}/c^2=0.37$ rad or 21°.

17.30. **(a)** $\tau = 2\pi(T+mc^2)/ceB$; $7.3\cdot 10^{-10}$ and $6.6\cdot 10^{-8}$ s; $r = \sqrt{T(T+2mc^2)}/eB = 3.5$ and 46 cm; **(b)** $T = mc^2 \times (\sqrt{1+(erB/mc^2)^2}-1)$; 2.9 MeV and 5.9 keV.

17.31. **(a)** $E = \dot{\Phi}/2\pi rc = 0.32$ V/cm; $T = ne\dot{\Phi}/c = 25$ MeV; **(b)** $l = \frac{c}{\alpha}(\sqrt{1+\alpha^2\tau^2}-1) = 0.9\cdot 10^3$ km, where $\alpha = eE/mc$ (see the solution of Problem 17.2) $T = eEl = 28$ MeV.

17.32. **(a)** $T = mc^2(\sqrt{1+(erB_m/mc^2)^2}-1) = 0.15$ GeV.

(b) $L = \int\limits_0^{T/4} v\,dt = \frac{c}{\omega}\arcsin\frac{1}{\sqrt{1+A^2}}$, $A = mc^2/erB_m$, where v is found from the formula $p = \frac{e}{c}rB$. In the case considered here $A \ll 1$, so that $\arcsin(1+A^2)^{-\frac{1}{2}} \approx \frac{\pi}{2}$ and $L = c/4\nu = 1{,}5\cdot 10^3$ km; $2.4\cdot 10^5$ revolutions.

17.33. **(a)** On the one hand, $\frac{dp}{dt} = -eE = \frac{e}{2\pi cr}\frac{d\Phi}{dt}$; on the other hand, dp/dt can be found, having differentiated the relation $p = \frac{e}{c}rB$ with respect to time at $r = \text{const}$. Comparing the expression obtained, we find that $\dot{B} = \langle\dot{B}\rangle/2$. In particular, this condition will hold with $B = \langle B\rangle/2$.

(b) Differentiating the expression $pc = erB$ with respect to time, where $pc = \sqrt{E^2-m^2c^4}$, we obtain

$$\frac{c}{v}\cdot\frac{dE}{dt} = e\left(B\frac{dr}{dt}+r\frac{\partial B}{\partial t}+r\frac{\partial B}{\partial r}\cdot\frac{dr}{dt}\right).$$

Since $\frac{dE}{dt} = fv = \frac{e\dot{\Phi}}{2\pi cr}v = \frac{evr\langle\dot{B}\rangle}{2c}$, it follows from the latter expression that $\frac{dr}{dt} = \frac{r(\langle\dot{B}\rangle-2\dot{B})}{2B(1-n)}$, $n = -\frac{r}{B}\cdot\frac{\partial B}{dr}$, where n is the fall-off index of the field. It can be seen that when $0<n<1$, the derivative $dr/dt>0$ (the orbital radius grows), provided $\langle\dot{B}\rangle > 2\dot{B}$, and vice versa.

17.34. Differentiating the expression $E = \dot{\Phi}/2\pi cr$ with respect to r (E is the modulus of the vector of the electric field strength) and taking into account that $\partial\dot{\Phi}/\partial r = 2\pi r\dot{B}(r)$, we obtain $\partial E/\partial r = 0$ and $\partial^2E/\partial r^2 > 0$.

17.35. **(a)** In the frame rotating about the field's axis with the angular velocity of the electron, the electron experiences the centrifugal force of inertia in addition to the Lorentz force. The resultant

force $f(r) = \frac{mv^2}{2} - \frac{e}{c}vB(r)$, and when $r = r_0$, $f(r_0) = 0$. The motion is stable if the force f is the restoring one, i.e. when $r > r_0$, $f < 0$ and vice versa. It is easy to see that this is the case when $B(r)$ diminishes slower than $1/r$, i.e. $n < 1$.

(b) Since the field falls off toward the periphery, it has the barrel shape, i.e. there is a radial component B_r outside the plane of symmetry. The latter component produces the vertical component of the Lorentz force $f_z = \frac{e}{c}vB_r$. In the vicinity of the plane of symmetry $B_r = (\partial B_r/\partial z)_0\, z$. Since $\text{rot}\,\mathbf{B} = 0$, $\partial B_r/\partial z = \partial B_z/\partial r$. Thereefore $f_z = \frac{e}{c}v\frac{\partial B}{\partial r}z$. If $n>0$ $\partial B/\partial r<0$ and the force f_z is always, directed toward the plane of symmetry.

17.36. **(a)** $\omega_r = \omega_0\sqrt{1-n}$; **(b)** $\omega_z = \omega_0\sqrt{n}$, where $\omega_0 = eB/mc$ (see the solution of Problem 17.15).

17.37. $E = mc^2\sqrt[4]{1.5r^3\dot{B}/ce} = 0.29$ GeV.

17.38. **(a)** $T = (e\rho B)^2/2mc^2$; 6, 12 and 12 MeV; **(b)** $\nu = \sqrt{T/2m}/\pi\rho$; 20, 14 and 10 MHz.

17.39. $V = 2\pi^2 m\nu^2\rho\Delta\rho/e = 0.2$ MV.

17.40. **(a)** $t = \pi^2 m\rho^2\nu/qV = 10\ \mu$s, where q is the particle's charge; **(b)** $L = \frac{1}{2\nu}\sum\limits_1^N v_n = \frac{1}{2\nu}\sqrt{2qV/m}\sum\limits_1^N\sqrt{n}$, where v_n is the velocity of the particle after the nth passing of the accelerating gap, N is the total number of passings. In the given case N is large ($N = T_{max}/qV$), and therefore, $\Sigma\sqrt{n} \approx \int\sqrt{n}\,dn$. Thus, $L \approx 4\pi^3 m\nu^2\rho^3/3qV = 0.2$ km.

17.41. $T = mc^2\,\Delta\tau/\tau_0$; 5.1 keV, 9.4 and 37 MeV.

17.42. $n = 2\pi f\,\Delta E/ceB = 9$.

17.43. **(a)** $\frac{\nu_0-\nu}{\nu_0} = \frac{T}{T+mc^2}$, 35 and 12%; **(b)** the angular velocity of the particle is related to its total energy E as $E = ceB/\omega$. Thus, $\frac{dE}{dt} = -\frac{ceB}{\omega^2}\cdot\frac{d\omega}{dt}$. On the other hand, $\frac{dE}{dt} = \frac{\varepsilon}{T} = \frac{\varepsilon\omega}{2\pi}$. From these formulas, we obtain: $\frac{d\omega}{dt} = -\frac{\varepsilon}{2\pi ceB}\omega^3$. After integration, we get $\omega(t) = \omega_0/\sqrt{1+At}$; $\omega_0 = eB/mc$; $A = \varepsilon\omega_0/\pi mc^2$.

17.44. **(a)** $r_t = \frac{c}{\omega_0}\sqrt{1-\left(\frac{mc\omega_0}{eB_m}\right)^3\frac{1}{\sin^2\omega t}}$;

(b) from 42.0 to 42.9 cm, $L = \omega_0\langle r\rangle/2\omega = 1.5\cdot 10^3$ km.

17.45. $\omega(t) = \frac{c}{r_0\sqrt{1+A^2(t)}}$, $A = mc^2/erB(t)$.

17.46. **(a)** From 0.35 to 1.89 MHz, 0.82 s; **(b)** $\delta E = 2\pi e r^2 \dot{B}/c = 0.19$ keV; **(c)** $1.5 \cdot 10^5$ km; $5.2 \cdot 10^6$ revolutions.

17.47. **(a)** $\nu_t = \frac{c}{\Pi} \cdot \frac{c}{\sqrt{1+A^2(t)}}$, $A = mc^2/erB(t)$, 0.20 and 1.44 MHz, 3.2 s; **(b)** $\delta E = er\Pi\dot{B}/c = 2.33$ keV; **(c)** $9 \cdot 10^5$ km; $4.3 \cdot 10^6$ revolutions.

17.49. **(a)** The length of the nth drift tube is $l_n = \frac{c}{2f} \times \sqrt{1-(mc^2/E_n)^2}$, where E_n is the total energy of the proton in the nth drift tube and $E_n = mc^2 + T_0 + n\Delta E$. In this case $T_0 + n\Delta E \ll mc^2$, and therefore $l_n = 4.9\sqrt{4+n}$ cm. The number of drift tubes is $N = 35$; $l_1 = 11$ cm, $l_{35} = 31$ cm; **(b)** $L = \sum l_n \approx 4.9 \int_1^N \sqrt{4+n}\, dn = 7.5$ m.

17.50. From 258 to 790 MHz; from $2.49 \cdot 10^9$ to $2.50 \cdot 10^9$ Hz.

17.51. **(a)** $E_x = (T_2 - T_1)/eL = 0.15$ MV/cm; **(b)** $v = c\sqrt{1+[mc^2/E(x)]^2}$, where $E(x) = mc^2 + T_1 + eE_x$; by a factor of 9.5, by 0.65%.

17.52. $5.5 \cdot 10^3$ GeV (see the solution of Problem 16.3).

APPENDICES

1. Units of measurements and their symbols

Unit	Symbol	Unit	Symbol	Unit	Symbol
ampere	A	gauss	G	newton	N
angstrom	Å	gram	g	oersted	Oe
barn	b	hertz	Hz	pascal	Pa
calorie	cal	joule	J	second	s
coulomb	C	kelvin	K	steradian	sr
dyne	dyn	maxwell	Mx	volt	V
electronvolt	eV	metre	m	watt	W
erg	erg	mole	mol	weber	Wb

Decimal prefixes for the names of units:

E	exa	(10^{18})	h	hecto	(10^2)	n	nano	(10^{-9})
P	peta	(10^{15})	da	deca	(10^1)	p	pico	(10^{-12})
T	tera	(10^{12})	d	deci	(10^{-1})	f	femto	(10^{-15})
G	giga	(10^9)	c	centi	(10^{-2})	a	atto	(10^{-18})
M	mega	(10^6)	m	milli	(10^{-3})			
k	kilo	(10^3)	μ	micro	(10^{-6})			

2. *K*- and *L*-absorption of *X*-ray radiation

Z	Element	Absorption edge, Å			
		K	L_I	L_{II}	L_{III}
23	V	2.268	—	23.9	24.1
26	Fe	1.741	—	17.10	17.4
27	Co	1.604	—	15.46	15.8
28	Ni	1.486	—	14.11	14.4
29	Cu	1.380	—	12.97	13.26
30	Zn	1.284	—	11.85	12.1
42	Mo	0.619	4.305	4.715	4.91
47	Ag	0.4860	3.236	3.510	3.695
50	Sn	0.4239	2.773	2.980	3.153
74	W	0.1785	1.022	1.073	1.215
78	Pt	0.1585	0.888	0.932	1.072
79	Au	0.1535	0.861	0.905	1.038
82	Pb	0.1405	0.781	0.814	0.950
92	U	0.1075	0.568	0.591	0.722

3. Some properties of metals

Metal	A, eV	Density, g/cm³	Crystal structure: type	Crystal structure: lattice constant, Å: a	Crystal structure: lattice constant, Å: c	Temperature: Debye θ, K	Temperature: melting, °C
Aluminium	3.74	2.7	cfc	4.04		374	658.7
Barium	2.29	3.75	csc	5.02		116	704
Beryllium	3.92	1.85	hcp	2.28	3.58	1100	1278
Bismuth	4.62	9.8	hex	4.54	11.84	80	271
Cesium	1.89	1.87	csc	6.05		60	28.5
Cobalt	4.25	8.9	hcp	2.51	4.07	397	1480
Copper	4.47	8.9	cfc	3.61		329	1083
Gold	4.58	19.3	cfc	4.07		164	1063
Iron	4.36	7.8	csc	2.86		467	1535
Lead	4.15	11.3	cfc	4.94		89	327.5
Lithium	2.39	0.53	csc	3.50		404	186
Magnesium	3.69	1.74	hcp	3.20	5.20	350	650
Molybdenum	4.27	10.2	csc	3.14		357	2620
Nickel	4.84	8.9	cfc	3.52		425	1452
Platinum	5.29	21.5	cfc	3.92		212	1775
Potassium	2.15	0.86	csc	5.25		132	62.3
Silver	4.28	10.5	cfc	4.08		210	960
Sodium	2.27	0.97	csc	4.24		226	97.5
Tin	4.51	7.4	tsc	5.82	3.18	141	231.9
Titanium	3.92	4.5	hcp	2.95	4.69	300	1720
Tungsten	4.50	19.1	csc	3.16		315	3370
Vanadium	3.78	5.87	csc	3.03		413	1715
Zinc	3.74	7.0	hcp	2.66	4.94	213	419.4

Notation. A is the work function; cfc — cubic face-centered; csc — cubic space-centered; hex — hexagonal; hcp — hexagonal close-packed; tsc — tetragonal space-centered.

4. Density of substances

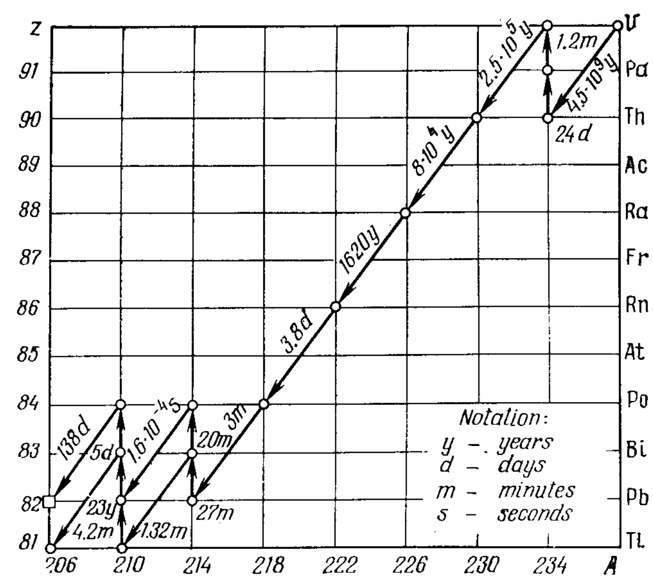
Substance	Density, g/cm³	Substance	Density, g/cm³
Air	$1.293\cdot10^{-3}$	Plutonium	19.8
Beryllium oxide BeO	3.03	Selenium	4.5
Boron	2.45	Silicon	2.35
Cadmium	8.65	Strontium	2.54
Germanium	5.46	Sulphur	2.1
Graphite	1.60	Tellurium	6.02
Heavy water D_2O	1.10	Thorium	11.7
Indium	7.28	Uranium	19.0
Mercury	13.6	Water	1.00
Paraffin CH_2	0.89	NaCl	2.18
Phosphorus	1.83	CsCl	4.04

Note. The densities of other metals are given in the foregoing table.

5. Constants of diatomic molecules

Molecule	Basic term	Internuclear distance d, 10^{-8} cm	Vibration frequency, $\bar{\nu}$, cm⁻¹	Anharmonicity coefficient x, 10^{-3}	Dissociation energy D, eV
H_2	$^1\Sigma$	0.741	4395.2	28.5	4.48
N_2	$^1\Sigma$	1.094	2359.6	6.15	7.37
O_2	$^3\Sigma$	1.207	1580.4	7.65	5.08
F_2	$^1\Pi$	1.282	1139.8	8.51	~ 1.6
P_2	$^1\Sigma$	1.894	780.4	3.59	5.03
S_2	$^3\Sigma$	1.889	725.7	3.93	~ 4.4
Cl_2	$^1\Sigma$	1.988	564.9	7.09	2.48
Br_2	$^1\Sigma$	2.283	323.2	3.31	1.97
I_2	$^1\Sigma$	2.666	214.6	2.84	1.54
HF	$^1\Sigma$	0.917	4138.5	21.8	5.8
HCl	$^1\Sigma$	1.275	2989.7	17.4	4.43
HBr	$^1\Sigma$	1.413	2649.7	17.1	3.75
HI	$^1\Sigma$	1.604	2309.5	17.2	3.06
CO	$^1\Sigma$	1.128	2170.2	6.22	~ 9.7
NO	$^2\Pi$	1.150	1906	7.55	5.29
OH	$^2\Pi$	0.971	3735	22.2	4.35

6. Radioactive Uranium Family



7. Properties of nuclides

Z	Nuclide	Nuclear spin	Surplus of atomic mass $M-A$, a.m.u.	Natural abundance, %	Type of decay	Half-life	Energy of α- and β-particles $T_{\beta\,max}$, MeV
	n	1/2	0.008665	—	β^-	11.7 min	0.78
1	H	1/2	0.007825	99.985			
	^{2}H	1	0.014102	0.015			
	^{3}H	1/2	0.016049	—	β^-	12.3 years	0.018
2	^{3}He	1/2	0.016030	$3\cdot10^{-4}$			
	^{4}He	0	0.002604	~ 100			
3	^{6}Li	1	0.015126	7.52			
	^{7}Li	3/2	0.016005	92.48			
4	^{7}Be	3/2	0.016931	—	*K*	53 days	
	^{8}Be	0	0.005308	—	2α	10^{-16} s	0.039
	^{9}Be	3/2	0.012186	100			
	^{10}Be	0	0.013535	—	β^-	$2.5\cdot10^{6}$ years	0.555
5	^{10}B	3	0.012939	20			
	^{11}B	3/2	0.009305	80			
6	^{11}C	3/2	0.011431	—	β^+	20.4 min	0.97
	^{12}C	0	0	98.89			
	^{13}C	1/2	0.003354	1.11			
	^{14}C	0	0.003242	—	β^-	5570 years	0.155
7	^{13}N	—	0.005739	—	β^+	10 min	1.2
	^{14}N	1	0.003074	99.63			
	^{15}N	1/2	0.000108	0.37			
8	^{15}O	—	0.003072	—	β^+	2.1 min	1.68
	^{16}O	0	—0.005085	99.76			
	^{17}O	5/2	—0.000867	0.037			
	^{18}O	0	—0.000840	0.204			
9	^{18}F	—	0.000950	—	β^+	1.87 h	0.649
	^{19}F	1/2	—0.001595	100			
	^{20}F	—	—0.000015	—	β^-	12 s	5.42
10	^{20}Ne	0	—0.007560	90.92			
	^{21}Ne	—	—0.006151	0.26			
	^{22}Ne	0	—0.008616	8.82			
11	^{22}Na	3	—0.005565	—	β^+	2.6 years	0.540
	^{23}Na	3/2	—0.010227	100			
	^{24}Na	4	—0.009033	—	β^-	15 h	1.39
2	^{23}Mg	—	—0.005865	—	β^+	11 s	2.95
	^{24}Mg	0	—0.014956	78.60			
	^{25}Mg	5/2	—0.014160	10.11			
	^{26}Mg	0	—0.017409	11.29			
	^{27}Mg	1/2	—0.015655	—	β^-	9.5 min	1.75 and 1.59
	^{26}Al	—	—0.013100	—	β^+	6.7 s	3.20
	^{27}Al	5/2	—0.018465	100			
	^{28}Al	3	—0.018092	—	β^-	2.3 min	2.86
14	^{28}Si	0	—0.023073	92.27			
	^{29}Si	1/2	—0.023509	4.68			
	^{30}Si	0	—0.026239	3.05			
	^{31}Si	—	—0.024651	—	β^-	2.65 h	1.47
15	^{30}P	—	—0.021680	—	β^+	2.5 min	3.24
	^{31}P	1/2	—0.026237	100			
	^{32}P	—	—0.026092	—	β^-	14.3 days	1.71

Z	Nuclide	Nuclear spin	Surplus of atomic mass $M-A$, a.m.u.	Natural abundance, %	Type of decay	Half-life	Energy of α and β-particles $T_{\beta\,max}$, MeV
16	^{32}S	0	—0.027926	95.02			
	^{33}S	3/2	—0.028540	0.75			
	^{34}S	0	—0.032136	4.21			
	^{35}S	3/2	—0.030966	—	β^-	87 days	0.167
17	^{35}Cl	3/2	—0.031146	75.4			
	^{36}Cl	2	—0.031688	—	β^-, *K*	$3.1\cdot10^{5}$	0.714
	^{37}Cl	3/2	—0.034104	24.6			
18	^{36}Ar	0	—0.032452	0.34			
	^{37}Ar	3/2	—0.033228	—	*K*	32 days	
	^{39}Ar	—	—0.035679	—	β^-	265 years	0.565
	^{40}Ar	0	—0.037616	99.60			
19	^{39}K	3/2	—0.036286	93.08			
	^{42}K	2	—0.037583	—	β^-	1.52 h	3.55 and 1.99
24	^{51}Cr	7/2	—0.055214	—	*K*	28 days	
25	^{55}Mn	5/2	—0.061946	100			
27	^{58}CO	2	—0.064246	—	*K*, β^+	72 days	0.47
	^{59}Co	7/2	—0.066811	100			
	^{60}Co	4	—0.066194	—	β^-	5.2 years	0.31
29	^{63}Cu	3/2	—0.070406	69.1			
	^{65}Cu	3/2	—0.072214	30.9			
30	^{65}Zn	5/2	—0.070766	—	*K*, β^+	245 days	0.325
35	^{82}Br	6	—0.083198	—	β^-	36 h	0.456
38	^{88}Sr	0	—0.09436	82.56			
	^{89}Sr	5/2	—0.09257	—	β^-	51 days	1.46
	^{90}Sr	0	—0.09223	—	β^-	28 years	0.535
39	^{90}Y	2	—0.09282	—	β^-	64 h	2.24
47	^{107}Ag	1/2	—0.09303	51.35			
53	^{127}I	5/2	—0.09565	100			
	^{128}I	1	—0.09418	—	β^-, *K*	25 min	2.12 and 1.67
79	^{197}Au	3/2	—0.03345	100			
	^{198}Au	2	—0.03176	—	β^-	2.7 days	0.96
81	^{204}Tl	—	—0.02611	—	β^-	4.1 years	0.77
82	^{206}Pb	0	—0.02554	23.6			
	^{207}Pb	1/2	—0.02410	22.6			
	^{208}Pb	0	—0.02336	52.3			
83	^{209}Bi	9/2	—0.01958	100			
	^{210}Bi	4	—0.01589	—	α	$2.6\cdot10^{6}$ years	4.97
84	^{210}Po	—	—0.01713	—	α	138 days	5.3
86	^{222}Rn	—	0.01753	—	α	3.8 years	5.49
88	^{226}Ra	0	0.02536	—	α	1620 years	4.777 and 4.589
90	^{232}Th	0	0.03821	100	α	$1.4\cdot10^{10}$ years	4.00 and 3.98
	^{233}Th	—	0.04143	—	β^-	22 min	1.23
92	^{234}U	0	0.04090	0.006	α	$2.5\cdot10^{5}$ years	4.76 and 4.72
	^{235}U	7/2	0.04393	0.71	α	$7.1\cdot10^{8}$ years	4.20-4.58
	^{236}U	0	0.04573	—	α	$2.4\cdot10^{7}$ years	4.45 and 4.50
	^{238}U	0	0.05076	99.28	α	$4.5\cdot10^{9}$ years	4.13 and 4.18
	^{239}U	—	0.05432	—	β^-	23.5 min	1.21
94	^{238}Pu	—	0.04952	—	α	89.6 years	5.50 and 5.45
	^{239}Pu	1/2	0.05216	—	α	$2.4\cdot10^{4}$ years	5.15-5.10

8. Neutron cross-sections

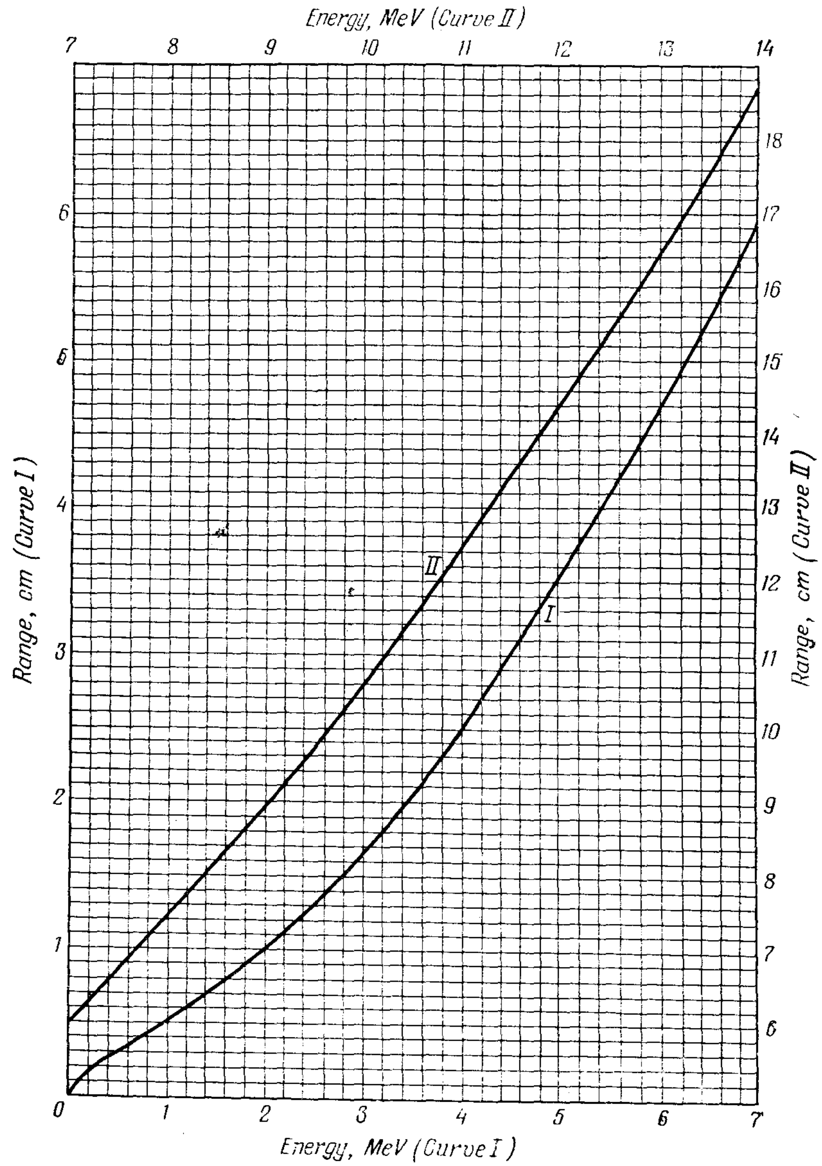
Element	Nuclide	Natural abundance, %	Halflife of nuclide produced	Cross-section, b		
				absorption σ_a	activation σ_{act}	scattering $\langle\sigma_{sc}\rangle$
H	^{2}H	0.015	12.3 years	$5\cdot10^{-4}$	$5.7\cdot10^{-4}$	7
Li	—	—	—	71	—	1.4
	^{6}Li	7.52	—	945 ($n\alpha$)	$2.8\cdot10^{-2}$	—
	^{7}Li	92.48	0.85 s	—	$3.3\cdot10^{-2}$	—
Be	^{9}Be	100	$2.7\cdot10^{6}$ years	10^{-2}	$9\cdot10^{-4}$	7
B	—	—	—	755	—	4
	^{10}B	20	—	3813 ($n\alpha$)	0.5	—
	^{11}B	80	0.03 s	—	$5\cdot10^{-2}$	—
C	—	—	—	$3.8\cdot10^{-3}$	—	4.8
	^{12}C	98.89	—	—	$3.3\cdot10^{-3}$	—
	^{13}C	1.11	5570 years	$0.5\cdot10^{-3}$	$9\cdot10^{-4}$	—
O	—	—	—	$2\cdot10^{-4}$	—	4.2
F	^{19}F	100	11 s	$<10^{-2}$	$9\cdot10^{-3}$	3.9
Na	^{23}Na	100	15 h	0.53	0.53	4
Al	^{27}Al	100	2.3 min	0.23	0.21	1.4
V	—	—	—	4.98	—	5
	^{50}V	0.24	—	250	—	—
	^{51}V	99.76	3.76 min	—	4.5	—
Cu	—	—	—	3.77	—	7.2
	^{63}Cu	69.1	12.8 h	4.5	4.5	—
	^{65}Cu	30.9	5.15 min	2.2	1.8	—
Ag	—	—	—	63	—	6
Cd	—	—	—	2540	—	7
	^{113}Cd	12.26	—	20 000	—	—
In	—	—	—	196	—	2.2
	^{115}In	99.77	54.2 min	—	155	—
I	^{127}I	100	25 min	6.22	$5.6\cdot10^{-3}$	3.6
Au	^{127}Au	100	2.7 days	98.8	96	9.3
U	—	—	—	7.68	—	8.3
	^{238}U	99.28	23.5 min	2.75	2.74	11.2

σ_a and σ_{act} are the cross-sections for thermal neutrons (2200 m/s); $\langle\sigma_{sc}\rangle$ are the cross-sections averaged over sufficiently wide energy interval.

9. Constants of fissionable nuclides
(due to thermal neutrons, 2200 m/s)

Nuclide	Natural abundance, %	Cross-section, b		Mean number of neutrons per fission	
		absorption σ_a	fission σ_f	instantaneous ν	delayed μ
^{233}U	—	588±4	532±4	2.52	0.0066
^{235}U	0.71	694±3	582±4	2.47	0.0158
^{239}Pu	—	1025±8	738±4	2.91	0.0061

10. Free Path vs. Energy Dependence for α-particles in Air

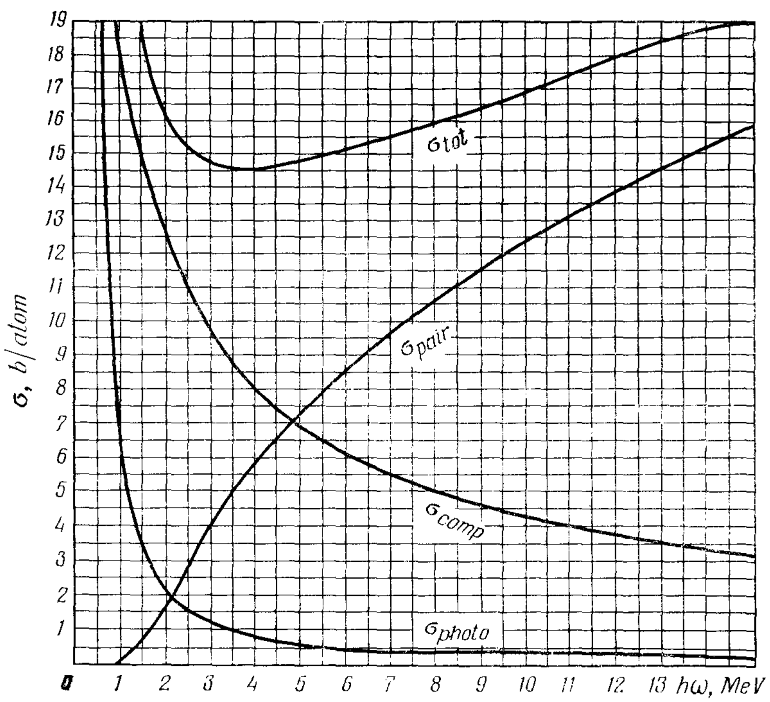


11. Attenuation and absorption coefficients for γ-quanta

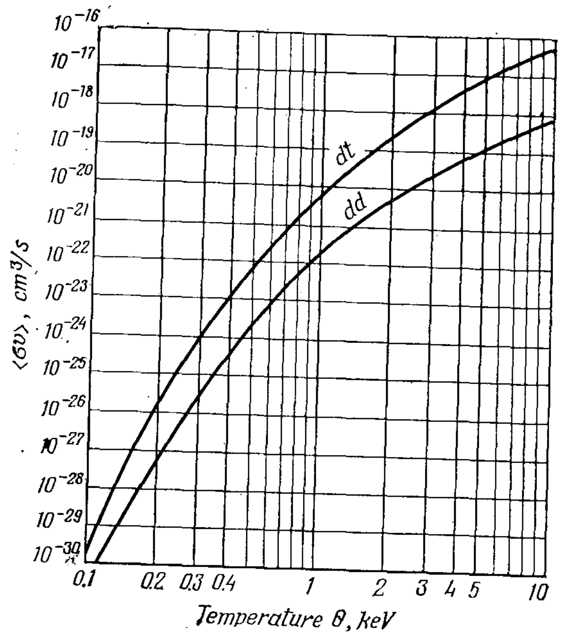
Energy MeV	Aluminium		Lead		Water		Air	
	μ/ρ	τ/ρ	μ/ρ	τ/ρ	μ/ρ	τ/ρ	μ/ρ	τ/ρ
0.1	0.169	0.0371	5.46	2.16	0.171	0.0253	0.155	0.0233
0.2	0.122	0.0275	0.942	0.586	0.137	0.0299	0.123	0.0269
0.4	0.0927	0.0287	0.220	0.136	0.106	0.0328	0.0953	0.0295
0.6	0.0779	0.0286	0.119	0.0684	0.0896	0.0329	0.0804	0.0295
0.8	0.0683	0.0278	0.0866	0.0477	0.0786	0.0321	0.0706	0.0288
1.0	0.0614	0.0269	0.0703	0.0384	0.0706	0.0310	0.0635	0.0276
1.5	0.0500	0.0246	0.0550	0.0280	0.0590	0.0283	0.0515	0.0254
2.0	0.0431	0.0227	0.0463	0.0248	0.0493	0.0260	0.0445	0.0236
3.0	0.0360	0.0201	0.0410	0.0238	0.0390	0.0227	0.0360	0.0211
4.0	0.0310	0.0188	0.0421	0.0253	0.0339	0.0204	0.0307	0.0193
6.0	0.0264	0.0174	0.0436	0.0287	0.0275	0.0178	0.0250	0.0173
8.0	0.0241	0.0169	0.0459	0.0310	0.0240	0.0163	0.0220	0.0163
10.0	0.0229	0.0167	0.0189	0.0328	0.0219	0.0154	0.0202	0.0156

μ/ρ and τ/ρ are the coefficients of mass attenuation (for a narrow beam) and absorption, cm^2/g.

12. Interaction Cross-Sections for γ-Quanta in Lead



13. The Graph of ⟨σv⟩ vs. Plasma Temperature



14. The Values of Some Definite Integrals

$$\int_0^\infty \frac{x^n\,dx}{e^x-1} = \begin{cases} 2.31, & n=1/2 \\ \pi^2/6, & n=1 \\ 2.405, & n=2 \\ \pi^4/15, & n=3 \\ 24.9, & n=4 \end{cases} \qquad \int_0^\alpha \frac{x^3\,dx}{e^x-1} = \begin{cases} 0.225, & \alpha=1 \\ 1.18, & \alpha=2 \\ 2.56, & \alpha=3 \\ 4.91, & \alpha=5 \\ 6.43, & \alpha=10 \end{cases}$$

$$\int_0^\infty x^n e^{-x}\,dx = \begin{cases} n!, & n>0, \text{ an integer} \\ \sqrt{\pi}/2, & n=1/2 \end{cases} \qquad \int_0^1 e^{-x^2}\,dx \approx 0.843$$

$$\int_0^\infty x^n e^{-x^2}\,dx = \begin{cases} \sqrt{\pi}/2, & n=0 \\ \frac{1}{2}\left[\left(\frac{n-1}{2}\right)!\right] & n \text{ is an odd integer} \\ \frac{1\cdot 3\cdot 5\ldots(n-1)}{2^{n/2}}\,\frac{\sqrt{\pi}}{2}, & n \text{ is an even integer} \end{cases}$$

Differentiation of an integral with respect to parameter:

$$\frac{\partial}{\partial\alpha}\int_{x_1(\alpha)}^{x_2(\alpha)} f(x,\alpha)\,dx = \int_{x_1}^{x_2}\frac{\partial f}{\partial\alpha}\,dx + f(x_2)\frac{\partial x_2}{\partial\alpha} - f(x_1)\frac{\partial x_1}{\partial\alpha}.$$

The values of the error integral $J(\alpha)=\sqrt{\frac{2}{\pi}}\int_0^{\alpha} e^{-x^2/2}\,dx$

α	$J(\alpha)$	α	$J(\alpha)$	α	$J(\alpha)$
0.1	0.0797	0.9	0.6319	1.7	0.9109
0.2	0.1518	1.0	0.6827	1.8	0.9281
0.3	0.2358	1.1	0.7287	1.9	0.9426
0.4	0.3108	1.2	0.7699	2.0	0.9545
0.5	0.3829	1.3	0.8064	2.25	0.9756
0.6	0.4515	1.4	0.8385	2.50	0.9876
0.7	0.5161	1.5	0.8664	2.75	0.9940
0.8	0.5763	1.6	0.8904	3.00	0.9973

15. Radioactivity and dose units

Quantity	Name and symbol		Conversion factors
	Off-system	SI	
Activity, A	Curie (Ci)	Becquerel (Bq) 1 Bq = 1 dis/s	1 Ci = $3.700\cdot10^{10}$ Bq
Exposure dose, D_{ex}	Roentgen (R)	Coulomb per kilogram (C/kg)	1 R = 258 μC/kg
Absorbed dose, D	Rad (rad)	Gray (Gy) 1 Gy = 1 J/kg	1 rad = $\begin{cases}100\ \text{erg/g}\\10^{-2}\ \text{Gy}\end{cases}$
Equivalent dose, D_{eq}	rem (rem)	Sivert (Sv) 1 Sv = 1 Gy/Q.F.	1 rem = $\begin{cases}1\ \text{rad/Q.F.}\\10^{-2}\ \text{Sv}\end{cases}$

* Here Q.F. denotes the quality factor.

The conversion of doses:

$$D_{eq}\,(\text{rem}) = \text{Q.F.}\cdot D\,(\text{rad})$$

Maximum permissible doses corresponding to 100 mrem a week.

Radiation	Radiation energy	Dose rate for 36-hour working week	Q.F.
X-ray and γ-radiation	< 3 MeV	0.78 μR/s	1
β-particles and electrons	< 10 MeV	20 particles/(cm²·s)	1
Neutrons, thermal	0.025 eV	750 neutrons/(cm²·s)	3
Neutrons, fast	1-10 MeV	20 neutrons/(cm²·s)	10

16. Conversion factors for some measurement units

1 Å = 10^{-8} cm	1 eV = $1.6\cdot10^{-19}$ J	1 Ω = $\frac{1}{9\cdot10^{11}}$ CGSE uni
1 b = 10^{-24} cm²	1 C = $3\cdot10^{9}$ CGSE unit	
1 year = $3\cdot11\cdot10^{7}$ s	1 A = $3\cdot10^{9}$ CGSE unit	1 A/m = $4\pi\cdot10^{-3}$ G
1 N = 10^{5} dyn	1 V = 1/300 CGSE unit	1 Wb = 10^{8} Mx
1 J = 10^{7} erg	1 F = $9\cdot10^{11}$ cm	1 H = 10^{9} cm

17. Fundamental Physical Constants

Velocity of light	$c = 2.998\cdot10^{8}$ m/s
Gravitational constants	$\gamma = 6.67\cdot10^{-8}$ cm³/(g·s²)
Avogadro constant	$N_A = 6.02\cdot10^{23}$ mol⁻¹
Loschmidt's number	$n_0 = 2.69\cdot10^{19}$ cm⁻³
Universal gas constant	$R = 8.314$ J/(K·mol)
Gas volume at S.T.P.	$V_0 = 22.42\cdot10^{-3}$ m³
Boltzmann constant	$k = 1.38\cdot10^{-16}$ erg/K
Planck constant	$\hbar = 1.054\cdot10^{-27}$ erg·s
Elementary charge	$e = \begin{cases}1.6\cdot10^{-19}\ \text{C}\\4.80\cdot10^{-10}\ \text{CGSE unit}\end{cases}$
Specific charge of electron	$e/m = \begin{cases}1.76\cdot10^{11}\ \text{C/kg}\\e/m = 5.273\cdot10^{17}\ \text{CGSE unit}\end{cases}$
Faraday constant	$F = 96487$ C/mol
Stefan-Boltzmann's constant	$\sigma = 5.67\cdot10^{-8}$ W·m⁻²·K⁻⁴
Wien's displacement constant	$b = \lambda_{pr}T = 2.90\cdot10^{-3}$ m·K
Rydberg constant	$R_\infty = \frac{m_e e^4}{4\pi c\hbar^3} = 1.0973731\cdot10^{5}$ cm⁻¹; $R^*_\infty = 2\pi c R_\infty = 2.07\cdot10^{16}$ s⁻¹
First Bohr radius	$r_1 = \frac{\hbar^2}{m_e e^2} = 0.529\cdot10^{-8}$ cm
Electron's binding energy in a hydrogen atom	$E = \frac{m_e e^4}{2\hbar^2} = 13.59$ eV
Compton wavelength	$\Lambda = \frac{h}{mc} = \begin{cases}3.86\cdot10^{-11}\ \text{cm}\ (e)\\2.10\cdot10^{-14}\ \text{cm}\ (p)\end{cases}$
Electronic radius	$r_e = \frac{e^2}{m_e c^2} = 2.82\cdot10^{-13}$ cm
Thomson scattering cross-section	$\sigma_T = \frac{8\pi}{3} r_e^2 = 6.65\cdot10^{-25}$ cm²
Fine structure constant	$\alpha = e^2/\hbar c = 1/137$
Bohr magneton	$\mu_B = \frac{e\hbar}{2m_e c} = 0.927\cdot10^{-20}$ erg/Oe
Nuclear magneton	$\mu_N = \frac{e\hbar}{2m_p c} = 5.05\cdot10^{-24}$ A·m²
Atomic mass unit, a.m.u. (1/12 of ¹²C atom mass)	1 a.m.u. = $\begin{cases}1.660\cdot10^{-24}\ \text{g}\\931.44\ \text{MeV}\end{cases}$

Particle	a.m.u.	Mass, e	MeV	Magnetic moment	Gyromagnetic ratio
Electron	$5.486 \cdot 10^{-4}$	$0.9108 \cdot 10^{-27}$	0.511	$1.00116\ \mu_B$	2.0022
Proton	1.007276	$1.6724 \cdot 10^{-24}$	938.23	$2.7928\ \mu_N$	5.5855
Neutron	1.008665	$1.6748 \cdot 10^{-24}$	939.53	$-1.913\ \mu_N$	−3.8263
Deuteron	2.013553	$3.3385 \cdot 10^{-24}$	1875.5	$0.8574\ \mu_N$	0.8574
α-particle	4.001506	$6.6444 \cdot 10^{-24}$	3726.2	0	—

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